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It Makes a Village: Residential Relocation after Charter School Admission

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Although numerous studies investigate how student achievement is impacted by educational vouchers and charter schools, there appears to be no research on how these programs impact the surrounding environment. This study examines residential relocation of families whose children attend a charter school. We develop a conceptual model that predicts where relocating families are likely to move, given *ex ante* distance and direction to the school. The model is parameterized using data from student mailing address changes. We find that families are almost twice as likely to relocate toward the school as would be expected if the school did not exert any attraction. Moreover, although families are not required to live near the school, the child's school exerts a significantly stronger attraction than parent workplaces. This result may have important implications for mitigating urban sprawl, fostering urban renewal and promoting sustainable real estate development.

Real estate professionals have long known that housing prices are higher in areas with good public schools. An indicator of the importance of schools to homebuyers is that while crime rates and transportation options are also understood to affect housing prices, only schools are a searchable field on Multiple Listing Service databases. It is no surprise that academic studies conducted in many countries show parents are prepared to pay substantially more for homes in better-performing school districts.

Most kindergarten through 12th grade education in the United States is provided by public schools where attendance is linked to home location based on district boundaries, or catchment areas. Students who live in a particular catchment area are assigned to a specific school. Families can exercise choice over which schools their children attend by buying a home in the catchment areas of their preferred schools. It is evident that people pay more for homes in catchment areas where school quality, as measured by student outcomes, is higher.

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However, it is not clear that parents are willing to pay more for higher-quality schools as measured by inputs—that is, the amount spent per pupil.

Occasionally, districts alter the boundaries of catchment areas to fill available school spaces or reduce overcrowding, eliminate spare capacity or promote other goals such as equalizing perceived school quality or promoting racial or ethnic integration. As one would expect, research suggests that homes reassigned to lower-performing catchment areas decline in value. Subsequent uncertainty about future student assignment policy also lowers housing prices. Cheshire and Sheppard (2004) note that buyers appear to be less eager to purchase a home in a high-quality school catchment zone if history suggests changing policies may result in the home being assigned to another school.

An unfortunate effect of the current assignment-by-catchment-area model can be seen in the pattern of development in most major urban areas. To choose a better school, parents must choose a better home—or at least a better catchment area. This home-to-school linkage has led to middle-income families migrating from catchment areas of underperforming urban schools to areas with higher-performing suburban schools. Baum-Snow (2007) observes that "between 1950 and 1990, the aggregate population of central cities in the United States declined by 17 percent despite population growth of 72 percent in metropolitan areas as a whole." Numerous factors have been cited as drivers of this long-term trend, but the most often cited culprit has been a middle-class migration from poorer inner-city schools to preferred suburban schools.

Growing discontent with the quality of assigned public schools—particularly inner-city schools—has led to some growth in attendance at private schools and in home schooling. Moreover, the U.S. Department of Education reports that between 1993 and 2007, the number of students enrolled in public school choice programs increased more than 57%. These programs allow students to enroll in schools other than those to which they are otherwise assigned.

This article presents a case study of residential relocation patterns for families whose children attend a charter school in the Raleigh-Durham area. There are at least two important characteristics of this school that make it a good laboratory for initiating an investigation of how families are likely to relocate when their children attend such schools. First, the rules for attending the school are very liberal. No students are assigned to the school, and there is no attendance zone that restricts admission other than the state's borders. There is also no tuition. Students are admitted without regard to academic ability, income or race. Thus, student attendance is by parental choice in a relatively unrestrictive and pure sense. Second, because the school has survived for more than a decade, a sufficient history exists to track the relocation patterns of numerous families.

We examine the residential relocation decision of families whose children attend this school using student mailing address changes. The data suggest that families attending the school are almost twice as likely to relocate toward the school as would be expected if the school did not exert an attraction. Furthermore, although there is no catchment zone, the child's school exerts a significantly stronger attraction than parents' work locations. These results provide important insights on the potential role of charter schools and other non-catchment-area based school choice plans in influencing residential growth and relocation patterns.

Before describing the details of the current research, we summarize two streams of literature that have developed around school assignment policy: real estate valuation and racial sorting. Our findings are clearly relevant to the first (albeit indirectly), and they may be important to the second. However, our methodology is markedly different from those used previously. To understand the need for this divergence, one must first understand the important questions these literatures have sought to address.

Literature Review

Many studies have examined the impact of school quality on home prices, but only a few have focused specifically on the effects of school choice on home values. For example, in 1997, Oslo, Norway, scrapped its zone-based school assignment system in favor of choice-based open enrollment. Before the change, a catchment area with pupil test scores significantly above average registered home prices 7–10% higher than average. After the policy change, about half the price premium for these homes disappeared (Machin and Salvanes 2010).

In 1990, Minnesota implemented a statewide system of interdistrict open enrollment that allowed students to attend schools outside their district. Students in poor-performing districts were able to attend schools in better-performing districts. Eight years later, home prices had appreciated more in districts where more students transferred out to preferred districts (Reback 2005). The explanation offered for the home value changes in both Minnesota and Oslo is that, after the policy change, families could access the premium-quality schools without paying for premium-priced homes in a preferred catchment.

Apparently, the impact of charter schools which operate without catchment areas on home values has not been studied, but the influence of publicly funded private schools on home prices, in parallel with catchment-based public schools, has been studied by Fack and Grenet (2010). They use data from Paris, France, which has a catchment-based school assignment system as well as a well-developed voucher-based private school system that operates without catchments. One-third of all middle schools and high schools in France are private. However, the distribution of private schools is not uniform, with some areas having few private schools and others having several. In areas with few private school options, homes are worth more in the catchment with desirable public schools. However, where many voucher-funded schools exist, public school assignment boundaries apparently have no impact on home prices. To the extent that charter schools are similar to publicly funded private schools, one might expect that the proliferation of such schools would also smooth home values across school district boundaries.

Of course, school assignment policies are viewed as critically important for social issues beyond local real estate valuation. In particular, legally enforced racial segregation and subsequent efforts to end this practice have occupied center-stage in the school assignment process for the last 60 years. A substantial academic literature has developed to connect school assignment policy with racial geographic sorting in urban areas. This important literature is too voluminous to cover in this article, but in order to fully understand how this charter school case study departs from the racial-sorting literature, it is appropriate to consider a few of the important ideas in the racial sorting literature.

Tiebout (1956) motivated the idea that a sorting equilibrium can arise as households "vote with their feet" by choosing residential locations with the most desired package of local public goods (*e.g.*, public schools). Numerous papers build on this paradigm. Recently, Baum-Snow and Lutz (2011) examine changes in racial sorting after judicial desegregation orders. They find white populations in southern central city school districts declined and black populations in non-southern central cities grew.

Weinstein (2012) investigates neighborhood racial sorting in response to changes in school assignments that resulted from the termination of court-ordered racial desegregation in the Charlotte-Mecklenburg Public School District. In 2001, these schools were ordered to dismantle a long-standing race-based assignment plan. Assignment zones were redrawn to give each student a guaranteed seat at a school close to her or his residence. Approximately half of families were reassigned to different schools, causing large changes in school racial compositions across the district. Over five years reassignments produced a new sorting of families. An increase in the fraction of black students in an elementary school produced a statistically significant increase in the percent of black families in the surrounding neighborhood.

The Contributions of This Paper

To examine relocation patterns of families who attend this charter school, we use methods originally developed to describe housing location choice relative to employee workplaces. Employers do not require employees to live in a catchment area as a condition of employment, and employees are free to change residences without jeopardizing their employment. Families whose children attend a non-catchment-based charter school are free to change residences also. They can live wherever they choose. This differs markedly from the relocation decision faced by families in the catchment-based systems studied in most prior literatures.

Previous research on housing location choices, as they relate to adult employment location, lends support to the hypothesis that families may choose to live closer to schools that their children attend. For example, Clark, Huang and Withers (2003) observe that people tend to relocate closer to their work locations when they move. Two-worker families consider the commutes of both parties when choosing to relocate. Interestingly, two-earner households are more likely to move closer to the wife's workplace than the husband's. Clark, Huang and Withers (2003) suggest this pattern may be attributable to females' greater need to balance the dual role of mother and worker. A similar logic would suggest that home-to-school commutes may also be an important relocation driver. We also know that people who live farther from work are more likely to relocate closer (Brown 1975). This suggests families who live far from school also may be more likely to move toward the school.

This study differs from previous investigations in at least three additional respects. First, while many previous studies reference "school choice plans" or a similar reference to "choice," the nature of the choices exercised by families in this study are very different from those mentioned in other studies. For example, Weinstein notes that in his Charlotte-Mecklenburg study "a district-wide ***public school choice plan*** was approved ... with school ***assignment zones*** dramatically redrawn to give each student a guaranteed seat at a school close to her residence, typically the closest (students could gain admission to other schools in the district through a lottery process) [emphasis added]." Notice that while the plan is referred to as a "choice" plan, in fact students are initially assigned to a neighborhood school. The school district's website notes that the district has discretion as to whether to approve a transfer request, and only students who attend a failing school are assured that they can transfer—after the school has been failing for three years. The Charlotte-Mecklenburg schools are more accurately described as adhering to a "neighborhood plan" than a "choice plan." In Charlotte, the only way to be guaranteed a particular school is to move into the assigned catchment area, *ex ante*. In contrast, the students who attend the charter school studied here are not assigned to the school. We are not familiar with any study that documents the magnitude of school attraction (even indirectly) in the complete absence of a school assignment criterion.

A second difference between this study and previous ones is that extant studies look at neighborhood composition (or home price levels) before and after changes in assignment policies. Then they infer that families have voted with their feet in a Tiebout sorting. While this is a reasonable inference, it seems likely that researchers would prefer to track individual families, if only the data were available. After all, biologists have used tracking tags to study animal migration for over 200 years. Unlike in prior studies, we observe family migration directly. This has the advantage of offering a much more statistically powerful test of the school's attraction level.

Third, extant studies generally focus on comparisons of school district assignment zones or census tracts with well-defined boundaries. This is a natural consequence of collected data inputs being pre-aggregated using these geographic boundaries. Moreover, because the vast majority of families in an assignment zone will probably send their children to the assigned school, it is reasonable to infer that changes in the local school will result in observable changes in the neighborhood. However, the students who attend the charter school studied here are scattered across numerous census tracts where they make up a very small fraction of the total school-age population. There are approximately 250 traditional public schools in the MSA, and the fraction of students attending this charter school is less than 1% of the total. Moreover, between 2000 and 2009, the Raleigh-Durham area was the fastest-growing large MSA in the country. Considering both of these factors, we clearly cannot attribute changes in specific neighborhoods or census tracts to migration by charter school families. On the other hand, the granularity of the data allows us to make inferences that would be impossible to coax from aggregated census data.

Data, Hypothesis and Descriptive Interpretations

Our data are provided by a charter school in the Raleigh-Durham, North Carolina, area. By state law, admission to charter schools is conducted by lottery. Because North Carolina capped the number of state charter schools at 100 during the period studied in this article, it was not uncommon for the demand for charter schools to exceed the available seats. This is the case for the school in question. Application to the school entitled the applicant to participate in the lottery, but there was no guarantee that the student would be admitted.

Preference is granted to applicants who have a sibling already enrolled at the school. If there are more seats available in the class than the number of sibling students who apply, then all of the sibling students are admitted and a lottery is held for the remaining seats. If there are fewer seats than sibling students, then the lottery is held for the sibling students only and no outside applicants are admitted.

Each applicant must complete an application containing, among other data, the mailing address of the applicant's family. Once the student is admitted, this application is retained in the student's permanent record. Using the permanent record files, we have assembled the initial mailing addresses for all students attending the school. The school continuously updates student mailing addresses for general purposes, and by comparing the address on each student's application to his/her subsequent mailing list address, we are able to determine which students have moved since being admitted to the school. In addition, by matching the last names and mailing addresses for students, we are able to determine which students are members of a single family. Moreover, we can identify which student in any family was the first admitted sibling. The data on these students are of interest in this research.

Because siblings are granted priority admission, a family who has one student admitted to the school can expect that siblings will gain admission in a later year. This may be important for families with multiple children, even if younger children are not yet of school age. Enrolling a child in the school creates a pathway for enrolling all other school-aged siblings once they are ready to attend the school. In other words, the family secures the right for each child to attend the school once the first child is admitted. Thus, admission of the first child to the school confers a valuable right that may impact the family's residential location choice.

With this in mind, admission of the family's first child would appear to be a triggering event most likely to alter a family's optimal residential location choice. Therefore, for each family, we identify both the family residence location prior to the first admission to the school and the subsequent mailing address as of January 2009.

The data we have collected reveal 662 families had at least one student attending the school. The application mailing addresses for the first-admitted child in four instances cannot be ascribed to a true place of residence because a Post Office Box is given. The other addresses described are presumed to be true residential addresses.

We geocoded addresses using the ArcGIS 9.2 "Geocode Addresses Tool," using street centerline data for address ranges. Specifically, we used the North Carolina Department of Transportation's 2007 Integrated Statewide Road Network database.[1] We also geocoded the school's location. The result of the geocoding is a shape file of points, with each point representing the address location (longitude and latitude) for a single record in the data table. Any addresses that did not properly geocode had points created based upon manual searches using Mapquest and Google Earth. The attribute data table of each point contained the record ID, student address and geographic latitude/longitude coordinates.

Using the January 2009 mailing addresses of all students, we identified families which moved after the family's first child was admitted to the school. For these families, we repeated the process and geocoded the new addresses.

Finally, we use Hawth's Tools, an ArcGIS 9.2 extension, to calculate the linear distance from each address to the school. We also calculate bearing and turn angle metrics, which are discussed later in the article.[2] Hawth's Tools are designed specifically for ecology-related analyses such as this. We also access Google Maps to calculate the nonlinear road-commuting distance and the estimated commuting time from each address to the school.

We expect that family relocation decisions are likely to be determined by commuting time and distance rather than linear distance. However, the geographic model that we construct later in the article uses trigonometric functions that presume linear movements. In order to obtain some comfort that linear distance provides a reasonable proxy for families' more likely decision variables of nonlinear road-commuting distance and commuting time, we have calculated the correlation between each of these three measures. These correlations are presented in Table 1.

Notice that all of these variables are very highly correlated. In particular, the drive distance is highly correlated with the linear distance. The very high level of correlation appears to be related to the fact that the school location is common to each commute. Given that the last leg of the commute follows the same few paths for all commuters, the linear distance maps very closely to the drive distance.

[1] See http://www.lib.ncsu.edu/gis/ncdot.html and http://www.ncdot.org/it/gis/Data-Distribution/DOTData/default.html.

[2] The Hawth's Tool module used is "Calculate Movement Paramenters." Documentation can be found at the following: http://www.spatialecology.com /htools/moveparamssimple.php.

Table 1 ■ Correlation of linear distance, drive distance and drive time for accepted applicants (first in family).

	Linear Distance	Drive Distance	Drive Time
Linear Distance	1.0000		
Drive Distance	0.9901	1.0000	
Drive Time	0.9554	0.9638	1.0000

Note: This table presents the correlation coefficients between linear distance, drive distance and drive time between home and school for the home address shown on the application of the first child accepted to the school from each family.

Table 2 ■ Original linear distance in miles from home to school for accepted applicants (first in family).

Summary Statistics	Original Linear Distance
Mean	5.7788
S.D.	4.9696
Q1	0.2612
Q5	0.9200
Q25	2.5725
Median	4.5993
Q75	7.2459
Q95	14.3403
Q99	24.8256
Min.	0.1030
Max.	56.5721
N	658

Note: This table shows the summary statistics of the original linear distance (in miles) from home to school for the first accepted applicant in a family.

We are able to identify a residential address at the time of application for 658 of the 662 families admitted to the school. Table 2 provides descriptive statistics concerning linear distance, in miles, from each family's original address to the school's location. Admitted applicants, on average, lived 5.77 miles from the school, and the median distance from the school was 4.59 miles. Less than 1% of the admitted students lived within a quarter of a mile from the school. Approximately 95% lived within 15 miles.

Which Families Moved?

Comparing the application addresses to the subsequent mailing addresses, we find that 176 of the families changed addresses after they were admitted to the school; the remainder did not change mailing addresses. We assume that a change of mailing address constitutes a change of residence, but this need not be the case. For instance, a family might use a business address or a post office address for receiving personal correspondence. In that case, the change will be misinterpreted as a change of residence. School administrators also point out that a small number of students have divorced parents with joint custody. We cannot systematically identify these students, and we have no means of determining what impact these family arrangements might have on the data. In any event, noise that is introduced by these factors should bias against finding school commute to be an important factor in relocation decisions.

Assuming that families make relocation decisions on the basis of commute time, we might expect that families who live a long distance from the school would be more likely to relocate. To test this hypothesis, we specify the following probit model:

$$P(Moved_i = 1|x_i) = \Phi(x_i'\beta) = \Phi(\beta_0 + \beta_1 Distance_i + \beta_2 Years_i). \tag{1}$$

The marginal effects are

$$\frac{\partial}{\partial t_j} P(Moved_i = 1|x_i = t) = \frac{\partial}{\partial t_j}\Phi(t'\beta) = \phi(t'\beta)\beta_j, \tag{2}$$

where $\Phi(\cdot)$ is the cumulative standard normal distribution function and $\varphi(\cdot)$ is standard normal density. ($Moved_i = 1$) indicates that a family i moved after admission, and ($Moved_i = 0$) indicates that the family did not move. $Distance_i$ is the pre-move linear distance from the school. We expect that families with longer home-to-school commutes are more likely to move in order to reduce the commute time and distance. If this is true, β_1 will be positive. $Years_i$ is the number of years that the student has been enrolled at the school. Students who have attended the school for a longer period of time are more likely to have moved, without regard to motivation. Thus, the coefficient β_2 should be positive.

Table 3 presents the results of the hypothesized model with variations. Below the partial effect of each independent variable, z-values are reported in parentheses. Elasticities with respect to each independent variable are also calculated with z-statistics shown underneath. Both the partial effects and elasticities are measured at the mean value.

In the first specification, the only independent variable considered is *Distance*. While the sign on the partial effect is positive and statistically significant, the magnitude of the partial is quite small. The second specification incorporates

Table 3 ■ Probit regression predicting the probability of moving.

Dep. Var. = Moved (1/0)	Model (1) Marginal Effect [dy/dx]	Model (1) Elasticity [$d(\ln y)/d(\ln x)$]	Model (2) Marginal Effect [dy/dx]	Model (2) Elasticity [$d(\ln y)/d(\ln x)$]	Model (3) Marginal Effect [dy/dx]	Model (3) Elasticity [$d(\ln y)/d(\ln x)$]
Distance	0.0050**	0.1772**	0.0044**	0.1685**	0.0044**	0.1683**
	(2.38)	(2.36)	(2.03)	(2.02)	(2.02)	(2.01)
Years			0.0529***	0.8711***		
			(8.36)	(7.84)		
Admitted Grade					−0.0523***	−0.5294***
					(−7.05)	(−6.73)
Current Grade					0.0532***	1.4150***
					(8.05)	(7.58)
Log Pseudolikelihood	−375.9008		−341.4323		341.4213	
Pseudo R^2	0.0072		0.0982		0.0982	
Predicted Prob.	0.2682		0.2465		0.2464	
N	650		650		650	

Note: This table reports marginal effects and elasticity from probit regressions predicting the probability of moving. The dependent variable (*Moved*) is a binary variable that equals one if the family moves and zero otherwise. The independent variables include original commute distance (*Distance*), years in school (*Years*), original grade (*Admitted Grade*) and current grade (*Current Grade*). Each specification builds on the previous one. The partial derivatives and elasticities of the dependent variable with respect to the independent variables are evaluated at the mean for each independent variable. Robust z-statistics are reported in parentheses. ***, ** and * denote significance at the 1%, 5% and 10% levels, respectively, in a two-tailed test.

Years. The average number of years attended by the students in the sample is 4.06.

As expected, the number of years that the student has attended the charter school is highly correlated with the probability of a move. Obviously, the more time that has elapsed between the two observation points, the more likely it is that a move will have occurred. The distance that the family originally commuted to school is also positively correlated with the move probability.

The third specification decomposes the time that the student has been enrolled at the school into the student's *Current Grade* and the student's *Admitted Grade*. The difference between *Current Grade* and *Admitted Grade* is the value of *Years* in the second specification. The negative partial effect on *Admitted Grade* indicates that the younger the student was when he or she was admitted, the more likely the family was to relocate. This is consistent with families choosing to relocate when they expect that their children will be enrolled at the school for a long period of time. For families that expect to be affiliated with the school for many years, the relative benefits of moving increase. The positive coefficient on *Current Grade* indicates that older students are more likely to have moved since enrolling.

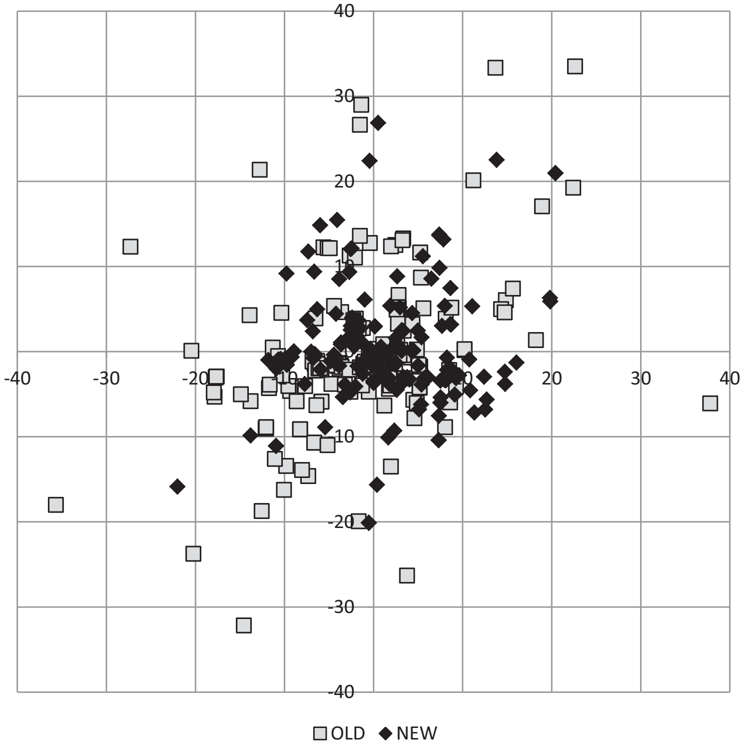
Did the Movers Move Closer? Some Nonparametric Tests

We now focus our attention on the 176 families that moved after the first child was enrolled in the school. To help the reader visualize the data we present Figure 1, which depicts *ex ante* (OLD) and *ex post* (NEW) residences of relocating families relative to the school's location at the center of the figure. The grid is for an area covering 6,400 km^2 (2,471 mi^2). The figure does not include four observations that would lie outside the graph borders (1 "NEW" and 3 "OLD" observations). Notice that the black NEW residences are more tightly clustered than the lighter shaded OLD residences.

More rigorously, let d_O be the distance between the family's original home and the school,[3] and let d_N be the distance between the family's new home and the school. Thus, we calculated the direction of the move relative to the school as $(d_O - d_N)$. If $(d_O - d_N) > 0$, the family moved closer to the school. In fact, the

[3] After five years, the school opened a second location approximately 2,500 yards away from the original building. The upper grades transferred to this building. Our analysis considers the original building as the school's geographic location. Of the 176 moving students at least 154 attended classes at this original location. If neither these later-admitted students nor their younger siblings ever attended classes at the original location, this should bias against finding an attraction to the original location. However, because these locations are near one another, the bias is probably small.

Figure 1 ■ Heat map of *ex ante* (gray-shaded) and *ex post* (black-shaded) residence locations of movers.



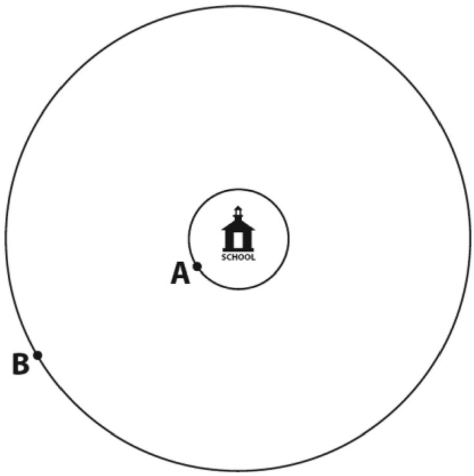
Note: This figure plots a heat map of *ex ante* and *ex post* residence locations of families who moved. The gray-shaded points show *ex ante* locations, and the black-shaded points show *ex post* locations.

average value of $(d_O - d_N)$ was 1.48 miles. The one-tailed t-test probability of obtaining this mean, assuming that the null hypothesis (H_O: mean = 0) is true, would be $p = 0.0045$.

Applying the sign test, 99 of the 176 movers moved in the direction of the school, and 77 moved away from the school. If the true underlying $\Pr(d_O - d_N > 0) = 0.5$, the chance of observing 99 or more positive values of $(d_O - d_N)$ is $p = 0.0566$. Similarly, the Wilcoxon sign-rank test rejects the null (H_O: mean = 0) with a one-tailed p-value of 0.023.

The above tests make the implicit assumption that conditional upon a family moving, we would expect $\Pr(d_O - d_N > 0) = 0.5$ if the family is not attracted

Figure 2 ■ Distance to school and move probability for two individuals.



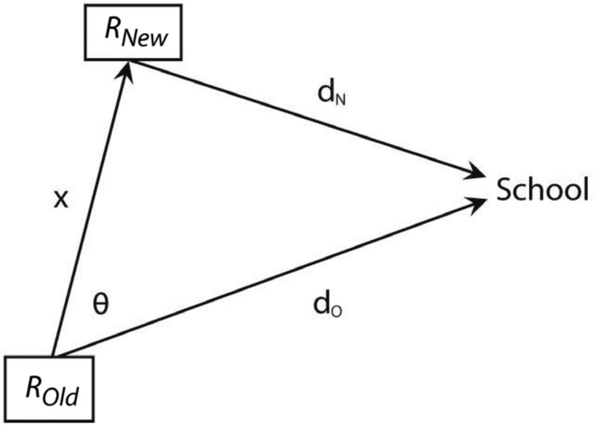
Note: This figure demonstrates the need to consider *ex ante* distance when considering the probability that families will move closer to school. Family B has a greater probability of moving closer than its original location because the circle on which B rests includes a greater area. The area available for family A to move closer to the school is much smaller.

to the school. However, this assumption is inappropriate. In fact, if a family is indifferent to the distance from the school, the mean of $(d_O - d_N)$ should be negative! To illustrate this point, consider two childless individuals in Figure 2, neither of whom has any interest in, nor affiliation with, the school shown at the middle of the figure.

If Individual A moves, she or he is highly unlikely to move closer to the school because the area inside the small circle represents a small fraction of the total potential move locations. Individual B has a higher probability of moving closer to the school simply because there are more addresses inside the larger circle that satisfy the condition $d_N < d_O$. Even for Individual B, the probability that $(d_O - d_N > 0)$ is less than half. $\Pr(d_O - d_N > 0) = 0.5$ is only asymptotically true. For example, if an uninterested party lives 1,000 miles due west of the school, then approximately half of the possible relocation moves would take him or her slightly east of the starting location, and approximately half the moves would take him or her west.

We will later establish an approximate benchmark for the move probabilities that a disinterested party actually faces. For the moment, it is sufficient to recognize that the farther a relocating family originally lives from the

Figure 3 ■ A vector structure of the school–residence relationships.



Note: This figure plots a vector structure of the school–residence relationships. R_{Old} is the old residence of the student prior to enrolling in the school. d_O is the distance from R_{Old} to the school. R_{New} is the new residence of the student, and d_N is the new commuting distance to the school. The distance moved from R_{Old} to R_{New} is designated as vector *X*. θ is the angle formed by moving from vector d_O to vector *X*. If a student moved directly toward the school, θ would be 0.

school, the more likely it will relocate closer to the school, because the area ($A = \pi d_0^2$) of condition-satisfying moves that are closer to the school grows geometrically with d_0. With this in mind, we repeat the Wilcoxon sign-rank test while weighting each observation by πd_0^2. Testing the null hypothesis H_O: $(d_O - d_N) \times (\pi d_0^2) = 0$, we reject the null with a one-tailed *p*-value of 0.0001.

A Model of School Attraction

The foregoing frequency distributions and probit analyses are helpful in describing the relationship between school location and relocation choice. However, if we wish to fully understand the magnitude of the school's attraction in residential relocation decisions, a two-dimensional spatial model of the relocation decision is useful. Ideally, a model of school attraction will (1) provide testable hypotheses concerning the probability of moving closer to or further from the school and (2) provide testable hypotheses concerning the effect of distance on school site attraction.

In order to simplify exposition of the model that will follow, let us first consider a simple conceptualization of one family's residential relocation. Figure 3 presents a vector structure of the school–residence relationships.

In the Figure 3 diagram, the student lives at the residence R_{Old} prior to enrolling in the school. The distance the student lives from the school is identified as d_O. After being admitted to the school, the student moves to a new residence, designated as R_{New}. The distance moved from R_{Old} to R_{New} is designated as vector X. After moving to R_{New}, the new commuting distance to the school is designated by the vector d_N. Summarizing the distances involved in this move, the student moved X miles from R_{Old} to R_{New}, and the commute distance to the school changed from d_O to d_N.

In addition to the distances that have been identified, another important aspect of this conceptualization concerns the angle θ. θ is the angle formed by moving from vector d_O to vector X. If a student moved directly toward the school, the value of θ would be 0. For movements in a counter-clockwise direction from the original school bearing, the values of theta are between $-\pi$ and 0 ($-\pi < \theta < 0$). In the Figure 3 example, the value of θ would be approximately $-\pi/4$, corresponding to a 45° angle moving counter-clockwise. Similarly, for movements in a clockwise direction from the original school bearing, the value of theta is between 0 and π ($0 < \theta < \pi$). The importance of π will be seen in the further development of the model.

We are interested in the relationship between distances from the student's residence before and after the move. The conceptualization of this relationship can now be structured as a model with two parameters in which each student's move is described by the vector X, which has both a length and a direction. Thus, the distribution of these moves across the full sample is a joint distribution of directions and lengths for all Xs.

This brings us to a formal model of the relationships conceptualized in Figure 3. Quigley and Weinberg (1977), Clark and Burt (1980) and Clark, Huang and Withers (2003) consider relocations as a function of move distances from workplaces (analogous to this study of moves related to school location). Unlike the preceding studies, which model move distances using an exponential distribution, we adopt the gamma distribution because it can be fitted to our data more successfully.

$$g\,(X;\varphi,\alpha) = \frac{\alpha^{\varphi}}{\Gamma(\varphi)} X^{\varphi-1} e^{-\alpha X}, \quad X > 0 \text{ and } \varphi,\alpha > 0. \tag{3}$$

This γ distribution is parameterized in terms of a shape parameter φ, as well as the rate parameter α. The function $\Gamma(\varphi)$ is defined to satisfy $\Gamma(\varphi) = (\varphi - 1)!$ for all positive integers φ, and to smoothly interpolate the factorial between integers.

A second assumption of our model is that the move directions for students follow a von Mises distribution (Gaile and Burt 1976). The von Mises distribution is also known as the circular normal distribution. Accordingly, it can be viewed as an analogue to the normal distribution that is useful for analyzing two-dimensional data. The parameters of the von Mises distribution are μ and κ, which are analogous to the normal distribution's μ and σ^2. Actually, k is analogous to the inverse of σ^2, $(1/\sigma^2)$.

The assumption that student movements are, on average, in the direction of the school is captured as $\mu = 0$ (an assumption that is subject to subsequent testing). For $\mu = 0$, the density function is defined as

$$v(\Theta) = \frac{1}{2\pi I_0(k)} e^{k\cos(\Theta)}, \quad -\pi < \Theta < \pi, k \geq 0, \tag{4}$$

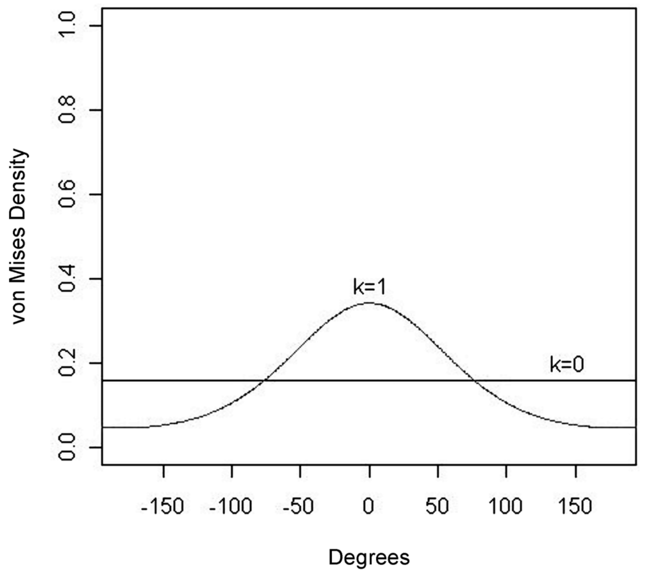
where Θ is the move direction described in Figure 3, measured in radians. I_0 is a modified Bessel function of the first kind and order zero.

Figure 4 clarifies why the von Mises distribution is also described as the circular-normal distribution. Notice that for $k = 1$, a graph of the density function looks very similar to a normal distribution. However, unlike for the normal distribution, the horizontal axis in Figure 4 does not extend from $-\infty$ to ∞. Instead, the axis extends from -180° to $+180^\circ$. Of course, these two values represent the same point on the circle so that the horizontal axis actually wraps around the circle. For larger values of k, the concentration at the origin increases and the standard deviation decreases. For $k = 0$, which also is depicted in the figure, the distribution becomes a circular uniform distribution.

Figure 5 presents a series of rose diagrams that allow the reader to visualize the concentration of movement toward $\mu = 0$ for various values of k. Each rose diagram is generated from a theoretical von Mises distribution with alternative values of the concentration parameter k. For each diagram, moves that occur in common directions are aggregated into various bins. Rose diagrams resemble pie charts, except that each bin (sector) has an equal angle. Rather than alter the central angles to account for different numbers of observations in each sector, we extend each sector from the center of the circle by varying distances to illustrate the number of moves that occur in a particular direction. For $k = 0$ the move directions are uniform, but for $k = 2$ the moves are strongly concentrated toward $\mu = 0$.

In combining move directions and distances, we will assume that the move directions and distances are independent of one another. This assumption aids tractability but biases against finding confirming empirical support if the assumption is invalid. Thus, as noted by Clark, Huang and Withers (2003), "... if

Figure 4 ■ The density function of the von Mises distribution.



Note: This figure depicts the density function of the von Mises (circular–normal) distribution. The horizontal axis extends from $-180°$ to $+180°$. These two are the same point on the circle so that the horizontal axis wraps around the circle. For larger values of k, the concentration at the origin increases and the standard deviation decreases. For $k = 0$, the distribution becomes a circular uniform distribution.

the fit between observed and expected is good, we are confident of the results of the model." Accordingly, the joint probability distribution of movement distance and direction is described by

$$c(X, \theta) = g(X)v(\Theta). \tag{5}$$

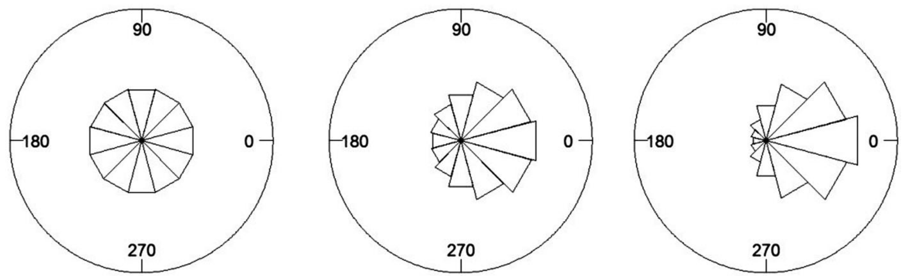
Given these assumptions we develop a model of the likelihood that a student will move into a particular area defined by two distances (X_1 and X_2) and two angles (Θ_1 and Θ_2),

$$P(X_1 < X < X_2, \theta_1 < \theta < \theta_2) = \int_{X_1}^{X_2} \int_{\theta_1}^{\theta_2} c(X, \theta)\, d\theta dX, \tag{6}$$

where

$$c(X, \theta) = g(X)v(\Theta) = \left(\frac{\alpha^{\varphi}}{\Gamma(\varphi)} X^{\varphi-1} e^{-\alpha X}\right)\left(\frac{1}{2\pi I_0(k)} e^{k\cos(\Theta)}\right).$$

Figure 5 ■ Rose diagrams of movement concentration toward $\mu = 0$ for various values of k; $k = 0$, $k = 1$ and $k = 2$.



$k = 0$ $k = 1$ $k = 2$

Note: This figure depicts theoretical rose diagrams for various concentrations of movement toward $\mu = 0$. For each diagram, moves that occur in common directions are aggregated into bins, where each bin includes 30°. For $k = 0$, the move directions are uniformly distributed; for $k = 2$, the moves are strongly concentrated toward $\mu = 0$.

Recall from Figure 3 that students move closer to the school for $d_N < d_O$. Thus, we are specifically interested in the region where $d_N < d_O$. Specifically, we wish to solve for $P(d_N < d_O)$. From the law of cosines

$$(d_N)^2 = (d_O)^2 + (X)^2 - 2\,(d_O X)\cos\theta. \tag{7}$$

Thus,

$$\begin{aligned} P\,(d_N < d_O) &= P((d_N)^2 < (d_O)^2) \\ &= P((d_O)^2 + (X)^2 - 2\,(d_O X)\cos\theta < (d_O)^2) \\ &= P(X < 2\,(d_O)\cos\theta) \\ &= \int\limits_{-\pi/2}^{\pi/2} \int\limits_{0}^{2(d_o)\cos\theta} c\,(X,\theta)\,dXd\theta, \end{aligned} \tag{8}$$

$$P(d_N < d_0) = 2\int\limits_{0}^{\frac{\pi}{2}} \int\limits_{0}^{2d_0 cos\theta} c\,(x,\theta)\,dxd\theta,$$

$$= 2\int\limits_{0}^{\frac{\pi}{2}} \int\limits_{0}^{2d_0 cos\theta} (\frac{\alpha^{\varphi}}{\Gamma(\varphi)} x^{\varphi-1} e^{-\alpha x})(\frac{1}{2\pi I_0\,(k)} e^{kcos\theta})dxd\theta$$

$$= \frac{\alpha^{\varphi}}{\pi I_0\,(k)\,\Gamma(\varphi)} \int\limits_{0}^{\frac{\pi}{2}} e^{kcos\theta} \int\limits_{0}^{2d_0 cos\theta} x^{\varphi-1} e^{-\alpha x} dxd\theta.$$

Let $t = cos\theta, dt = dcos\theta = -sin\theta d\theta$
because $cos^2\theta + sin^2\theta = 1, d\theta = \frac{1}{-sin\theta}dt = -\frac{1}{\sqrt{1-t^2}}dt.$

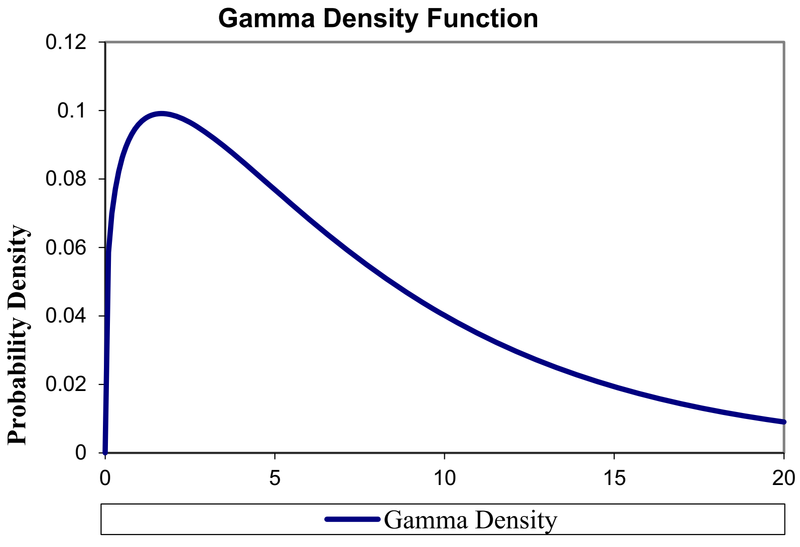
$$P(d_N < d_0) = \frac{\alpha^{\varphi}}{\pi I_0\,(k)\,\Gamma(\varphi)} \int\limits_{0}^{1} \frac{1}{\sqrt{1-t^2}} e^{kt} \int\limits_{0}^{2d_0 t} x^{\varphi-1} e^{-\alpha x} dxdt. \tag{9}$$

Equation (9) can be evaluated for various values of k and d_O using numerical integration. This allows us to establish the relationship between $P(d_N < d_O)$ and d_O.

Tests of School Attraction

Assuming that the observed move distances are drawn from the gamma distribution, we find maximum likelihood estimation (MLE) parameter estimates of $\alpha = 0.166$ and shape parameter $\phi = 1.28$. Figure 6 plots the fitted γ density

Figure 6 ■ The γ Density function of move distance.



Note: This figure plots the fitted γ density function against move distances. Move distances correspond to the lengths of the X vector in Figure 3 and to values for X in Equation (3).

function against the move distance. The mean of the γ distribution, $(\alpha^{-1})(\phi)$, is 7.73 miles.

The move distance corresponds to the length of the X vector in Figure 3, and it is also the value of X in the theoretical distribution from Equation (3). The fitted γ distribution (the solid curve) produces a modal move 1.7 miles from the original location. This seems to be a reasonable finding. Rather than changing homes within the same neighborhood, the γ function suggests that relocaters are more likely to move to a nearby neighborhood than immediately next door.[4]

Turning to our tests of move direction, the direction of each move in the sample can be represented by a vector with direction θ whose length is one (unit vector). The use of unit vectors conforms to the theoretical assumption that move direction and move length are independent. Summing all the sample vectors results in a vector R, where $\theta_R = \tan^{-1} \frac{1/n \sum \sin \theta_i}{1/n \sum \cos \theta_i}$ is a measure of mean

[4]The Kolmogorov–Smirnov Goodness-of-Fit test for the γ function yields a p-value $=$ 0.356, and we fail to reject the hypothesis that the move distances are drawn from this γ distribution.

move direction. The length of vector R also reflects the extent of clustering in the sample's mean direction. This clustering is analogous to the variance in nondirectional data. Standardizing by the number of observations in the sample yields an index $\bar{R}$ with a value between 0 and 1. $\bar{R} = \frac{R}{n} = \frac{\sqrt{(\sum \sin\theta_i)^2 + (\sum \cos\theta_i)^2}}{n}$. $\bar{R}$ is a function of the concentration parameter k by virtue of $\bar{R} = \frac{I_1(\hat{k})}{I_0(\hat{k})}$, where $I_0(k)$ is a modified Bessel function of the first kind and zero order.[5]

For the sample of relocating families in the current study, θ_R equals 0.136 radians, or 7.79°. The clustering index $\bar{R}$ equals 0.522, yielding concentration parameter $k = 1.218$.[6]

Given a move direction bias, we test the assumption that the move directions are biased toward the school. This test assumes the school is the attractor and tests whether or not we can reject that assumption. The 95% confidence interval around the school direction can be written as $0 \pm 1.96/\sqrt{nk\bar{R}} = 0 \pm 1.96/\sqrt{(176)(1.218)(0.522)} = 0 \pm 0.1853$ radians. Because $-0.1853 < \theta_R < 0.1853$, we accept the hypothesis (*i.e.*, cannot reject) that the move directions are concentrated toward the school.

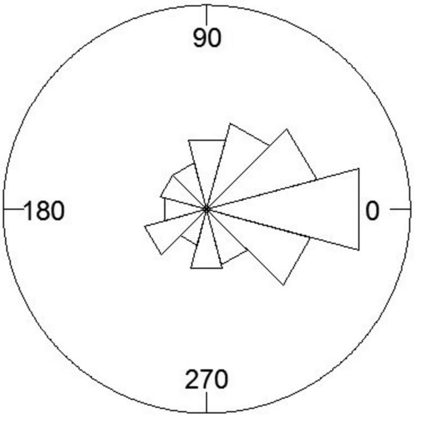
As a point of reference, previous studies by Clark and Burt (1980) and Clark, Huang and Withers (2003) consider workplace attraction. The first paper studied workplace attraction in the Milwaukee metropolitan area. This study found a concentration parameter $k = 0.638$. The second study conducted similar tests to gauge Seattle area work-place attraction and yielded a parameter $k = 0.668$. Notice that the school's attraction ($k = 1.218$) is significantly larger than reported work-place attraction measures.

To help the reader more clearly visualize the move pattern of relocating families, we present a rose diagram in Figure 7. Similar to those presented in Figure 5, this rose diagram aggregates moves that occur in common directions into several bins. However, while the diagrams in Figure 4 are produced from theoretical von Mises distributions, Figure 7 depicts actual empirical observations from the data.

[5]Solving for κ requires numerical approximation. We used the circular statistics package found at http://cran.r-project.org/web/packages/circular/circular.pdf.

[6]For the von Mises distribution parent population when n is large and $k = 0$ the statistic $2n\bar{R}^2$ is approximately χ^2 distributed with two degrees of freedom. In this test the value is 95.88, which is far above any reasonable cutoff value ($p = 0.05$, cutoff value $= 5.99$). Thus, we reject the null hypothesis of $k = 0$ (no bias). See Mardia (1972).

Figure 7 ■ Move directions with 12 bins.



Note: This figure presents the observed density of family moves as a rose diagram. The circle is segmented into twelve 30° bins. The right-most segment is centered on the school so that this bin contains all observations for families moving in a direction within 15° of $\theta = 0$. The length of each wedge is proportional to the square root of the number of observations. The fraction of the observations represented by the largest wedge is 32.4%, and the fraction represented by the smallest wedge shown is 1.70%.

Again, we have segmented the circle into twelve 30° bins. The right-most segment is centered on the school so that this bin contains all observations for families moving in a direction within 15° of $\theta = 0$. In order to make the constructed areas proportional to the frequencies, the length of each wedge is proportional to the square root of the number of observations. In this graph, the fraction of the observations represented by the largest wedge is 32.4%, and the fraction represented by the smallest wedge shown is 1.70%. In this framework, the magnitude of the family relocation bias seems obvious.

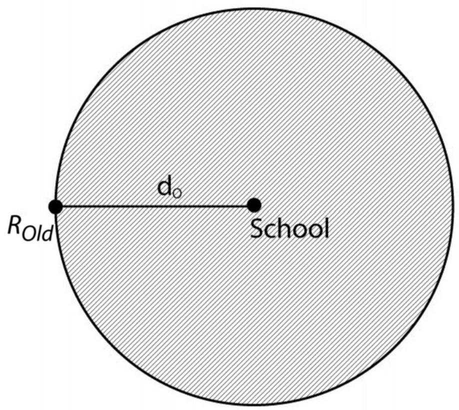
Imputed Probabilities of Toward-School Migration

Conditional upon a family moving, we are interested in assessing the probability that it will move toward the school. Figure 8 provides a simple graphic representation of the question. Given that the family's original home R_{Old} is a distance d_O from the school, we are interested in the probabilities that the family will move to a location that is closer to school—the shaded area in the figure.

Base Case Probabilities (k = 0)

There is some probability that the family would move closer to the school even if the school were not a relocation attractor. This is the probability when $k = 0$.

Figure 8 ■ The conditional probability of a family moving closer to school.



Note: The shaded area in the figure provides a graphic depiction of the space where a new residence will be closer to the school than the original residence at R_{Old}.

To obtain this baseline probability, we numerically solve Equation (9) for various values of d_O, given $k = 0$, $\alpha = 0.166$ and $\phi = 1.28$.

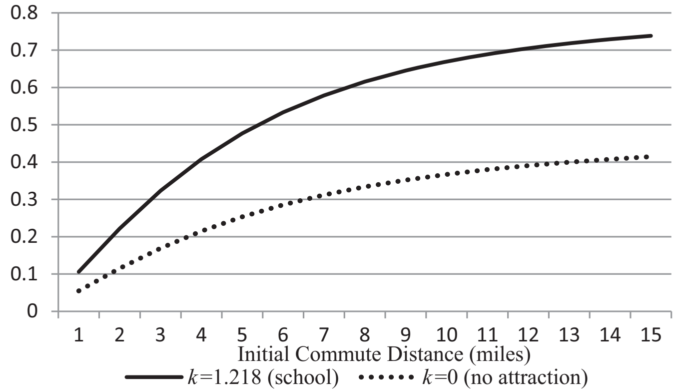
Although each mover must move either closer to the school or farther away from the school, the probability of moving closer is not 50%. For families already living near the school, the probability that they will move closer is small simply because the area inside the circle is small. For $k = 0$, $\alpha = 0.166$ and $\phi = 1.28$, a family living a mile from the school ($d_O = 1$) only has a 0.055 probability of moving closer. However, for $d_O = 10$, the probability of moving closer rises to 0.367. Only in the limit does the probability rise to 50%.

*Imputed Move Probabilities (*k = *1.218)*

Given the observed attraction that the school exerts, we next reassess the probability that a family will move closer by reevaluating Equation (9) for all values of d_O, given $k = 1.218$. The parameters α and ϕ are unchanged. For families already living a mile from the school, the probability of moving closer nearly doubles, rising from 0.055 to 0.106. For $d_O = 10$, the probability rises from 0.367 to 0.669.

Although the increase in probability can be estimated for longer initial commutes, only 8% of the movers had initial commutes of over 15 miles. With

Figure 9 ■ Imputed probabilities [$P(d_N < d_O)$] for $k = 0$ and $k = 1.218$.



Note: This figure graphs the increase in $P(d_N < d_O)$ for $1 \leq d_O \leq 15$ under the baseline assumption ($k = 0$) and under the assumption that $k = 1.218$, as observed from the actual data.

relatively few actual observations, we are not confident that the imputed probabilities would be meaningful for extreme values of d_O.[7]

Figure 9 provides a visual depiction of the increase in $P(d_N < d_O)$ for $1 \leq d_O \leq 15$ under the baseline assumption ($k = 0$) and under the assumption that $k = 1.218$ as observed from the actual data.

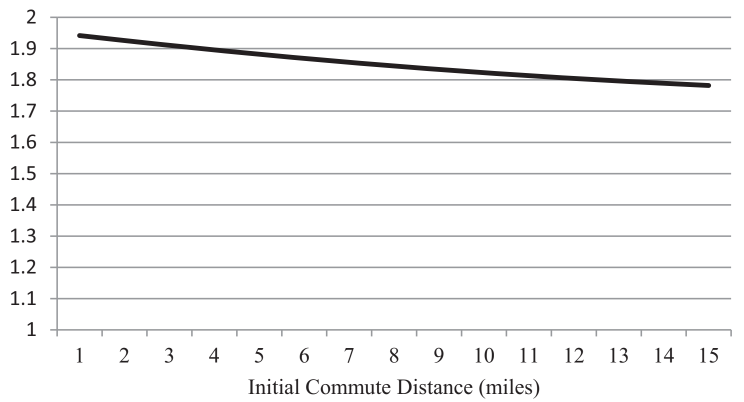
Figure 10 depicts the ratio of $P(d_N < d_O)|_{k\ =1.218}$ to $P(d_N < d_O)|_{k\ =0}$ for $1 \leq d_O \leq 15$. As noted above, for families already living a mile from the school, the probability of moving closer nearly doubles. Even for families living 15 miles away from the school, the probability of moving closer is almost 1.8 times greater.

Further Analysis

Returning to the rose diagram shown in Figure 7, we have also calculated the mean distance moved by the families in each of the 12 bins. The mean move distances are graphically depicted in Figure 11, with the values for the mean and standard deviations shown below the figure.

[7]For example, if we fit the model for $d_O = 100$, $P(d_N < d_O) = 0.815$, but no initial commutes were this long. It seems likely that the parameters of the fitted gamma distribution would be altered if we had observed such an observation.

Figure 10 ■ Increase in the probability of moving closer to the school: $\frac{P(d_N < d_O)|_{k=1.218}}{P(d_N < d_O)|_{k=0}}$.



Note: This figure graphs the ratio of $P\ (d_N < d_O)|_{k\ =\ 1.218}$ to $P(d_N < d_O)|_{k\ =0}$ for $1 \leq d_O \leq 15$ (initial commute distance).

The group names in the legend reflect the geographic bounds on each bin. The bounds are identical to those used to construct Figure 7. The first group is for movers in the direction of the school, which includes moves between $+15^\circ$ and -15° (345°). This group is labeled as "group <15&>345." The bins in the table are listed in a counter-clockwise direction from the school.

Obviously, families moving toward the school move much farther, on average, than those moving away. The mean distance moved toward the school is 11.3 miles, and the mean distance moved directly away from the school is only 1.7 miles. We conclude that the distance moved is affected by the direction, and the assumption that distance and direction are independent does not hold.[8]

Assessing the Direction of Causality

The previous sections of this article document that families who enrolled a child in this charter school tend to subsequently relocate closer to the school at an unexpectedly high rate. The presumption has been that the correlation is confirmation of the school as a relocation magnet. However, it is possible that the direction of causality is actually in the opposite direction. It is possible that families are applying to the school because they already intend to move close

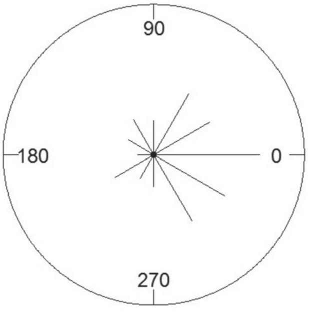
[8]We statistically reject independence of distance and direction based on a small-sample Analysis of Variance (ANOVA) test ($p = 0.0052$).

to the school. It is also possible that the direction of causality flows in both directions: some families apply because they plan to move closer, and other families move because they have been accepted already. In this section we will attempt to assess the direction of causality by (1) considering the timing of moves by families, relative to admission, (2) by surveying moving families to inquire as to their motivations and (3) by considering parental work locations, the most likely alternative attractors for moving families.

Quick Versus Slow Movers

We are fortunate to have survey data available for a subset of moving families. This survey data allow us to identify the year in which 89 of the 176 moving

Figure 11 ■ Move distances for 12 bins.



Mean Move Distances by Bin

Group	Mean	Std. Dev.	*N*
<15&> 345	11.2622	10.21312	54
15-45	6.841459	5.331118	28
45-75	7.466875	8.153274	17
75-105	3.645407	2.113573	11
105-135	4.283388	4.50675	5
135-165	3.08801	3.811275	5
165-195	1.692871	1.131968	4
195-225	4.719975	2.542676	9
225-255	2.829729	1.149862	3
255-285	3.343041	1.348591	8
285-315	8.121195	5.655127	7
315-345	8.644745	6.705769	25

Note: This figure depicts the mean distance moved by families in each of the 12 bins depicted in Figure 7. The mean and standard deviations for these values are shown below the figure. Group names in the legend reflect the geographic bounds on each bin. The first group is for moves most toward the school, which includes moves between $+15°$ and $-15°$ (345°). This group is labeled as group "<15&>345." The bins in the table are listed in a counter-clockwise direction from the school.

Table 4 ■ Quick vs. slow movers.

	Quick	Slow	No Data
θ (°)	4.1°	12.4°	3.5°
95% Lower limit	−10.9°	−1.7°	−16.6°
95% Upper limit	19.2°	26.4°	23.5°
Confidence interval range	30.1°	28.1°	40.1°
κ	4.253	1.470	0.889
Test satistic	22.89	51.57	28.71
Reject no bias cutoff = 5.99	Yes	Yes	Yes
Obs.	15	74	87

Note: This table presents test results on θ and κ for quick versus slow movers. Quick, the first column, examines families who moved within six months of admission. Slow, the second column, examines families moving more than six months after admission. No data families did not respond to the survey.

families changed addresses. Additionally, 85 of these 89 families provided parent work histories that are sufficient for us to identify where one or both parents worked at the time they moved.

In the first test, we split the sample into two groups: families that move shortly after being admitted, and those who wait more than six months before moving. To the extent that families are motivated to apply to the charter school because they expect to move toward the school anyway, we should see a high concentration parameter for "quick movers." "Slow movers" who take more than six months to relocate are more likely to be moving because they were already accepted to the school, then the school attracted them closer. If the concentration parameter is high for these movers, it suggests that the direction of causality runs in the direction we have previously hypothesized.

We will refer to the alternative θ_R values in this section as θ_{Quick} and θ_{Slow}. Concentration parameters will be referred to as κ_{Quick} and κ_{Slow}.

The first column of Table 4 reports results for the 15 responding families who moved within six months of admission. These families had a mean move direction of 4.1° and a concentration parameter $\kappa_{Quick} = 4.253$. The magnitude of κ_{Quick} is surprisingly large, and it is consistent with the hypothesis that these families already intended to move closer to the school before their child was admitted. In untabulated results, 31 families reported moving within 18 months of their child being admitted to the school (16 additional families). For this 31-family group, $\kappa = 2.143$.

Table 5 ■ Survey of mover motivations.

Did you apply to (the school) because you already expected to move closer to (the school) anyway?		
Answer Options	**Response Percent**	**Response Count**
Yes	11.1%	4
No	88.9%	32
When you decided to move, did you consider your shorter commute distance to the school as one of the factors?		
Answer Options	**Response Percent**	**Response Count**
Yes	41.7%	15
No	58.3%	21

Note: This table presents the survey results regarding self-reported motivations for family relocations.

The slow-to-move families (those moving more than six months after admission) have a concentration parameter $\kappa_{Slow} = 1.470$, which is also statistically significant. The behavior of these families is consistent with the school serving as a relocation attractor. For reasons we cannot explain, the families who did not respond to the survey have a lower κ value than those who responded.

Overall, we interpret the results in Table 4 as indicating that the direction of causality flows in both directions: some families apply because they plan to move closer, and other families move because they have already been accepted.

Survey of Mover Motivations

In a separate online survey, we asked families what motivated their moves. The survey asked only two questions:

- Did you apply to (the school) because you already expected to move closer to (the school) anyway?
- When you decided to move, did you consider your shorter commute distance to the school as one of the factors?

A total of 36 families responded to this survey. We tabulate the responses as Table 5.

Workplace Versus School Attraction

Finally, we examine the magnitude of parent–workplace attraction for the moving families. We do this for two reasons. First, because we are considering move causality, it makes sense to consider the most important alternative family-specific factors that might be relevant. Second, measuring workplace attraction for this sample allows us to consider whether the families in this study otherwise behave in a "normal" manner. In other words, have these families' movements been consistent with what has been previously observed and documented relative to work locations as studied by Clark and Burt (1980) and Clark, Huang and Withers (2003).

To address this question, we return to the original survey referenced in the "Quick vs. Slow Movers" subsection. Eighty-nine families responded to the survey, and 85 of these responses provided adequate data to assess relevant work addresses for one or both parents. The surveys requested information about (1) how long the family had lived at the current address, (2) how long the mother (and/or father) worked at their current employment address, (3) the street address where parents were employed and (4) the previous street address where parents were employed. From this data we are able to determine where a parent was working at the time they moved to their new (current) residence. We then repeat the procedures used to develop θ_R and κ: calculating θ_R angles relative to the school, the mother's work location and the father's place of employment. We will refer to the alternative θ_R values in this section as θ_{School}, θ_{Mother} and θ_{Father}. Concentration ratios will be referred to as κ_{School}, κ_{Mother} and κ_{Father}, respectively.

Of the 85 respondents, 55 reported that the mother worked outside the home at the time of the relocation. There were 59 fathers working outside the home at the same time. Several families reported that only one parent lived in the home, but we have not incorporated this information into the analysis. Results are shown in Table 6.

Of the 85 families for which we have survey data, the mean θ values are all within the 95% confidence interval. We cannot reject the hypothesis that the moves are biased toward both the school and toward both work locations. The concentration parameter for the school is 1.485, and the κ for the mother's work location is 0.867. In both cases, the test statistic rejects the null of no bias.[9]

[9] We note that the concentration parameter toward the mothers' work locations is very close to that previously reported for women in Seattle by Clark, Huang and Withers (2003). That study reports a value of 0.831. We cannot reject the hypothesis H_0: $\kappa_{\text{Father}} = 0$. Thus, we cannot reject the idea that these families have no bias toward the fathers' work locations. Clark, Huang and Withers (2003) found the concentration parameter for men to be 0.536.

Table 6 ■ School vs. work attraction.

	School	Mother Work	Father Work
θ (°)	9.8°	−12.6°	−19.9°
95% Lower limit	−1.7°	−40.1°	−90.5°
95% Upper limit	20.6°	11.5°	38.4°
Confidence interval range	22.3°	51.6°	128.9°
κ	1.485	0.867	0.386
Test statistic	59.999	17.405	4.236
Reject no bias cutoff = 5.99	Yes	Yes	No
Obs.	85	55	59

Note: This table presents the test results on θ and κ for school, mothers' work locations and fathers' work locations. Parent work locations were determined from survey questions concerning both work history and home address (residency) history. θ refers to the move direction relative to the location of interest, θ_{School}, θ_{Mother} and θ_{Father} . κ (concentration ratios) values are also calculated relative to each location. For 85 respondents we could determine job locations at the time of home relocation for 55 (59) mothers (fathers) who worked outside the home. Some families were single-parent households, some families had two working parents and some two-parent families had one spouse working outside the home. Differing family structures have not been incorporated into the analysis.

We compare within this sample the κ_{School}, κ_{Mother} and κ_{Father} values utilizing a bootstrap resampling approach.[10] At a 10% significance level, we find that $\kappa_{School} > \kappa_{Mother} > \kappa_{Father}$. The analysis extends the finding of Clark, Huang and Withers (2003) that women's job locations are a stronger relocation draw than men's work locations, but the school is a stronger draw than either.

Caveats Concerning Generalizing the Results

This study provides a conceptual foundation for considering environmental implications of school choice plans. However, the data considered are provided by a single North Carolina charter school. Careful interpretation requires that

[10]We treat the observed values as the sampling population and take repeated samples from the population. Using these repeated samples we calculate the statistic of interest and observe its variation from bootstrap sample to sample. We use this variability estimate as the estimate of our standard error. When testing for the difference between κ_{School} and κ_{Mother}, we take a random sample with replacement of size 85 from the home-to-school thetas as well as a random sample with replacement of size 55 from the home-to-mother's-work thetas. Using this sample we estimate κ_{School} and κ_{Mother} and then calculate their difference. We repeat this sampling process 10,000 times, then we calculate the standard deviation of the differences. The bootstrap-sample means produce a near-normal distribution. Using this standard deviation and assumed normality, we calculate a confidence interval for the difference in κ values. The confidence interval can be used to test the hypothesis that κ_{School} and κ_{Mother} are significantly different.

we consider what factors may be unique to this school and which are likely to be generalizable.

First, it is possible that this school is located in an area that is unusually attractive to families. If so, movement toward the school may be a function of other available amenities rather than the school itself. Likewise, the absence of obvious "negative amenities" may augment the school's apparent attractiveness.

A second factor that seems likely to be important to the school's attraction is that this school enrolls students from kindergarten through 12th grade. It also gives admission preference to the families of current students. Both of these policies seem likely to lead to greater family attraction because they create greater long-term family-school stability.

A third factor that may impact the school's attraction is the financial stability of the school itself. This charter school was founded by a successful businessman who has also founded other successful private schools. Families who were aware of this fact probably recognized that the school was likely to succeed, both academically and financially. A school with a short history, founded by a sponsor without a legacy of financial and/or academic success might not produce similar environmental impacts.

Fourth, while there is reason to believe that other types of schools may produce qualitatively similar attractions, the magnitude of the attraction might be greater for an independent charter school than for a tuition-dependent private school or a "magnet school" that is operated by an elected school board. Unlike a private school, this school is publicly funded and charges no tuition. Because the school is free, families may perceive that their connection to the school is likely to be more permanent than would be the case with a private school. Private school parents must continue to pay fees to retain the services of the school. Recognizing this cost of continuing the relationship, private school families may view their long-term connection to a private school as more uncertain. If so, we would expect the enrollment in a charter school to be more stable, and the attraction level to be greater.

Various school districts also utilize "magnet" programs, which allow families to enroll children in district-operated schools of their choice. It is an open question whether these magnet schools would exert environmental effects that are quantitatively similar to the subject school. The sometimes transitory nature of school-district policy may suggest otherwise. For example, in Wake County, North Carolina, where this school is located, the election of a new school board in 2009 led to uncertainty about the fate of the district's magnet programs. Charter and private schools are probably less subject to political turmoil that might undermine a family's long-term commitment to the school. A property right that allows all of a family's children to attend the school is likely to provide greater school-family stability than can be achieved when each school-board election may usher in new assignment policies. Districts with hotly contested school board elections may be unable to promise enough stability to create strong levels of family attraction for school-board-operated magnet schools.

Fifth, the quality of surrounding "traditional" public schools may have an impact on the success of a charter school and the magnitude of its impact on the surrounding area. In general, the Wake County, North Carolina, schools are considered to be above average. However, assignment policy in the county is designed to produce within-school diversity and to minimize between-school diversity. We cannot assess how our results would differ if the nearby traditional public schools' quality were unusually poor (or good). Addressing this question would require an examination that includes data from multiple charter schools.

Conclusion

This study is a first effort at developing a conceptual foundation for considering environmental implications of school choice plans. More narrowly, we develop a model of move distance (distributed gamma) and direction (distributed von Mises) to predict family relocation choice, relative to school location. The model is parameterized using data from student mailing-address changes. The fitted data suggest that families attending the school in question are almost twice as likely to relocate toward the school as could be expected if the school did not exert any attraction. Because move distance and direction in the sample are not independent, the theoretical model probably underestimates the true magnitude of the school's attraction. This result may have important implications for the potential role of charter schools and other non-catchment-area based school choice plans (such as private school tax credits, vouchers and magnet school programs) in mitigating urban sprawl, fostering urban renewal and promoting sustainable real estate development.

This study has implications even where various forms of school choice already exist. For example, Milwaukee's voucher program excludes students from families with incomes above 175% of the federal poverty level: $37,439 for a family of four in 2008–2009. The threshold is apparently intended to focus resources on students from poor families. However, an unintended consequence of restrictive eligibility may be to further concentrate poor families in the inner city, while middle-class families relocate to the suburbs. When one considers the greater environmental impact of the voucher policy, a better design might allow wealthier families to participate in the program. Retention of these families in the city would produce environmental externalities that are usually considered to be positive in terms of reducing sprawl, reducing pollution and promoting urban renewal.

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