

Spatial Autocorrelation: Covariance and Semivariance

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Quantitative Terrain Descriptors

Covariance and Semivariogram are *numeric* methods used to describe the character of the terrain (ex. Terrain roughness, irregularity)

Terrain character has important implications for:

1. the type of sampling strategy chosen
2. estimating DTM accuracy (after sampling and reconstruction)

Spatial Autocorrelation

The First Law of Geography

“Everything is related to everything else, but near things are more related than distant things.” (Waldo Tobler)

The value of a variable at one point in space is related to the value of that same variable in a nearby location

Ex. Moran's I, Geary's C, LISA

Positive Spatial Autocorrelation (Neighbors are similar)

Negative Spatial Autocorrelation (Neighbors are dissimilar)

$$R(d) = \frac{\text{Cov}(d)}{V}$$

R(d) = correlation coefficient of all the points with horizontal interval (d)

Covariance

The degree of similarity between pairs of surface points

The value of similarity is an indicator of the complexity of the terrain surface

Smaller the similarity = more complex the terrain surface

$$V = \frac{\sum_{i=1}^N (Z_i - M)^2}{N - 1}$$
$$\text{Cov}(d) = \frac{\sum_{i=1}^N (Z_i - M)(Z_{i+d} - M)}{N - 1}$$

V = Variance calculated from all N points

Cov (d) = Covariance of all points with horizontal interval d

Z_i = Height of point i

M = average height of all points

Z_{i+d} = elevation of the point with an interval of d from i

Semivariance

Expresses the degree of relationship between points on a surface

Equal to half the variance of the differences between all possible points spaced a constant distance apart

$$\gamma(h) = \frac{1}{2}[z(x_i) - z(x_j)]^2$$

As points are compared to increasingly distant points, the semivariance increases

If there is strong spatial dependence, points that are closer together will have a smaller semivariance

Semivariogram

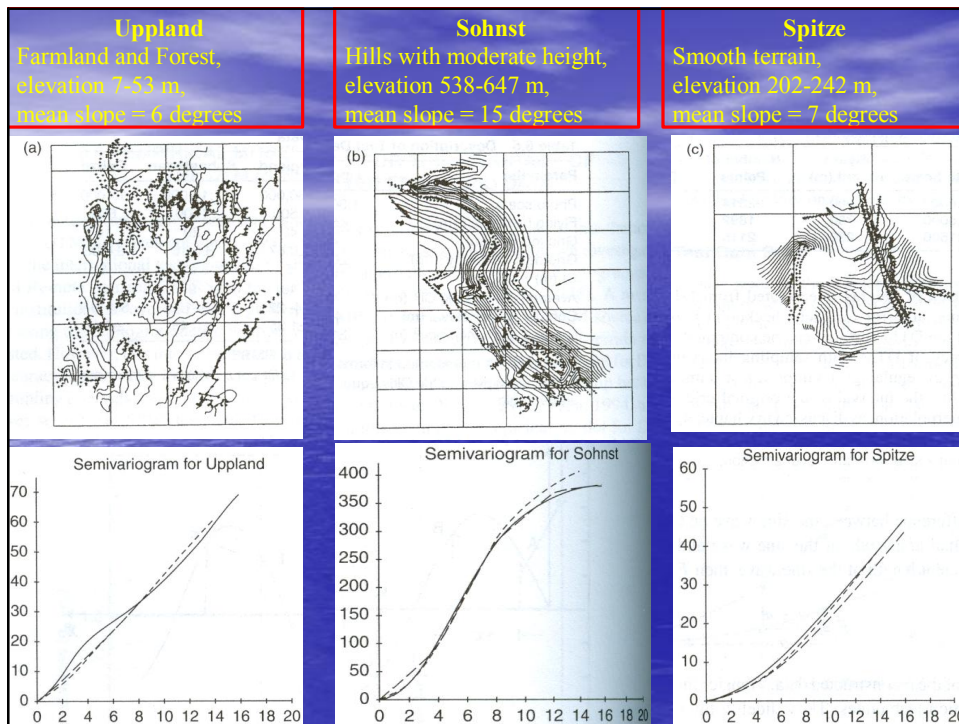
A plot of semivariance as a function of distance between the observation

Used in kriging to calculate estimates of the surface at specified locations

Measures the variability of data with respect to spatial distribution(quantifies similarity/disimilarity)

Analyzes variance between pairs of data points over a range of distances

$$2\gamma(d) = \frac{\sum_{i=1}^N (Z_i - Z_{i+d})^2}{N}$$



Quiz Questions

1. T/F Positive Autocorrelation means that neighboring points have similar elevations.
2. Explain how knowing the covariance and/or semivariance can help describe the terrain's surface.
3. Covariance and semivariograms are _____ descriptors of terrain character.
4. T/F One difference between covariance and semivariogram is they show opposite responses as the horizontal distance (d) increases.

References

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