- **Numeric Terrain Descriptor**
- **Fractal Geometry**
  - Introduced by Mandelbrot in 1981 to describe dimensionality that defies Euclidean geometry
    - "Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line."(Mandelbrot, 1983).
  - Derived from Latin *fractus,* meaning "broken" or "fractured"

Characteristics of Fractals

- Self-Similarity: small details viewed at any scale repeat elements of the overall pattern.
  - All scales equivalent; objects at different scales are indistinguishable
- Length of a curve is a function of measurement unit
  - Complex objects take up more space at finer scales and as finer details are measured
- Dimension of fractals exceeds topological dimension

Examples:
- Coastlines
- Mountainous Landscapes
- Tree Morphology

Fractal Dimension

- A statistical quantity of how completely a fractal appears to fill a space or how many points are in a given set
  - Line>>>plane
  - Plane>>>space
- The fraction describes the degree of perturbation from ideal (Euclidean) geometry

How Long is Britain's Coastline?

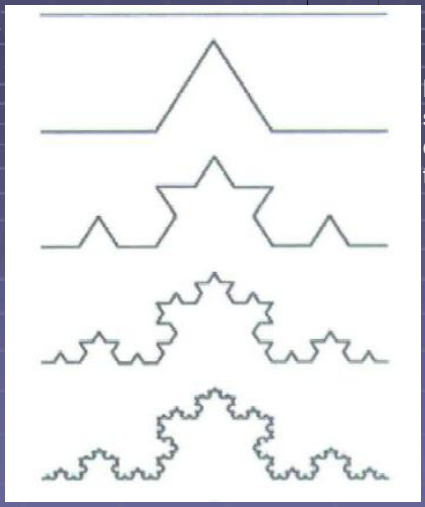
Unit = 200 km
length = 2400 km

Unit = 100 km
length = 2800 km

Unit = 50 km
length = 3400 km

The measured length increases without limit as the measurement scale decreases towards zero.

The Koch Curve



A line segment..

Broken into 4 self-similar segments, each 1/3 the size of the original

The process is repeated

After 5 iterations, the complexity increases. Process can be repeated into infinity …

Calculating the Fractal Dimension Of The Koch Curve

D = log N / log S

N=number of self-similar pieces
S=scale to which the small pieces compare to the original one
D=Fractal dimension

So: **Log 4/Log 3 = A fractal dimension of 1.231**

*The higher the number, the more complex the feature it describes.

Applications

- Modeling and analysis of earth's surface
- Modeling intercepted snow accumulation in forests
- Forestry research: crown analysis
- Measurement of coastlines or any features which are jagged, fragmented or discontinuous
- Digital image texture description and segmentation

Questions

- Describe self-similarity.
- True or False: Given the proper size measuring device, an accurate measurement of the coast of Britain is possible.
- True or False: In fractal theory, the length of a curve is function of the measurement unit.
- What are some applications of fractal geometry?

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