TIN & Surface Interpolation

Input point coverage

Thiessen polygon coverage

Bisected TIN

TRN
Triangulated Irregular Network (TIN)

- Node \((x, y, z)\)
- Edge
- Triangles
- Topology
  - The triangle number
  - The numbers of each adjacent triangle
  - The three nodes defining the triangle
  - The \(x, y\) coordinates of each node
  - The surface \(z\) value of each node
  - The edge type of each triangle edge (hard or soft)

Conversions between Surface Models

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Static vs. Dynamic Triangulation

- Static: all data are used in a single-pass process
- Dynamic: data are used in a multiple-pass process (i.e., subsequent editing of a TIN)

Ways of Forming Triangles

Basic requirements:
1. Unique formation
2. Equilateral triangles
3. Minimal edge length
TIN Formation

- Delaunay criterion (empty circumcircle principle): the circumscribing circle of any triangle contains no other points of data except the three defining it.

- Local equiangularity (max-min angle principle): minimizing the range of interior angles in the two triangles that form a convex quadrilateral (through Lawson’s Local Opt. Procedure).

- Minimum sum-distance principle:
- Minimum circumscribing circle radius:
- ...

Static Triangulation of Points

Algorithm:
1. Select an initial point
2. Search for the first edge
3. Form the first triangle (nearest point to the edge)
4. Search for a subsequent edge
5. Form a subsequent triangle
6. Repeat 4, 5 until all points become nodes of triangles
Searching for a Node Point

(a)                  (b)

Searching for a Subsequent Edge

Convex hull algorithm

Imaginary bnd algorithm
Constrained Triangulation

- Random points and feature-specific points of a DTM

- Approaches:
  - Point densification of features
  - Constrained Delaunay Triangulation

Triangulation from Contours

- Point densification of contours
- Contours as soft breaklines
Triangulation from DEM

- Selection of significant points from DEM
  - Very Important Points (VIP)
  - Maximum z-tolerance (d)

- Triangulation from VIP

Delineate TIN Data Area – ArcGIS 3D Analyst Tool
TIN model

TIN model with pixel center points
Spatial Interpolation

- Methods to construct the surface model
- Global vs. local
- Exact vs. best fit
- Information lost at the sampling stage can never be reconstructed (?)
Local Method

Neighbors
Distribution of control points
Extent of spatial autocorrelation

(a) find the closest points to the point to be estimated, (b) find points within a radius, and (c) find points within each of the four quadrants.

Distance Weighted Interpolation

• Neighbor points with observations
• Weighting functions

\[ z = \frac{\sum_{i=1}^{s} z_i \frac{1}{d_i^k}}{\sum_{i=1}^{s} \frac{1}{d_i^k}} \]
Types of Local Spatial Interpolation

Figure 6.11 Different types of moving surfaces for interpolation: (a) nearest; (b) averaging; (c) linear surface; and (d) curved surface.

Kriging

\[ \hat{Z}(s_0) = \sum_{i=1}^{N} \lambda_i Z(s_i) \]
DEM Resampling

• Down-scale (Fine to coarse)
  – Treat output grid as polygons
  – Data aggregation

• Up-scale (Coarse to fine)
  – Treat DEM as point data
  – Interpolation

Spatial Interpolation with Sparse Sample Points

• Convert contours to DEM
• Generate DEM from transects
Contours to DEM

Densification of Sample Points
Densification of Sample Points