Chapter 5: Electron Sources

1. All microscopes need an electron source to illuminate the specimen. There are stringent requirements for the beam of electrons and these are best met by only two types of source: Thermionic sources and field emission sources.

2. Thermionic sources are either tungsten hairpin filaments or lanthanum hexaboride (LaB_6) crystal needles, and field emitters are fine tungsten needles. Some field emitters are coated with low working function materials such as zirconium oxide (ZrO_2). These types of field emitters are called “Schottky” emitters.

3. The important points are that in general (a) the two types of electron sources (thermionic or field emission) can not be interchanged within the same microscope due to the vacuum requirement and the lens settings, (b) field-emission sources give “monochromatic” electrons; thermionic sources are less monochromatic and give “white” electrons.

- For high performance, high spatial resolution, rapid data acquisition, and reliable operation, an electron microscope requires an electron source with the following ideal properties: Small source size, low electron emission energy spread, high brightness (beam current per solid angle), low short-term noise and long-term stability.

- Schottky and cold-field emission are superior to thermionic sources in terms of source size, brightness, and lifetime. Both are up to 1000 times smaller and up to 100 times brighter than thermionic emitters. However, Schottky emission (SE) is preferred over cold field emission (CFE) because it provides higher stability and is easier to operate. Sources, such as CFEs, operate at room temperature having some disadvantages because they rapidly become covered with adsorbate molecules that arrive from the vacuum system walls. This type of emitter has to be cleaned from time to time by “flashing” to high temperature..

Thermionic Emission

1. If we heat any material to a high enough temperature, we can give the electrons sufficient energy to overcome the natural barrier that prevents them from leaving this material. This natural barrier is termed the "work function" (Φ) and usually has a value of a few electron volts.

2. The physics of thermionic emission is well explained by Richardson's Law in terms of the current density (J) from the source to the operating temperature (T).

$$J = AT^2 e^{-\Phi/kT}$$

Where J in A/m^2, T in Kelvin, and A is Richardson's constant (A/m^2K^2)

3. From the above equation we can see that we need to heat the source to a temperature T such that energy is greater than Φ. Then electrons will escape from the source and be available to form an electron beam.

4. However, not many materials can withstand this type of heating. The only viable thermionic sources are either refractory (high melting point) materials or those with an exceptionally low work function. For instance: W has a melting point of 3695 K, and LaB_6 has a low work function Φ of 2.4 eV.

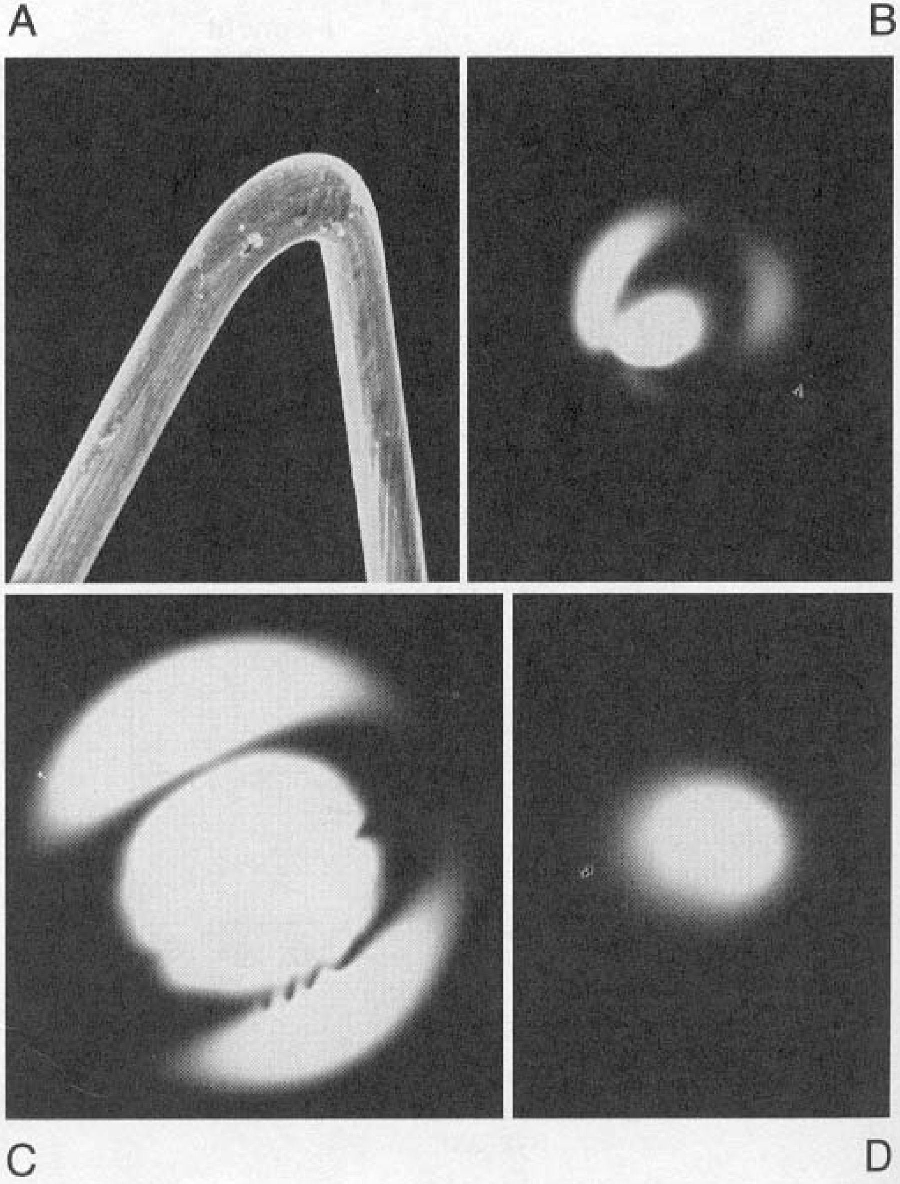


Figure 5.5. (A) The tip of a tungsten hairpin filament and the distribution of electrons when the filament is (B) undersaturated and misaligned, (C) undersaturated and aligned, and (D) saturated.

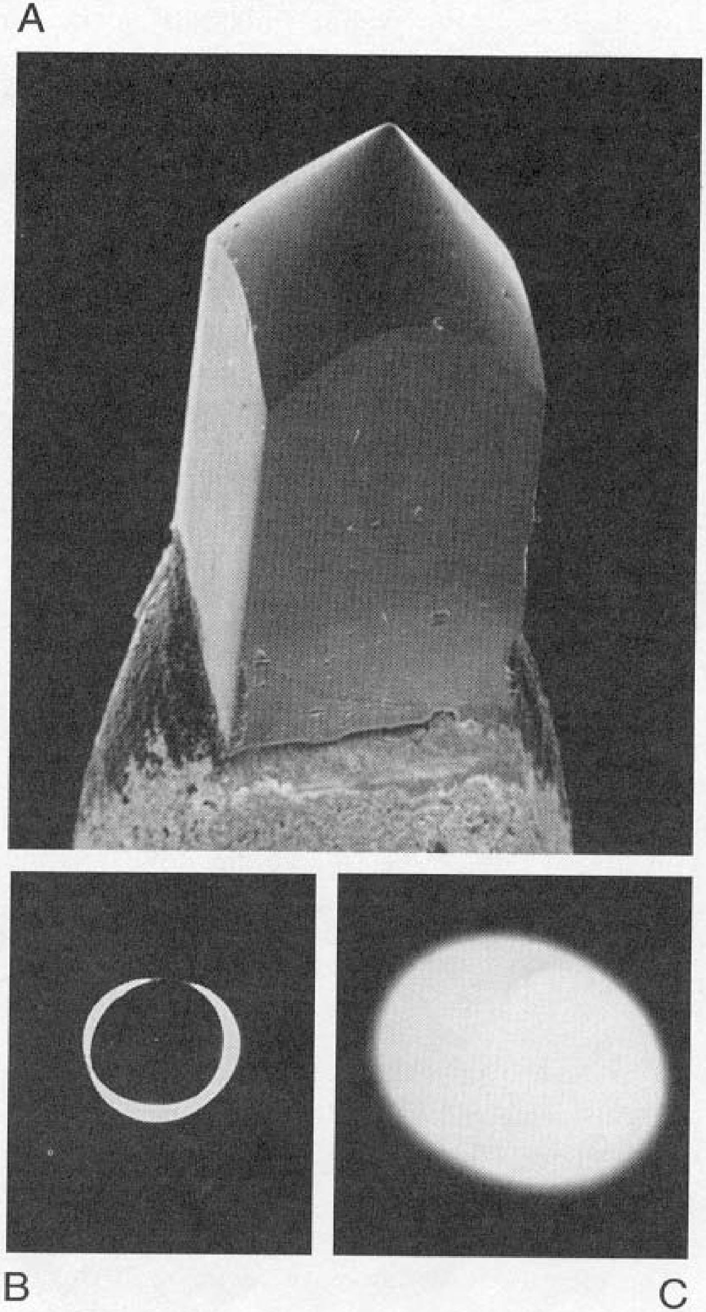
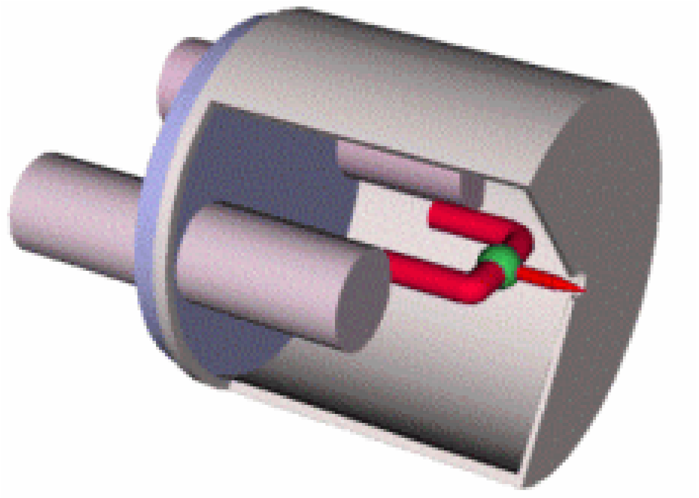


Figure 5.6. (A) An LaB_6 crystal and the electron distribution when the source is (B) undersaturated and aligned and (C) saturated.

Schottky Field Emission Sources:

In electron emission devices, especially electron guns, the thermionic electron emitter will be biased negative relative to its surroundings. This creates an electric field of magnitude F at the emitter surface. Without the field, the surface barrier seen by an escaping Fermi-level electron has height W equal to the local work-function. The electric field lowers the surface barrier by an amount ΔW, and increases the emission current. This is known as the “Schottky effect” or field enhanced thermionic emission.



As shown the image on the left, a single crystal tungsten wire with a sharp end etched to a small radius (red in the sketch) is mounted on a tungsten hairpin (also red). A current through the filament is used to maintain the tip at a temperature of 1750 - 1850 K. The tip just penetrates a hole in a cylindrical suppressor electrode mounted around the assembly. Electrons are emitted from the tip due to both thermal excitation and the electric field at the tip due to the potential difference between it and an extractor electrode (not shown). Electrons from the filament are repelled by the potential on the suppressor. Electrons from the tip are used by the subsequent column to form a focused beam.

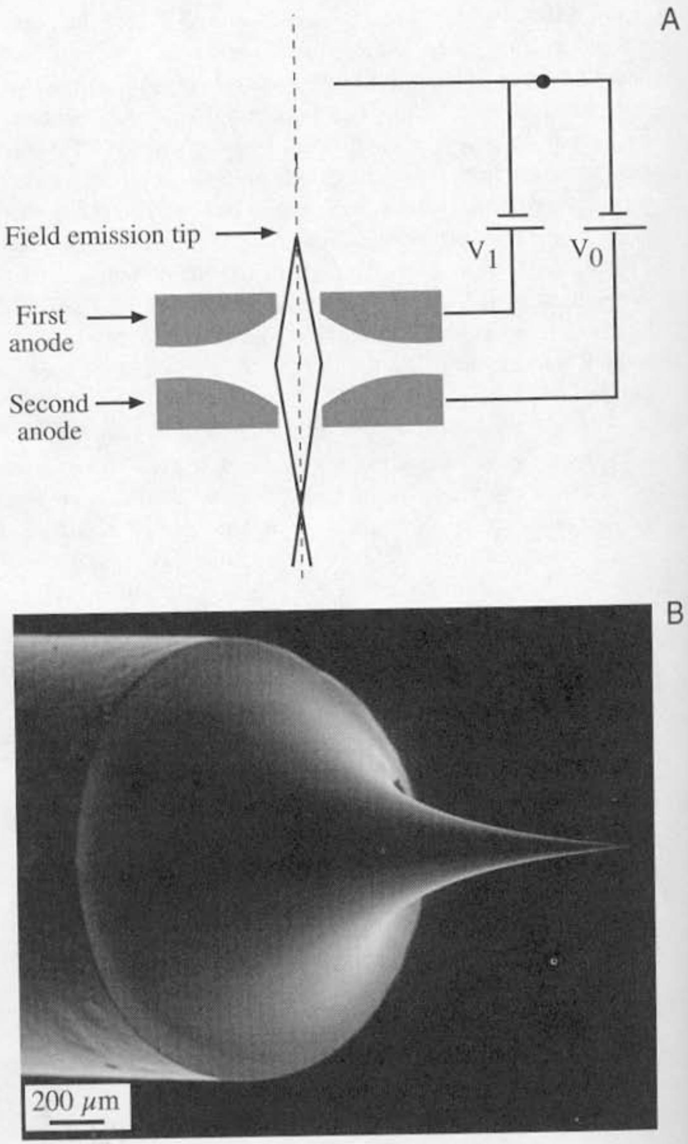


Figure 5.7. (A) Electron paths from a field-emission source showing how a fine crossover is formed by two anodes acting as an electrostatic lens. Sometimes an extra (gun) lens is added below the second anode. (B) An FEG tip, showing the extraordinarily fine W needle.

Electron Field Emission

- The emission of electrons from a metal or semiconductor into vacuum (or a dielectric) under the influence of a strong electric field. In field emission, electrons tunnel through a potential barrier rather than escaping over it, as in thermionic or photoemission.
- Field emission is most easily obtained from sharply pointed metal or semiconductor needles. The smallest controllable tip radius is about 100 nm.
- The small optical source size and very high current densities of field-emission cathodes make them attractive electron sources for electron microscope and microprobe applications because, for focused beam sizes below about 500 nm, field-emission sources provide higher currents than thermionic cathodes.
- Field-emission sources operate on a fundamentally different principle than thermionic sources. When an electric field is applied to a material with a sharp point, the strength of an electric field E at that point is very strong.

  Based on $E = V/r$, where E in V/cm, and r (in cm) is the radius of the point source.

Field Emission

For a metal field emitter at low temperature, the process can be understood in terms of the illustration (Fig. 5.0). The metal can be considered a potential box, filled with electrons to the Fermi level, which lies below the vacuum level by several electronvolts. The distance from Fermi to vacuum level is called the work function, Φ. The vacuum level represents the potential energy of an electron at rest outside the metal, in the absence of an external field. In the presence of a strong field E, the potential outside the metal will be deformed along the line AB, so that a triangular barrier is formed, through which electrons can tunnel. Most of the emission will occur from the vicinity of the Fermi level where the barrier is thinnest. Since the electron distribution in the metal is not strongly temperature-dependent, field emission is only weakly temperature-dependent and would occur even at the absolute zero of temperature.

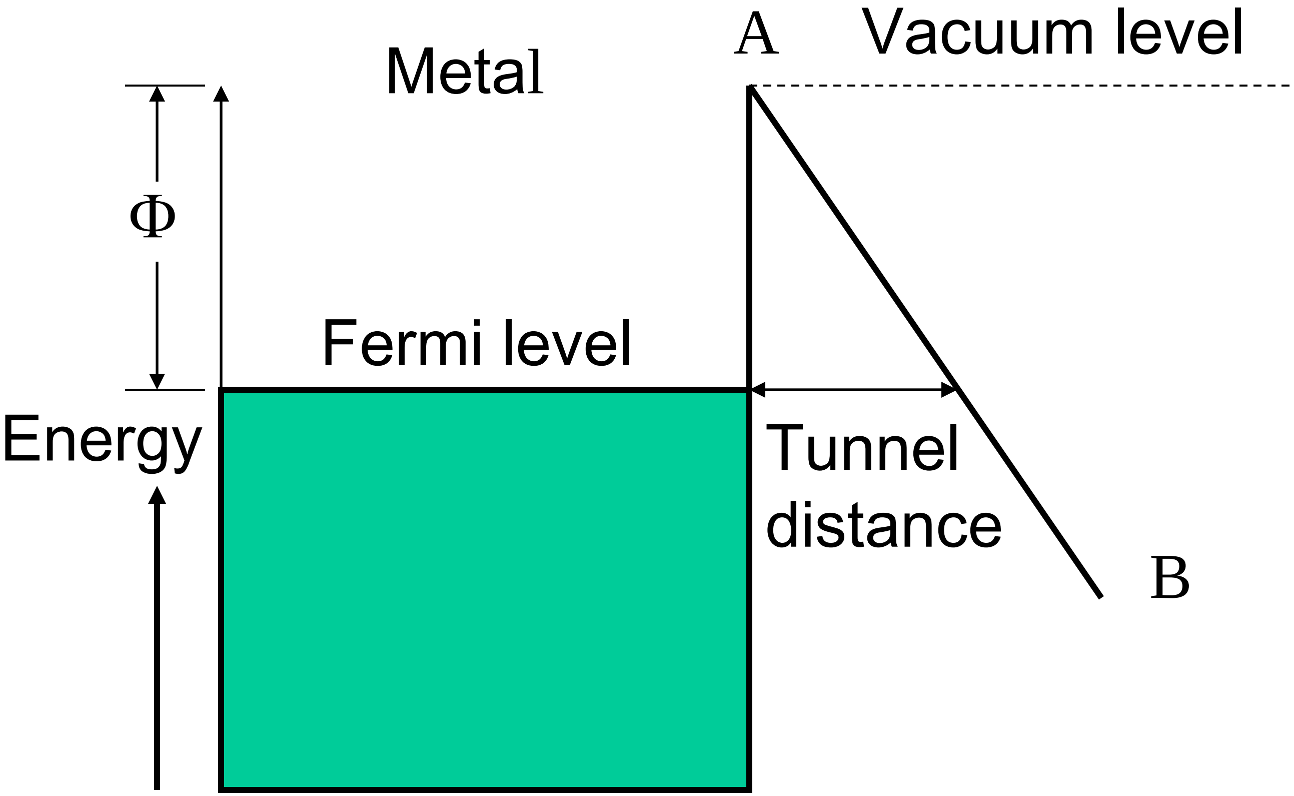


Fig. 5.0 Diagram of the energy-level scheme for field emission from a metal at absolute zero temperature.

Continue: Electron Field Emission

- The phenomenon of field emission was first reported by R.W. Wood in 1897.
- Fowler and Nordheim, in 1928, provided the first generally accepted explanation of field emission in terms of the newly developed quantum mechanics ("Fowler-Nordheim" equation).

$$J = BE^2 \exp\{-6.8 \times 10^7 \phi^{3/2} / E\}$$

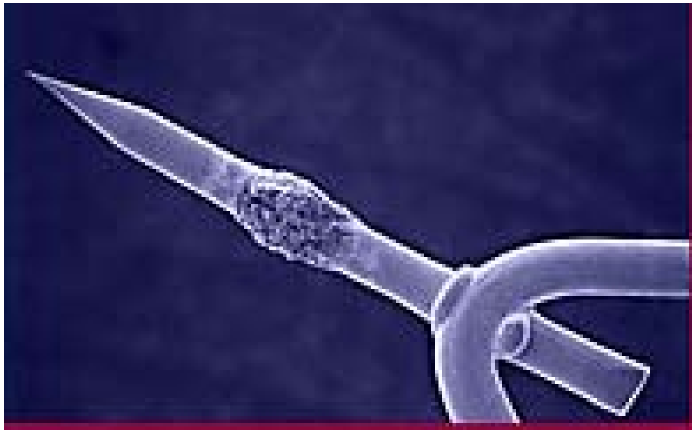
J: emission current density (A/cm^2)
B: field-independent constant of dimensions (A/V^2)
E: applied field (V/cm)
ϕ: work function (eV)

- E.W. Muller in 1936 invented the field emission microscope which is very useful for the measurement of heats of desorption, work function changes, and diffusion energies of adsorbates.

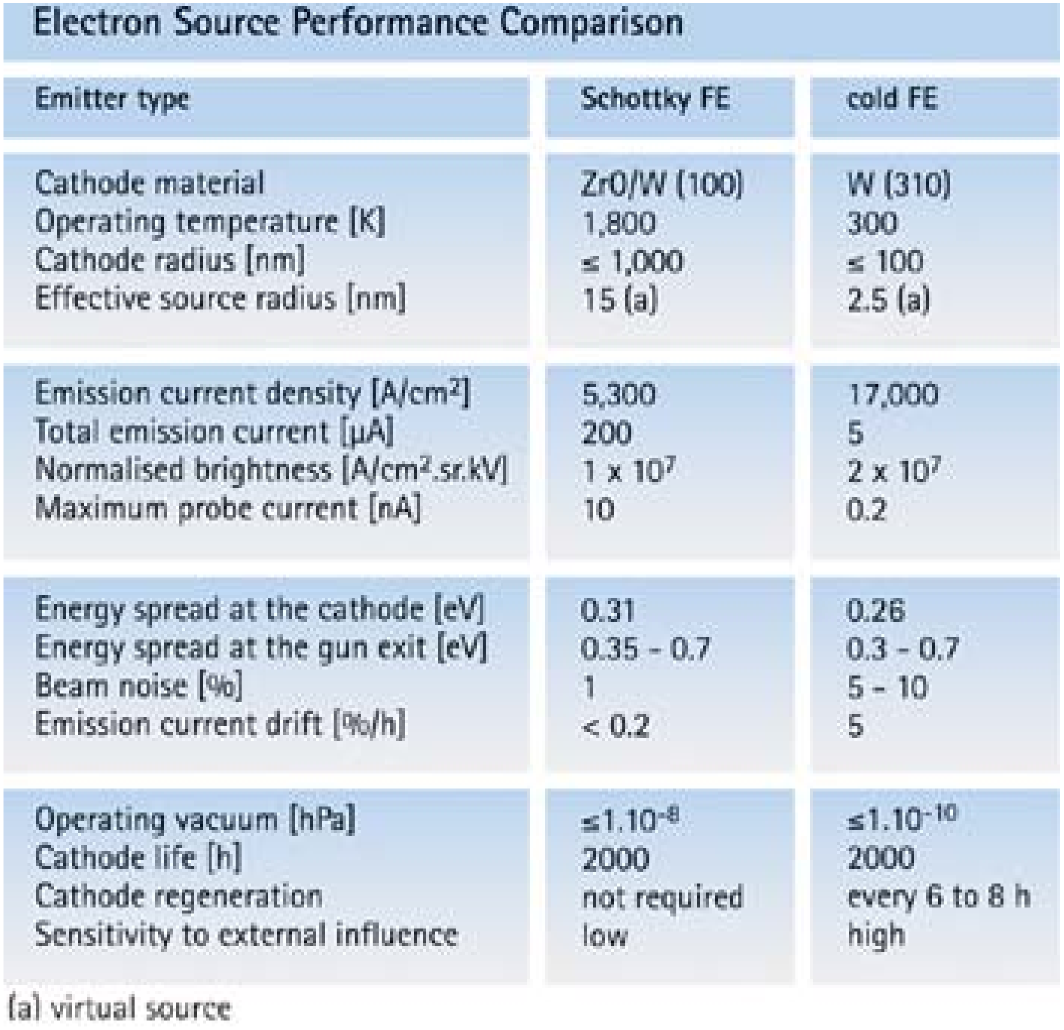
- To allow field emission, the surface has to be free of contaminants and oxide. We can achieve this by operating in UHV conditions ($<10^{-11}$Torr), in this case, the tungsten is operated at ambient temperatures and the process is called “cold field emission.”

- Alternatively, we can keep the surface in a pristine condition at a poorer vacuum by heating the tip. The thermal energy, in fact, assists in electron emission when the electrons don’t tunnel through the barrier. For such “thermal field emission,” surface treatments with ZrO_2 improve the emission characteristics, particularly the stability of the source. Such “Schottky” emitters are becoming popular.

- To evaluate the quality of the electron beam generated by either thermionic emission or field emission, parameters such as brightness, coherency, and stability are important.



- The Schottky emitter consists of a single-crystalline tungsten wire, 1 mm long and 125 mm in diameter, and with {100} planes perpendicular to the wire axis. One end of the wire is etched down to a tip with a diameter of ~1 mm. At the other end the wire is spotwelded to a polycrystalline tungsten wire of approximately the same diameter. This polycrystalline tungsten loop is fixed to two poles which are embedded in a cylindrical ceramic base of ~1 cm in diameter. About halfway along the wire a reservoir of ZrO_x is attached. This gives the {100} planes a much lower work function than other crystallographic orientations. To get emission from the source it is operated at an elevated temperature (1800 K) in a vacuum of ~10^{-9} mbar and the electric field at the tip end is of the order of ½-1 V/nm.



Electron Source Performance Comparison

Emitter type	Schottky FE	cold FE
Cathode material	ZrO/W (100)	W (310)
Operating temperature [K]	1,800	300
Cathode radius [nm]	≤ 1,000	≤ 100
Effective source radius [nm]	15 (a)	2.5 (a)
Emission current density [A/cm^2]	5,300	17,000
Total emission current [µA]	200	5
Normalised brightness [A/cm^2.sr.kV]	1×10^7	2×10^7
Maximum probe current [nA]	10	0.2
Energy spread at the cathode [eV]	0.31	0.26
Energy spread at the gun exit [eV]	0.35 - 0.7	0.3 - 0.7
Beam noise [%]	1	5 - 10
Emission current drift [%/h]	< 0.2	5
Operating vacuum [hPa]	$\le 1.10^{-8}$	$\le 1.10^{-10}$
Cathode life [h]	2000	2000
Cathode regeneration	not required	every 6 to 8 h
Sensitivity to external influence	low	high

(a) virtual source

Emitter Tip Deformation

Deformation of the emission area is one of the factors affecting long-term emission stability and usability. In all electron columns, residual gases are present. When a high energy electron hits a residual gas molecule, a positive ion can be created. This ion is accelerated back to the emitter and bombards the emission area. Ion bombardment will mechanically deform an emitter's surface. Because Schottky emitters operate at 1800 degrees C, the surface mobility is high enough to anneal such deformations in a reasonable time. The room temperature CFE will not anneal such deformations. To repair the CFE, it is necessary to periodically "flash" the emitter. The flashing process is simple heating of the emitter to allow deformations to be annealed and to remove the adsorbed molecules, just as occurs automatically with Schottky emitter use. The CFE flashing process not only interrupts work in progress, it eventually leads to end-of-life for the cold field emitter. Each time a CFE is flashed in the absence of an electric field, the emitter radius grows slightly. Ultimately, the tip radius grows so large to the extent that sufficient electric field cannot be achieved. Schottky emitters do not grow at these elevated temperatures because the Schottky emitter endform is in thermal-field equilibrium.

Beam Noise

- Beam noise is the time-dependent fluctuation in beam current. Describing the relationship simply, beam noise is inversely proportional to the emission area. Emission area is dependent on emitter radius. If all conditions are the same, the smaller the emission area, the higher the noise. Cold field emission is more noisy than Schottky emission simply because of the emission area (i.e. radius) size differences.

- Another contributing factor to noise is emitter temperature. Schottky emission noise is caused by the surface Brownian motion (the random movement of microscopic particles suspended in a liquid or gas, caused by collisions with molecules of the surrounding medium. Also called Brownian movement) of the W and Zr emitter atoms at 1800 degrees C. The CFE is operated at room temperature, and one might think the noise caused by the surface Brownian motion of the W emitter atoms at 25 degrees C would be less. Unfortunately, in all real vacuum systems, residual gas adsorbs onto the CFE. It is the surface Brownian motion (which can be significant at room temperature) of these absorbed gases that is partly responsible for the noise in cold field emission.

Brightness

- Brightness is the current density per unit solid angle of the source.
- Electron sources differ considerably in their size and, as a result, the electrons leave the source with a range of angles.
- Brightness is particularly important when we are using very fine electron beams, as we do in analytical and scanning microscopy.
- To define the brightness, we consider an electron source having the following characteristics:
  - A diameter d_o
  - Giving off a certain emission current i_e
  - The electrons diverging from the source with a semiangle α_o

These parameters are actually defined at the gun crossover. See Fig. 5.1

Therefore, the current density = $i_e/\pi(d_o/2)^2$ and the solid angle of the source = $\pi\,\alpha_o^2$

The brightness is defined as $\beta = i_e/\pi(d_o/2)^2\,\pi\,\alpha_o^2 = 4i_e/(\pi d_o\alpha_o)^2$, where the units of β are usually A/ cm^2 Sr (Steradian).

- An important factor embodied in the brightness definition is that β increases linearly with increasing accelerating voltage for thermionic sources ($J \sim T \sim i_e$).
- From the definition, obviously, the higher the value of β, the more electrons we can put into an electron beam of a given size, and the more information we can extract from the specimen and the more we can damage sensitive specimens.
- The brightness is very important in analytical electron microscopy, which is the technique of quantitative analysis of the many signals that come from a specimen irradiated by an electron beam. As we go to higher magnification in HRTEM, the screen intensity becomes less because we are viewing only a fraction of the illuminated area of the specimen.
- The electron density can be increased by using the brightest available source. Then images can be recorded with reasonably short exposure times.

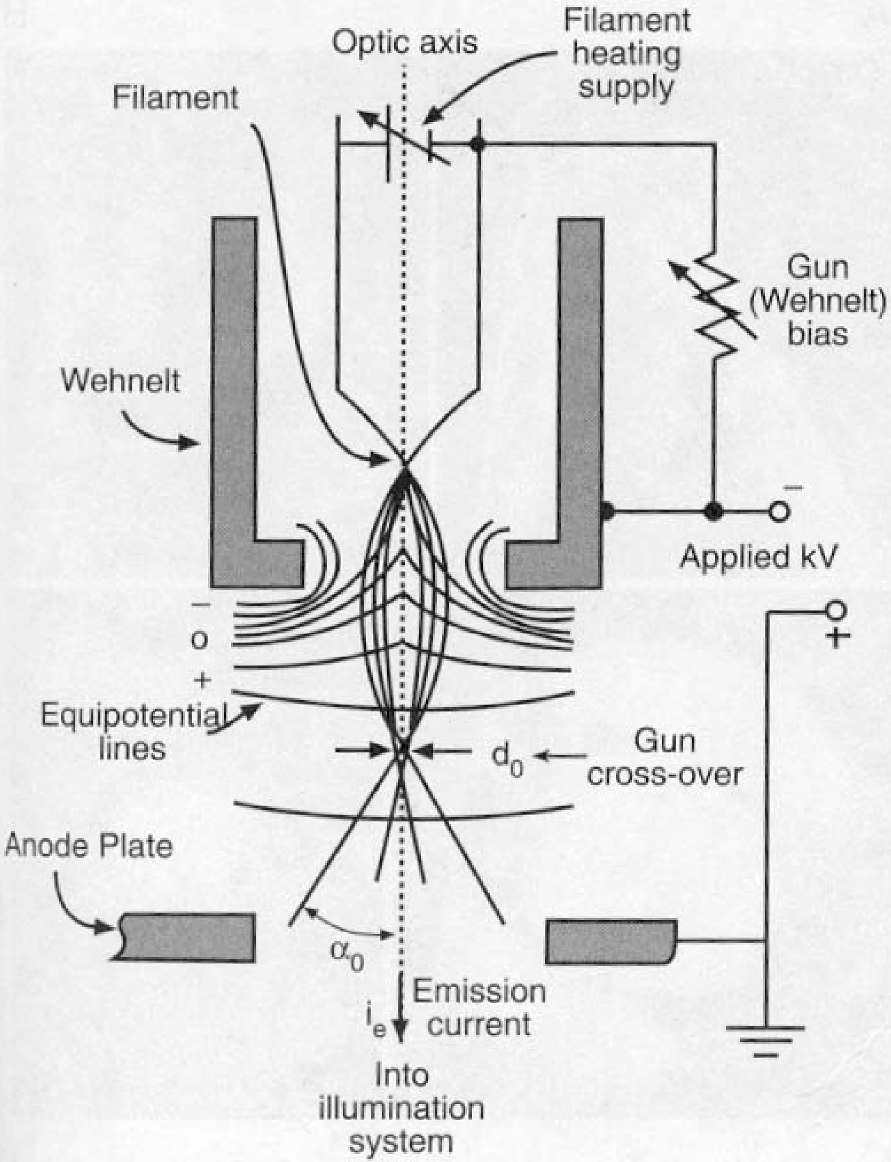


Figure 5.1. Schematic diagram of a thermionic electron gun. A high voltage is placed between the filament and the anode, modified by a potential on the Wehnelt which acts to focus the electrons into a crossover, with diameter d and convergence/divergence angle α

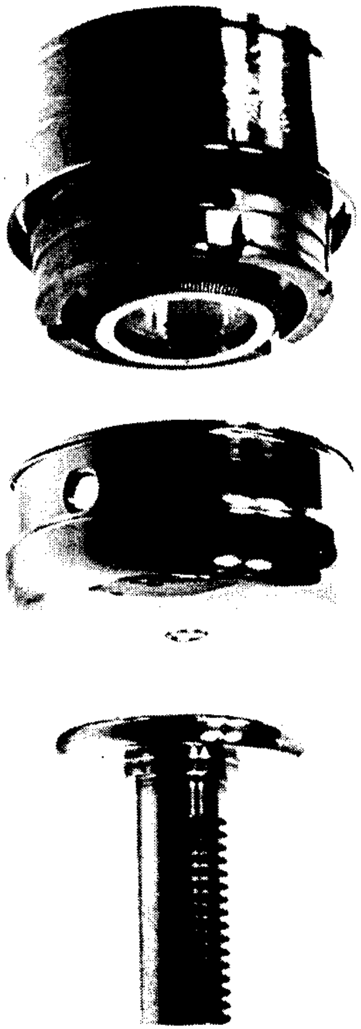


Figure 5.2. The three major parts of a thermionic gun, from top to bottom: the cathode, the Wehnelt cylinder, and the anode, shown separated. The Wehnelt screws onto the cathode (filament) support and both are attached to the high-tension cable which contains power supplies for heating the filament and biasing the Wehnelt. The anode sits just below the Wehnelt, in the top of the TEM column.

Temporal Coherency and Energy Spread

- The coherency of a beam of electrons is a way of defining how well the electron waves are “in step (phase)” with one another.
- To get a coherent beam of electrons we must create one in which all the electrons have the same wavelength, just like monochromatic light. We refer to this aspect of coherency as “temporal coherency.”
- If the “wave packets” are all identical they have the same coherence length. A definition of the coherence wave length λ_c is

  $$\lambda_c = \upsilon h/\Delta E$$

  Where υ is the electron velocity, ΔE is the energy spread of the beam, and h is Planck’s constant.
- The above equation suggests that we must have an electron source emitting electrons with small energy spread. It also means that we have to have stable power supplies to the source and a stable high-voltage supply so that all the electrons have a small ΔE, thus giving a well-defined wavelength.
- In practice, the typical ΔE values for the three sources are in the range 0.1 to 3 eV (which is remarkably small compared with a total energy of 100 to 400 keV).

Spatial Coherency and Source Size

1. Spatial coherency is related to the size of the source. Perfect spatial coherence would imply that the electrons were all emanating from the same point at the source. So source size governs spatial coherence and smaller source size gives better coherency.

2. We can define the distance d_c, the effective source size, for coherent illumination to be

   $$d_c << \lambda/2\alpha$$

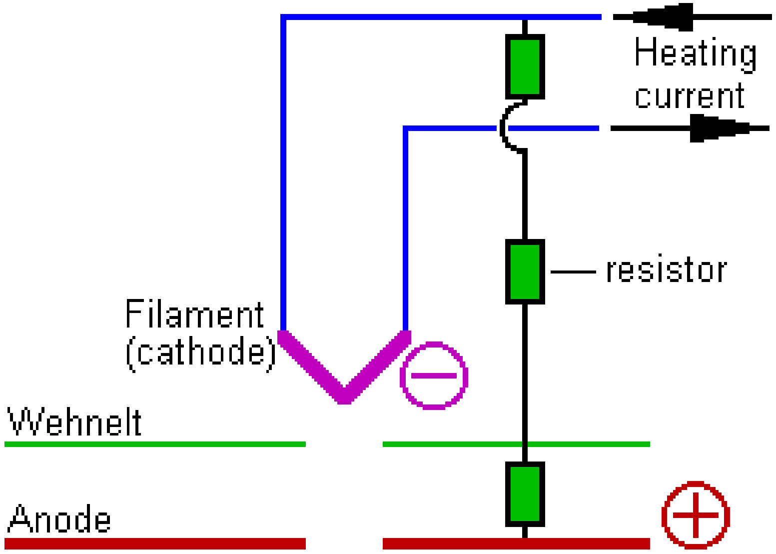
   Where λ is the electron wavelength and α is the angle subtended by the source at the specimen. We can control α by inserting an aperture in the illumination system.

Electron gun

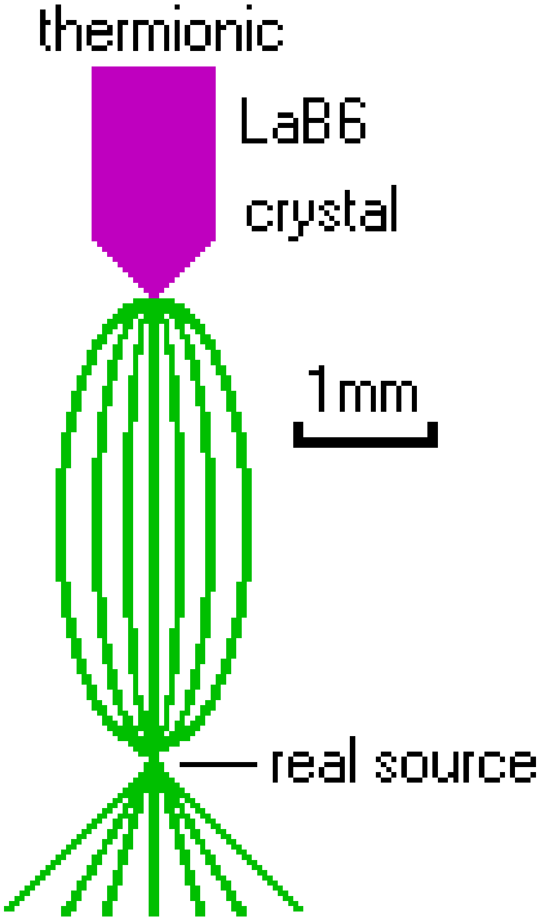
- The electron beam is generated in the electron gun. Two basic types of gun can be distinguished: the thermionic gun and the field emission gun (FEG). Thermionic guns are based on two types of filaments: tungsten (W) and lanthanum-hexaboride (LaB_6) (cerium-hexaboride, CeB_6, can also be used instead of LaB_6; its performance is roughly the same as that of LaB_6). On modern instruments the different types of thermionic filaments can be used interchangeably. The FEG employs either a (thermally-assisted) cold field emitter - as on the Philips EM 400-FEG - or a Schottky emitter - as on the more recent generations of FEG microscopes (CM20/CM200 FEG, CM30/CM300 FEG, Tecnai F20 and F30).

Thermionic gun

The thermionic gun (so-called triode or self-biasing gun) consists of three elements: the filament (cathode), the Wehnelt and the anode. The Wehnelt has a potential that is more negative - the bias voltage - than the cathode itself. The bias voltage is variable (controlled by the Emission parameter) and is used for controlling the emission from the filament. A high bias voltage restricts the emission to a small area, thereby reducing the total emitted current, while lowering the bias voltage increases the size of the emitting area and thus the total emission current.



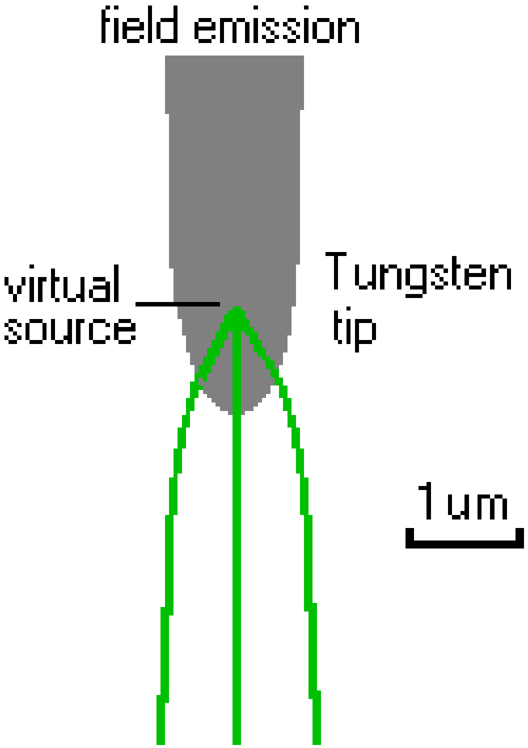
The emitted electrons that pass through the Wehnelt aperture are focused into a cross-over between the cathode and anode. This cross-over acts as the electron source for the optics of the microscope.



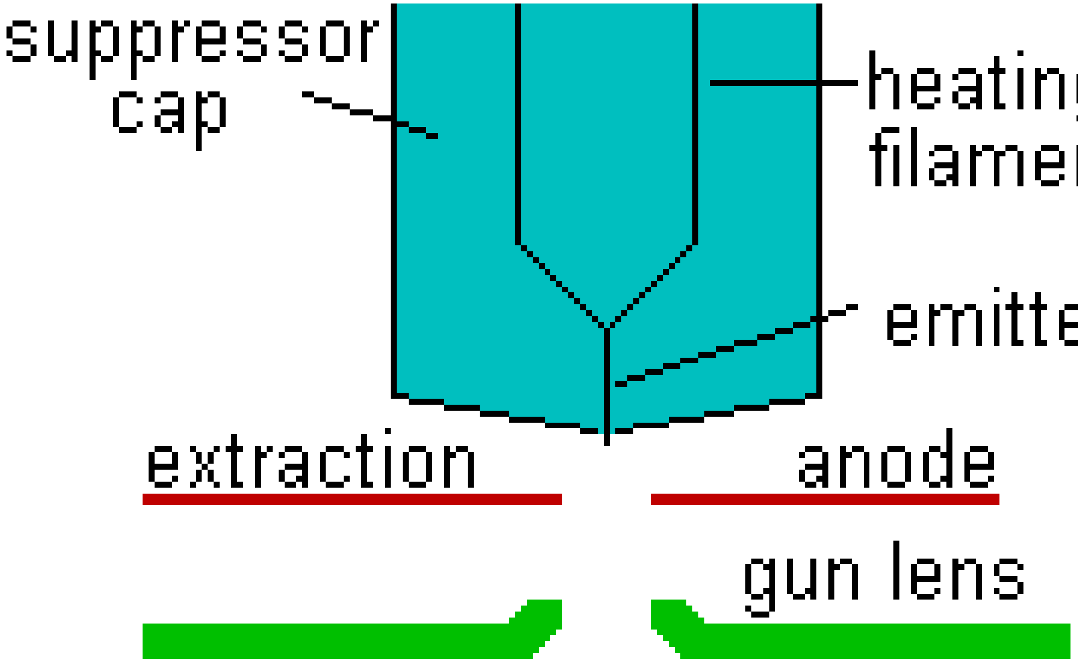
The size of the cross-over is determined by the type of filament, the electric field between cathode and anode, and by the exit angles of the electrons from the filament. At low bias voltages, electrons are emitted from a larger area of the curved tip of the filament, causing a higher divergence of emission angles and thus a larger source size. Higher emission therefore not necessarily improves the brightness (a performance parameter of the emitter, measured in A/cm²Sr). In addition, higher emission increases the Coulomb interaction between electrons - the so-called Boersch effect - (some get accelerated, others decelerated) which increases the energy spread.

Field Emission Gun

In the case of a Field Emission Gun (abbreviated FEG), electron emission is achieved in a different way than with thermionic guns. Because a FEG requires a different gun design as well as much better vacuum in the gun area (~10e^{-8} Pa instead of the ~10e^{-5} Pa necessary for thermionic guns), it is found only on dedicated microscopes (Tecnai F20, F30). The FEG consists of a small single-crystal tungsten needle that is put in a strong extraction voltage (2-5 kV). In the case of a cold FEG or thermally-assisted cold FEG, the needle is so sharp that electrons are extracted directly from the tip. For the Schottky FEG (as used on the Tecnai microscopes) a broader tip is used which has a surface layer of zirconia (ZrO_2). The zirconia lowers the work function of the tungsten (that is, it enhances electron emission) and thereby makes it possible to use the broader tip. Unlike the thermionic gun, the FEG does not produce a small cross-over directly below the emitter, but the electron trajectories seemingly originate inside the tip itself, forming a virtual source of electrons for the microscope.



The FEG emitter is placed in a cap (suppressor) which prevents electron emission from the shaft of the emitter and the heating filament (very similar to the Wehnelt of the thermionic gun). Electron emission is regulated by the voltage on the extraction anode. Underneath the extraction anode of the FEG is a small electrostatic lens, the gun lens. This lens is used to position the first cross-over after the gun in relation to the beam-defining aperture (usually the C2 aperture). If the gun lens is strong, the cross-over lies high above the aperture while a weak gun lens positions the cross-over close to the aperture, giving a high current but at the expense of aberrations on the beam. A strong gun lens is therefore used where small, intense and low-aberration electron probes are needed (diffraction, analysis and scanning), while a weak gun lens is used when high currents are important (TEM imaging). In the latter case, the beam is spread and the aberrations do not affect the area within the field of view.



The high brightness of FEGs comes about because of two reasons:

1. The small size of the tip ensures that large numbers of electrons are emitted from a small area (high A/cm²).
2. The electrons come out of the tungsten crystal with a very restricted range of emission angles (high A/Sr).

FEGs also have a low energy spread due to their low working temperature and emission geometry (small virtual source size, but much larger actual size of the emitting area).

Comparison of Electron Guns

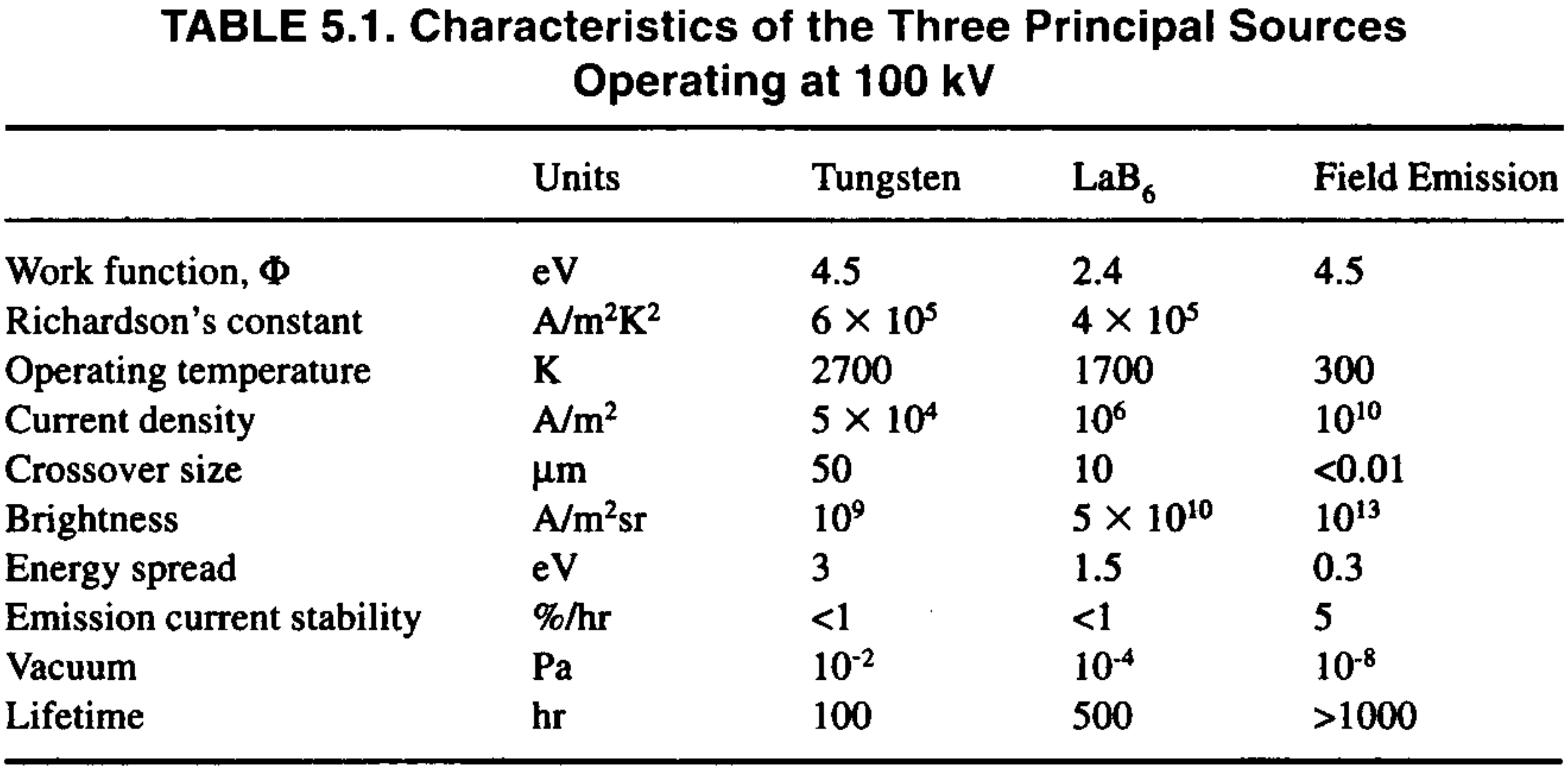


TABLE 5.1. Characteristics of the Three Principal Sources Operating at 100 kV

	Units	Tungsten	LaB_6	Field Emission
Work function, Φ	eV	4.5	2.4	4.5
Richardson's constant	A/m^2K^2	6×10^5	4×10^5	
Operating temperature	K	2700	1700	300
Current density	A/m^2	5×10^4	10^6	10^{10}
Crossover size	μm	50	10	<0.01
Brightness	A/m^2sr	10^9	5×10^{10}	10^{13}
Energy spread	eV	3	1.5	0.3
Emission current stability	%/hr	<1	<1	5
Vacuum	Pa	10^{-2}	10^{-4}	10^{-8}
Lifetime	hr	100	500	>1000

The Reasons for Choosing the Highest kV

- The gun is brightest.
- The wavelength is shortest; the resolution is potentially better.
- The cross section for inelastic scatter is smaller; the heating effect is smaller.

As materials scientists, you should always operate the microscope at the maximum available kV, unless there is a definite reason to use a lower kV. Of these reasons,the most obvious is avoiding beam damage.

Chapter 6: Lenses, Apertures, and Resolution

In an electron microscope, we change the focus and magnification by changing the strength of the lens itself. So electron lenses differ fundamentally from glass lenses in that one lens can be adjusted to a range of strengths.

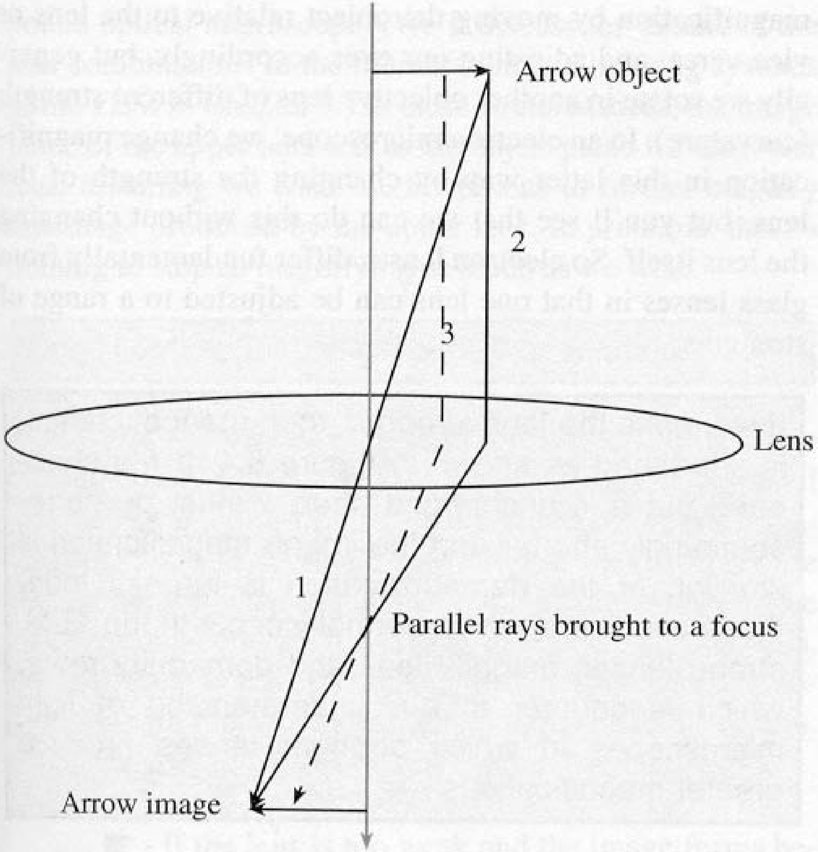


Figure 6.2. How to draw a ray diagram: first construct ray 1 through the middle of the lens, then ray 2, parallel to the optic axis, to determine the lens strength. Finally, draw line 3 parallel to 2 to define the focal plane where the parallel rays are focused. Thus an asymmetric object is imaged off axis and rotated through 180°.

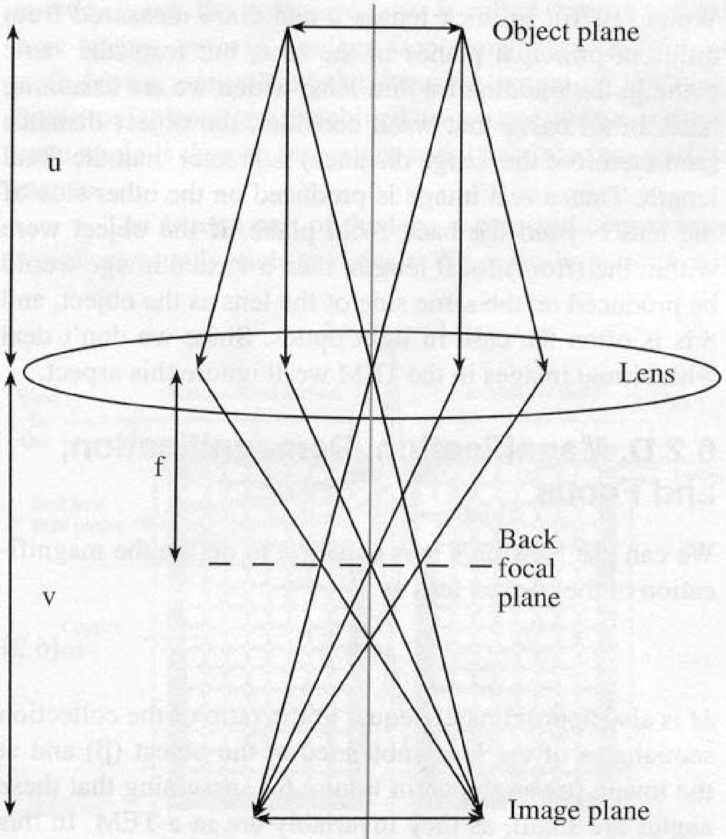


Figure 6.3. A complete ray diagram for a finite object, symmetrically positioned around the optic axis. All rays emerging from a point in the object (distance *u* from the lens) that are gathered by the lens converge to a point in the image (distance *v* from the lens) and all parallel rays are focused in the focal plane (distance *f* from the lens).

The Lens Equation, Magnification and Demagnification, and Focus

Newton's lens equation:

$1/u + 1/v = 1/f$ (where u is the object distance, v is the image distance, and f is the focal length)

We can use Newton's lens equation to define the magnification of the convex lens as $M = v/u$

Sometimes we may want to demagnify an object (for example, when we want to form a small image of the electron source, to create the finest possible beam at the specimen). If that is the case, we define the demagnification as $1/M$

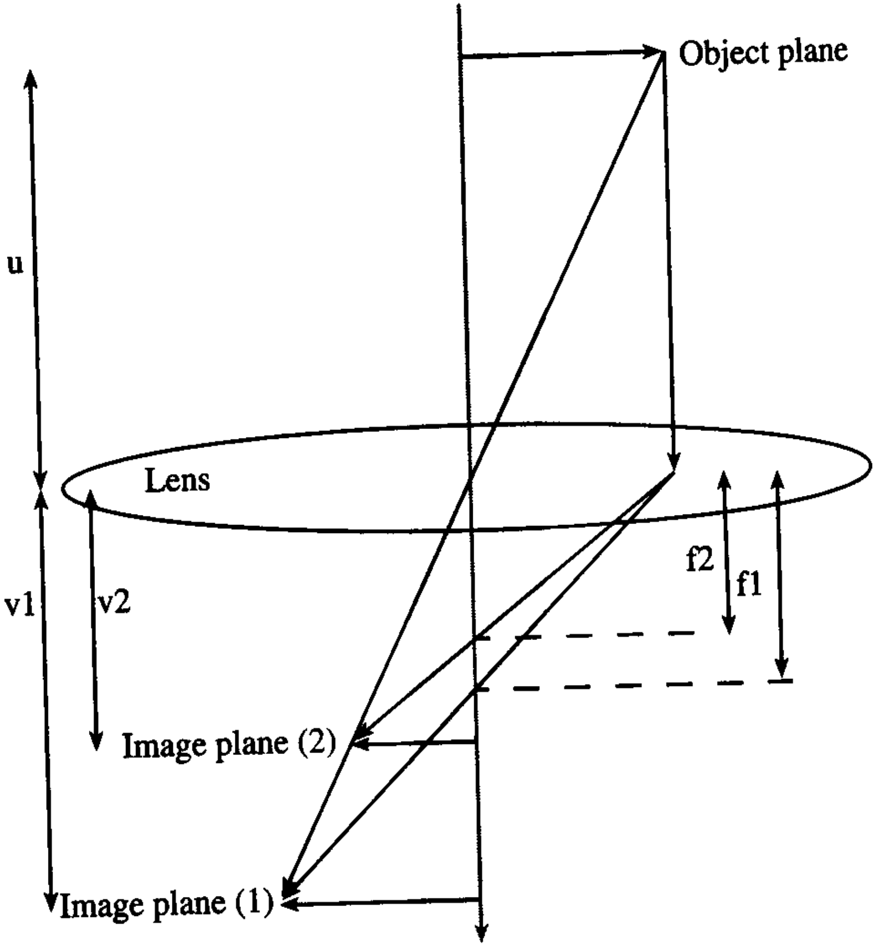
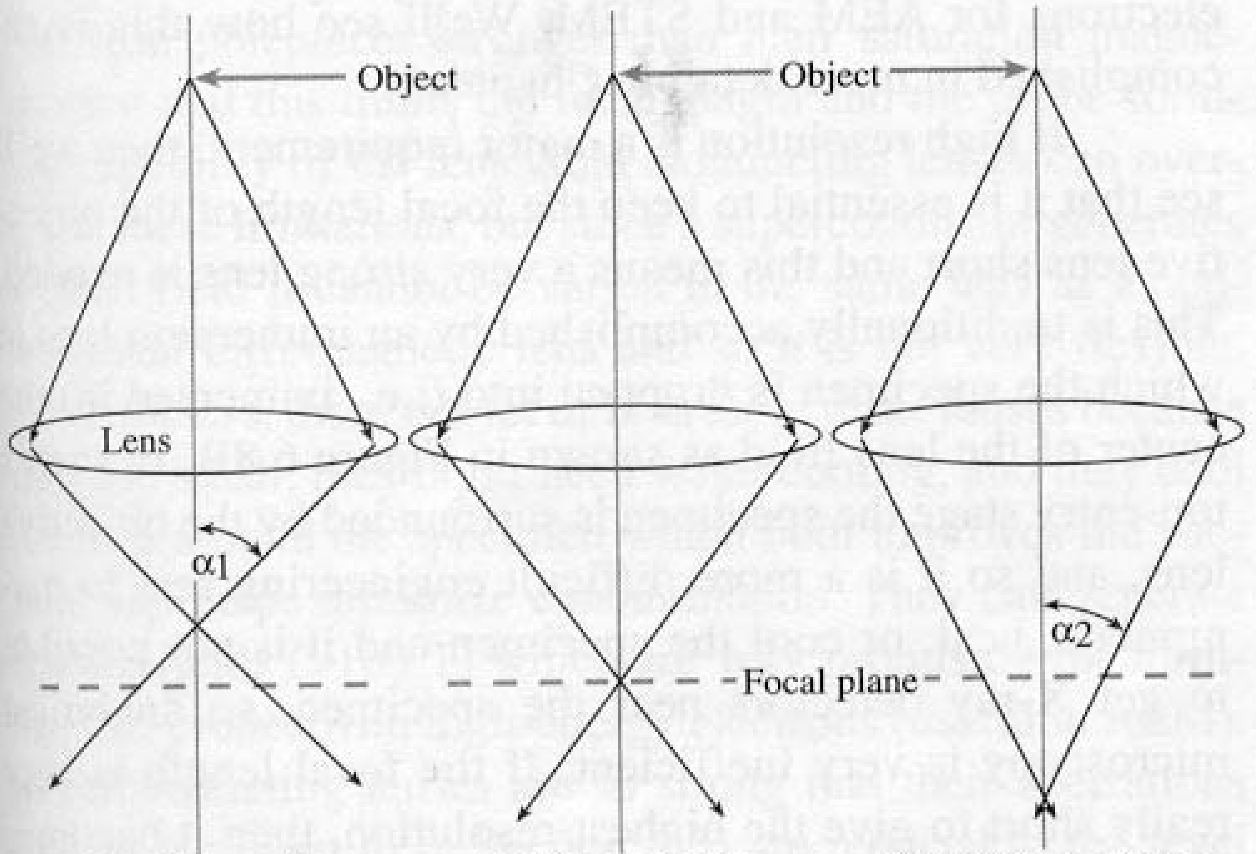


Figure 6.4. Strengthening the lens shortens the focal length *f*. So a weaker lens (*f1*) produces a higher magnification of the object than a stronger lens (*f2*) since the image distance *v* increases, but the object distance is unchanged.

Focus

If the lens is too weak and the image forms below the desired image plane, the image will be out of focus and the lens is said to be underfocused.

If the lens is too strong and the image forms above the image plane,
then we say the lens is overfocused.



a) Overfocused lens b) Focused lens c) Underfocused lens

Figure 6.5. (a) Ray diagram illustrating the concepts of overfocus, in which a strong lens focuses the rays before the image plane, and (c) underfocus, where a weaker lens focuses after the image plane. It is clear from (c) that at a given underfocus the convergent rays are more parallel than the equivalent divergent rays at overfocus ($\alpha_2 < \alpha_1$).

Electron Lenses

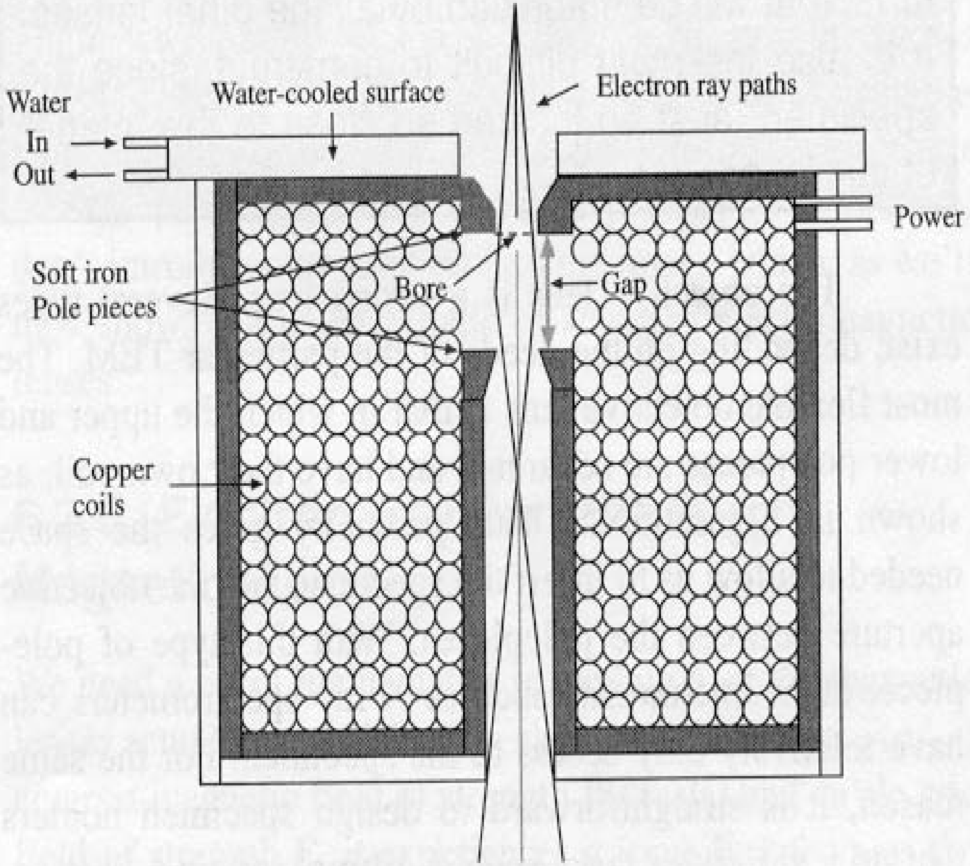


Figure 6.6. Schematic diagram of a magnetic lens. The pole pieces surround the coils and, when viewed in cross section, the bore and the gap between the polepieces are visible. The magnetic field is weakest on axis and increases in strength toward the side of the polepiece, so the electrons are more strongly deflected as they travel off axis.

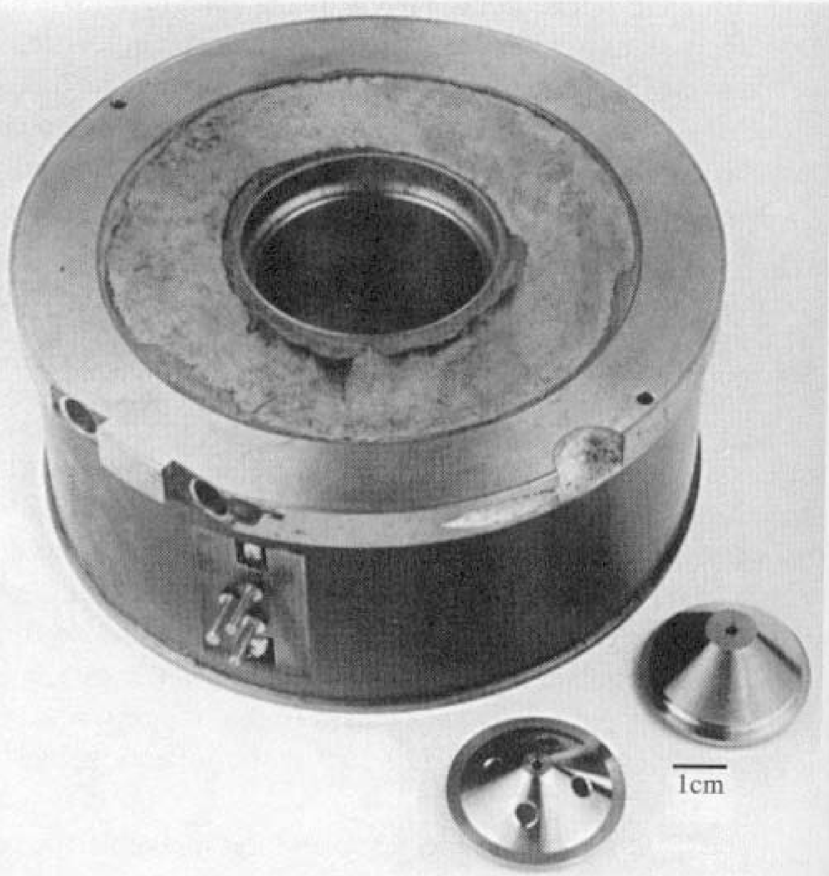


Figure 6.7. A real lens: the cylindrical shape conceals the copper wire coils. The two conical polepieces beside the lens sit inside the central hole in the lens. The three-pin electrical connections provide current to the coil to magnetize the polepieces, and cooling water is circulated in and out of the two holes in the top plate of the lens to dissipate the resistive heat generated.

Different Kinds of Lenses

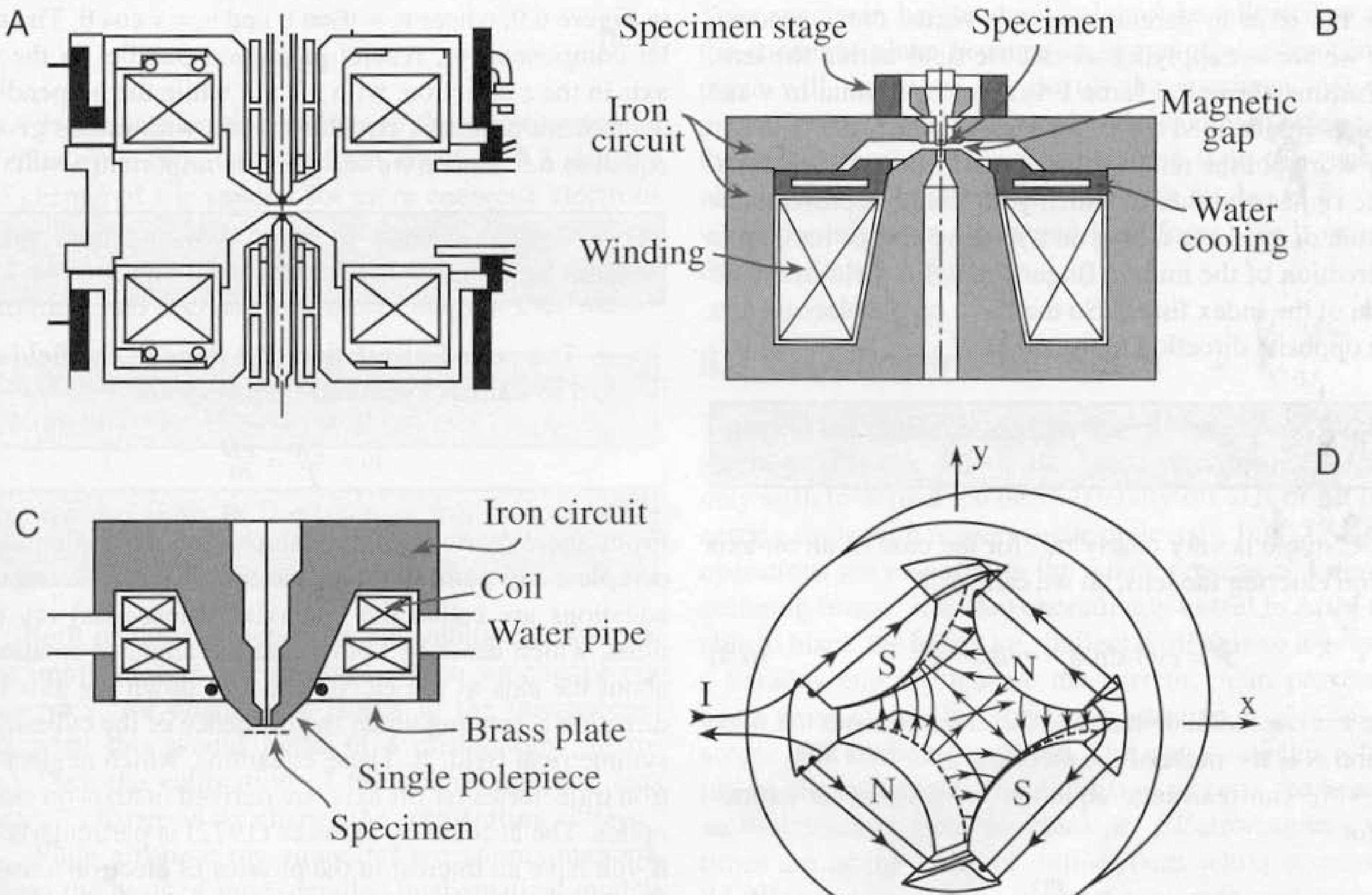


Figure 6.8. A selection of different lenses: (A) a split polepiece objective lens, (B) a top-entry immersion lens, (C) a snorkel lens, and (D) a quadrupole lens.

- The objective lens is the most important lens in the TEM, since it forms the images and diffraction patterns that will be magnified by all the other lenses. It is also the most difficult to construct, since the specimen must be located so close to the “plane” of this lens.

- The objective lens is a strong lens. The most flexible objective lens is that in which the upper and lower polepieces are separated and have their own coil, as shown in Fig. 6.8A. This geometry gives the space needed to allow us to insert the specimen and the objective aperture between the polepieces. With this type of polepiece, other instruments such as X-ray spectrometers can have relatively easy access to the specimen. For the same reason, it is straightforward to design specimen holders that do a variety of tasks such as tilting, rotating, heating, cooling, and straining.

- With split polepieces it is possible to make the upper polepiece behave differently than the lower polepiece. The most common application of this is to excite the upper objective polepiece very strongly. This kind of lens is ideal for an AEM/STEM because it can produce both the necessary broad beam of electrons for TEM and a fine beam of electrons for AEM and STEM.

- If high resolution is a major requirement, it is essential to keep the focal length of the objective lens short and this means a very strong lens is needed which can be accomplished by an immersion lens.

Image Rotation and the Eucentric Plane

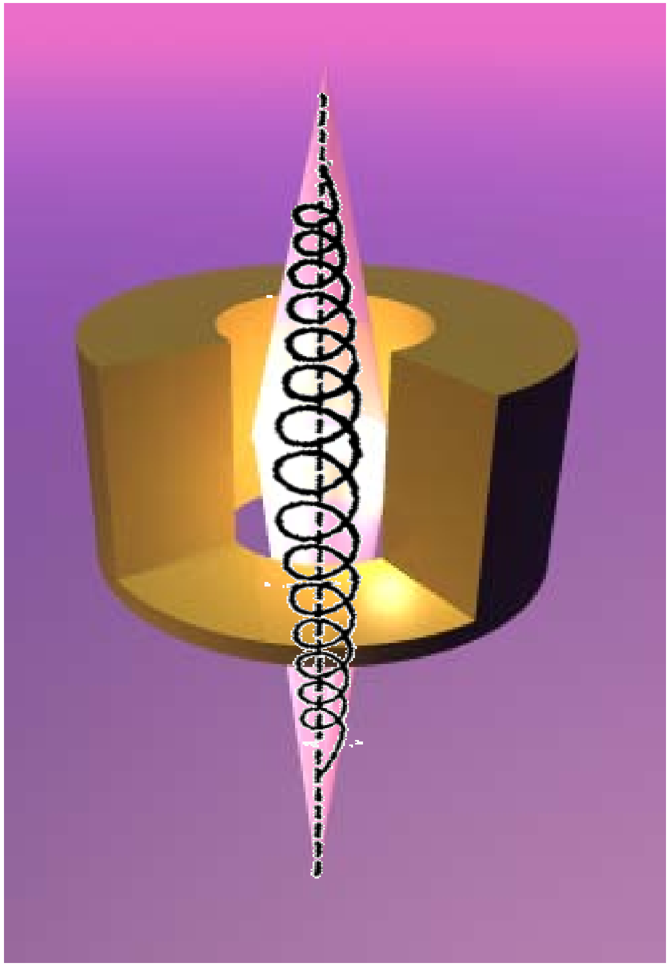
- The electrons follow a helical path as they traverse the field along the axis of the lens. Its effects are seen in the routine operation of the TEM because the image, or diffraction pattern, rotates on the viewing screen as you try to focus or if you change magnification. This rotation may require calibration. The manufacturer may have compensated for it by including an extra lens.

- Fig. 6.4 suggests that if we change the strength of the lens, the position of the focal plane and the image plane will also change. Because of this we have to define a standard object plane for the main imaging lens of the microscope and we call this the *eucentric* plane.

- Specimen height should always be adjusted to sit in the *eucentric* plane because an image of an object in this plane will not move as you tilt the specimen. All other planes in the imaging system are defined with reference to the *eucentric* plane. If your specimen is in the *eucentric* plane, then the objective lens strength is always the same when the image on the screen is in focus.

The Electromagnetic Lens

- The electrons move through the lens in a helical path, a spiral, not a straight line.
- One effect is that the image in an TEM will appear to rotate if you vary the accelerating voltage.



Apertures and Diaphragms

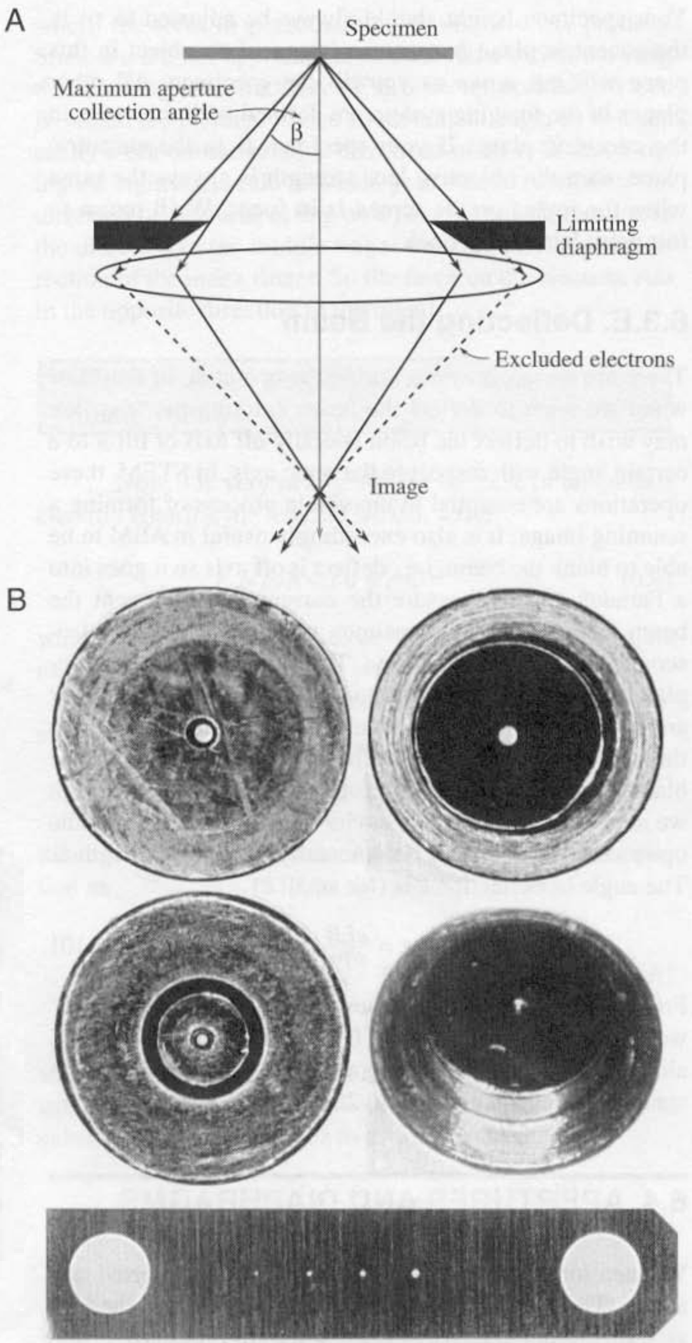


Figure 6.10. (A) Ray diagram illustrating how a diaphragm restricts the angular spread of electrons entering the lens. Only electron paths less than a semiangle β subtended by the aperture at the object are allowed through the lens (full ray paths). Electrons from the object scattered at angles >β are stopped by the diaphragm (dashed ray paths). (B) A selection of diaphragms: the top and middle left are upper and lower views, respectively, of a conventional objective diaphragm; the top/middle right are views of a "top-hat" (thick) C2 diaphragm; below is a metal strip containing several apertures. Each diaphragm is ~3 mm across.

Spherical Aberration, Chromatic Aberration, and Astigmatism

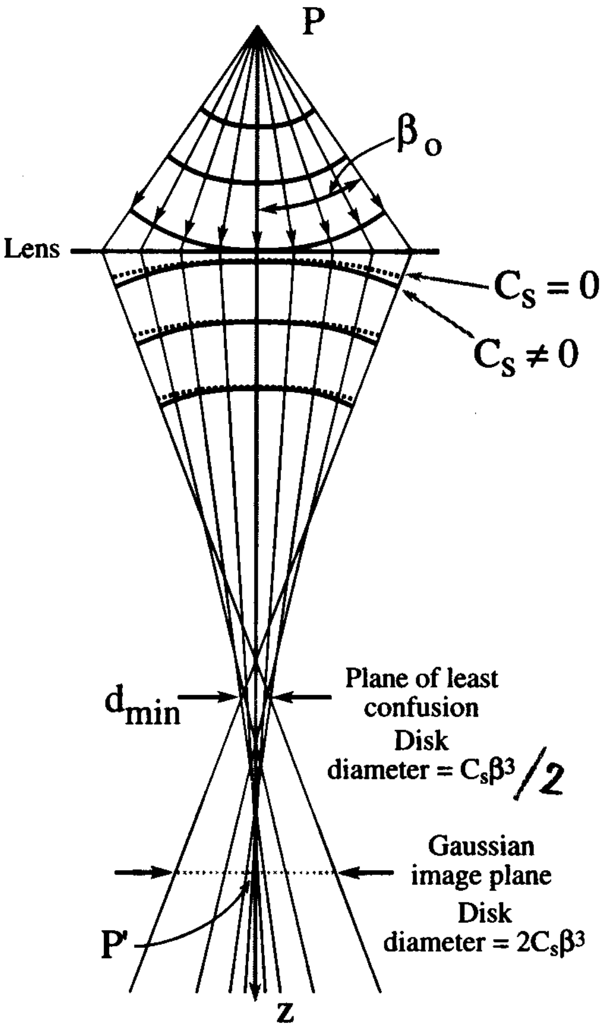


Figure 6.11. Spherical aberration in the lens causes wavefronts from a point object P to be spherically distorted. The point is thus imaged as a disk with a minimum radius in the plane of least confusion and a larger disc at P' in the Gaussian image plane.

Chromatic Aberration

The objective lens bends electrons of lower energy more strongly and thus electrons from a point in the object once again form a disk image. The radius r_{chr} of this disk is given by:

$$r_{chr} = C_c \Delta E \beta / E_o$$

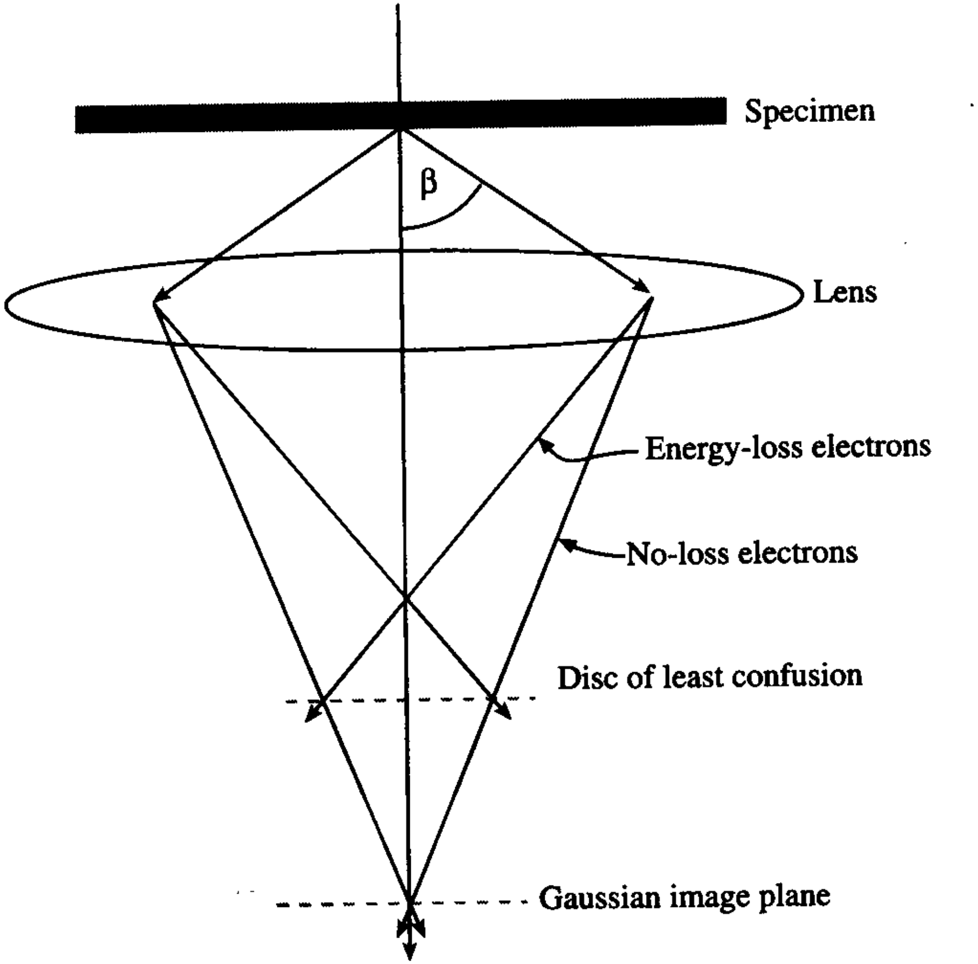


Figure 6.12. Chromatic aberration results in electrons with a range of energies being focused in different planes. Electrons emerging from the specimen with no loss of energy are less strongly focused than those that suffered energy loss in the specimen, so a point is imaged as a disk.

Astigmatism

- Astigmatism occurs when the electrons sense a nonuniform magnetic field as they spiral around the optic axis.

- This aberration arises because we can't machine the soft iron polepieces to be perfectly cylindrically symmetrical down the bore. The soft iron may also have microstructural inhomogeneities which cause local variations in the magnetic field strength. Even if these difficulties were overcome, the apertures we introduce into the lens may disturb the field if they are not precisely centered around the axis. Furthermore, if the apertures are not clean, the contamination charges up and deflects the beam. So there are a variety of contributions to astigmatism, which distorts the image by an amount r_{ast} where

$r_{ast} = \beta \Delta f$ and Δf is the maximum difference in focus induced by the astigmatism.

Resolution

The resolution is defined as the “minimum” resolvable distance in the object. The theoretical resolution is expressed as the distance apart of the two incoherent point sources is defined as the theoretical resolution of the lens r_{th} and is given by the radius of the Airy disk:

$$r_{th} = 0.61\lambda/\beta$$

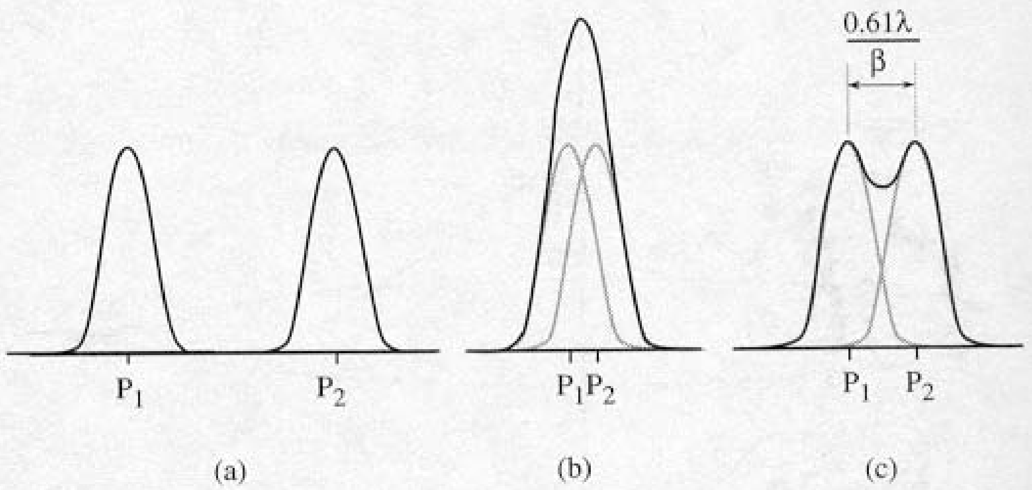
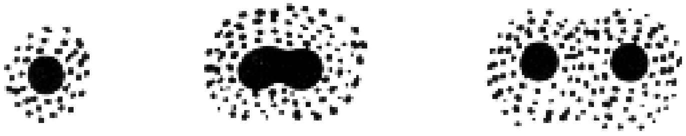
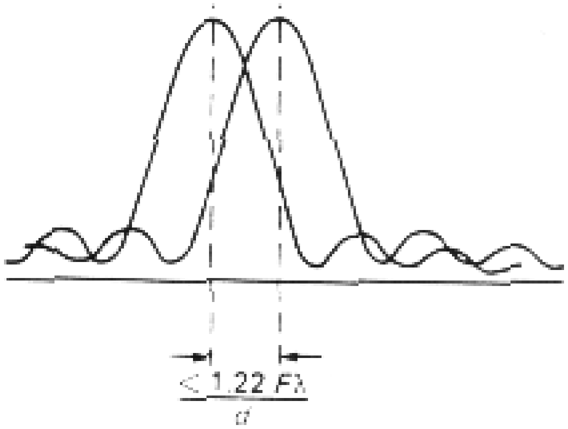


Figure 6.13. (a) The Airy disk intensity profile from two point sources P_1 and P_2 defines the resolution of a lens. In (b) the two Airy disks are so close that they cannot be distinguished, but in (c) the two are separated such that the maximum in the image of P_1 overlaps the minimum in P_2. This is the definition of resolution defined by the Rayleigh criterion.

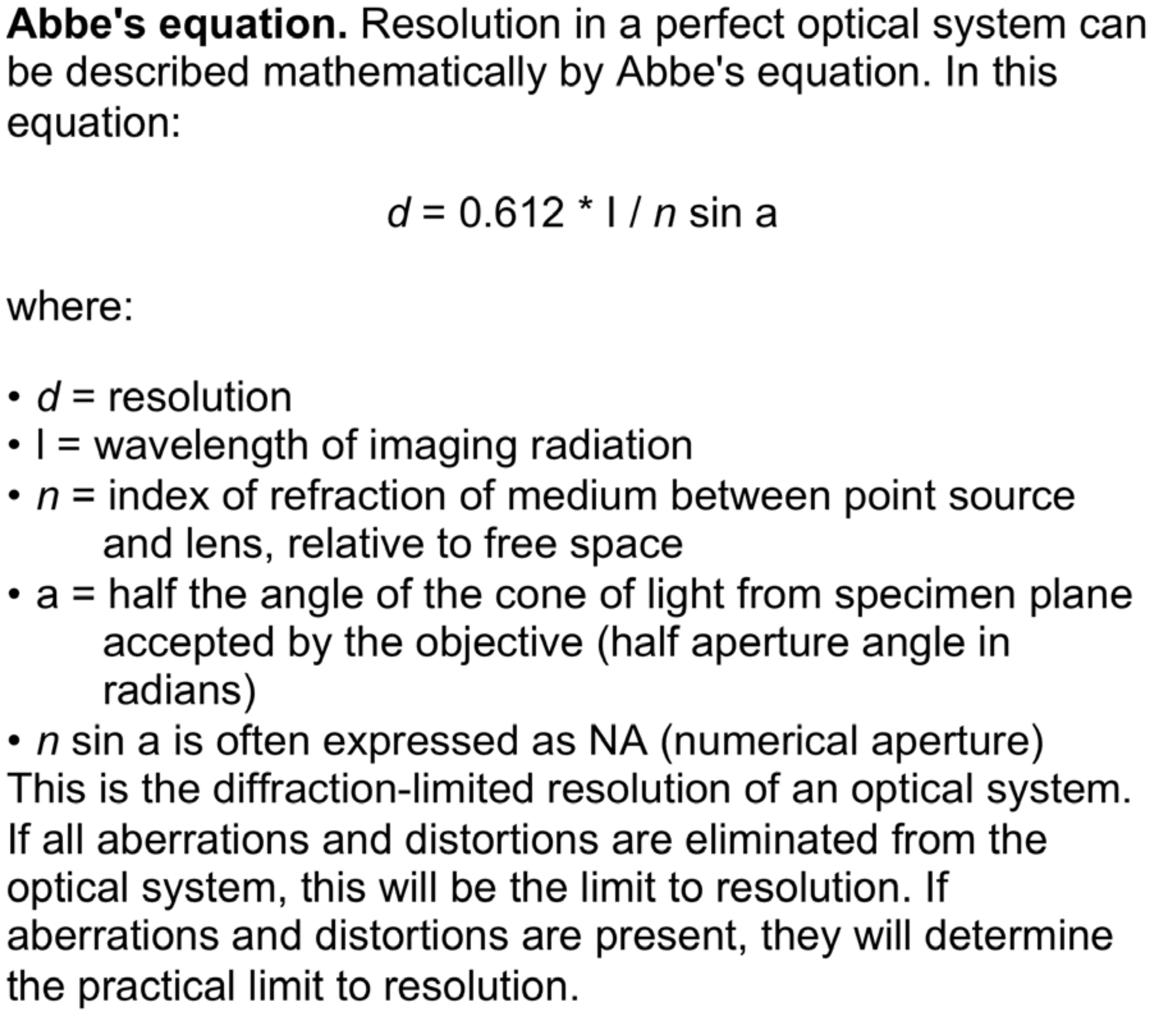
Resolution

Resolution. Notice the primary, secondary and tertiary wavefronts generated by the Airy disc. (Of course, these continue to emanate at higher orders, but their affect on optical phenomena diminishes in importance with each higher order.) The resolution is typically described as the distance between the first order peak and the first order trough (designated "r" above). Resolution is empirically described as the ability to discriminate between two points. If an object is just below the level of resolution, the peaks generated by the two points will make the object appear to be a single point.





Resolution – Abbe’s Equation



Abbe's equation. Resolution in a perfect optical system can be described mathematically by Abbe's equation. In this equation:

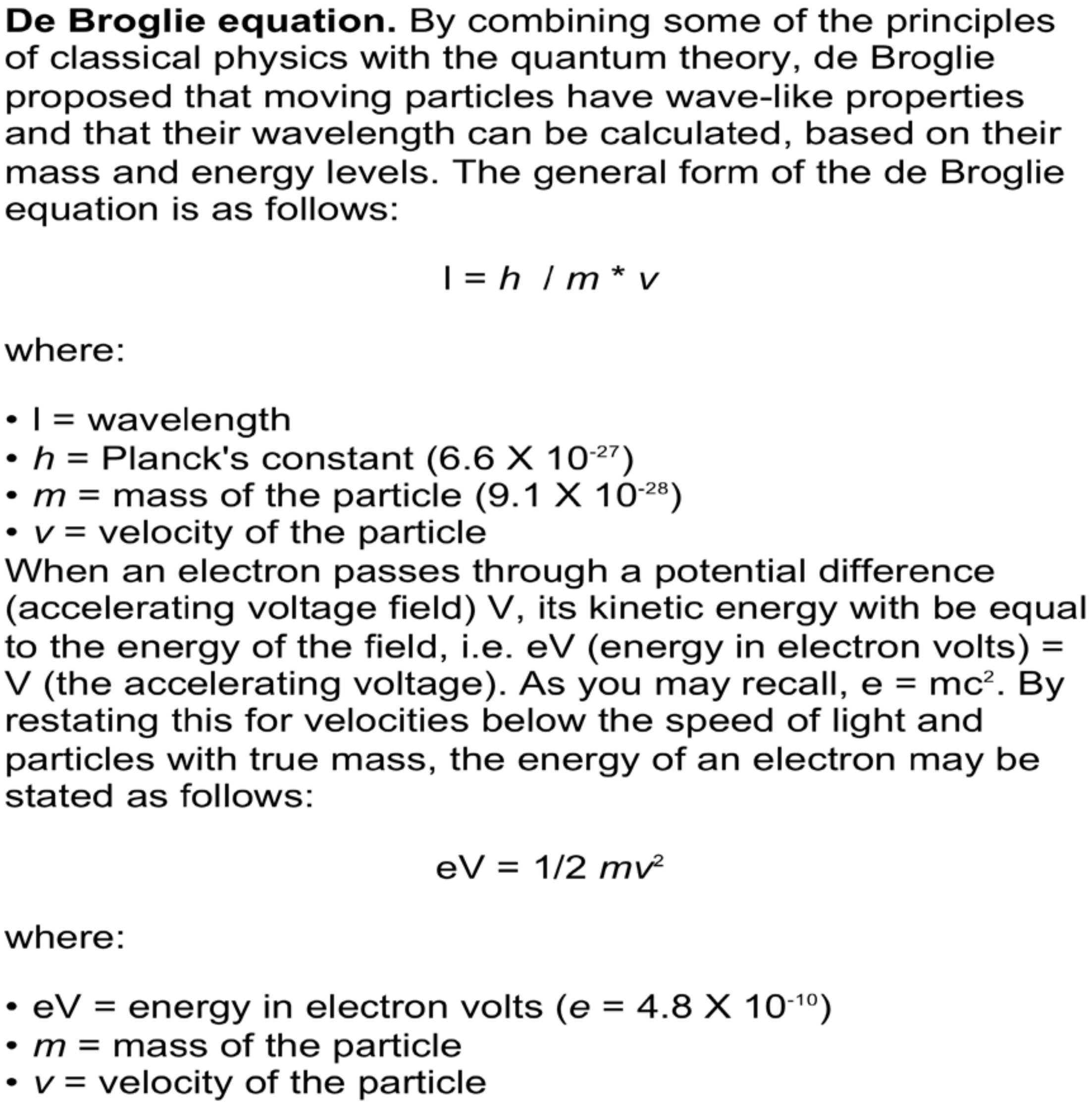
$$d = 0.612 * l / n \sin a$$

where:

- *d* = resolution
- l = wavelength of imaging radiation
- *n* = index of refraction of medium between point source and lens, relative to free space
- a = half the angle of the cone of light from specimen plane accepted by the objective (half aperture angle in radians)
- *n* sin a is often expressed as NA (numerical aperture)

This is the diffraction-limited resolution of an optical system. If all aberrations and distortions are eliminated from the optical system, this will be the limit to resolution. If aberrations and distortions are present, they will determine the practical limit to resolution.

De Broglie Equation



De Broglie equation. By combining some of the principles of classical physics with the quantum theory, de Broglie proposed that moving particles have wave-like properties and that their wavelength can be calculated, based on their mass and energy levels. The general form of the de Broglie equation is as follows:

$$l = h \ / m * v$$

where:

- l = wavelength
- h = Planck's constant (6.6×10^{-27})
- m = mass of the particle (9.1×10^{-28})
- v = velocity of the particle

When an electron passes through a potential difference (accelerating voltage field) V, its kinetic energy with be equal to the energy of the field, i.e. eV (energy in electron volts) = V (the accelerating voltage). As you may recall, $e = mc^2$. By restating this for velocities below the speed of light and particles with true mass, the energy of an electron may be stated as follows:

$$eV = 1/2 \, mv^2$$

where:

- eV = energy in electron volts ($e = 4.8 \times 10^{-10}$)
- m = mass of the particle
- v = velocity of the particle

De Broglie Equation - Continue

By using some assumptions about the velocity of the particle and its mass, it is possible to express either wavelength (l) or velocity (*v*) in terms of the accelerating voltage (V). By further substituting the values of *h* and *m* above, the equation for l reduces to the following:

$$l = 1.23 \text{ nm} / V^{1/2}$$

One caveat is that as the velocity of the electron approaches the speed of light, Einstein's special equations of relativity need to be used for greater accuracy as the mass and momentum of electrons increases with velocity.

Equation for resolution in TEM: This value for l can then be substituted into Abbe's equation. Since angle a is usually very small, for example 10^{-2} radians (a likely figure for TEM), the value of a approaches that of sin a , so we replace it. Since n (refractive index) is essentially 1, we eliminate it, and we multiply 0.612 by 12.3 to obtain 0.753. Therefore, the equation reduces to the following:

$$d = \frac{0.753}{a\,V^{1/2}}$$

where:

- *d* = resolution in nm
- a = half aperture angle
- V = accelerating velocity

Now, solving for 100,000 volts, the result is 0.24 nm or 2.4 Å. This improves with higher accelerating voltage and gets worse with lower voltages. (Using Einsteinian calculations, the resolution is: 0.22 nm or 2.2 Å.) Each lens and aperture has its own set of aberrations and distortions. If aberrations and distortions are present, they will determine the practical limit to resolution.