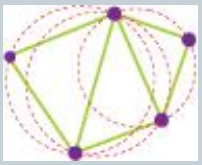
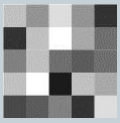
Delaunay Triangulation

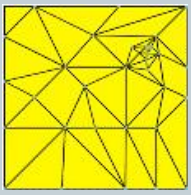


... a technique for creating a mesh of contiguous, nonoverlapping triangles from a dataset of points

Boris Nikolaevich Delaunay

Triangulated Irregular Network





A TIN surface is a good way to represent surface morphology.

It is a vector based model constructed by triangulating a set of points.

Terrain is modeled as a large network of connecting non-overlapping triangles

TIN in Arc

- In initial creation all points need to have a X,Y,Z value
- CREATETIN command generates a TIN from ARC/INFO coverages, ASCII files, Sample Points, DEM Rasters using the Delaunay Method (specific algorithms)
  - -Breakline features influence triangulation (hard, soft, fault)
  - - COVER, GENERATE, LATTICE are all methods of how TIN Z values are interpreted.

Where to Start?

Delaunay Triangulation is used to create a TIN surface from a collection of points

The geometric center of the data points (elfick 1979)

The shortest of all possible lines between any two data points (yeoli 1977)

A line segment on the imaginary boundary (McCullagh and Ross 1980)

A line segment on the boundary convex hull (Tsai 1993; Gosper 1998)

Choosing a starting point and a search principle (to determine other triangles) is specified the triangulation is relatively easy.

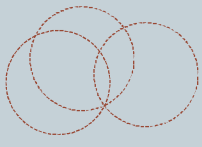
Where to go Next?

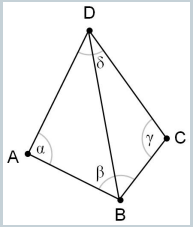
- Bowyer- Watson Algorithm for Dynamic Triangulation
  - - triangulation by gradually adding more points, simple incremental algorithm
  - - starts with large course triangles and continually add points/ breaklines
  - - after each additional point is added the triangle is checked to make sure it meets Delaunay criteria
  - -- if NOT it uses Edge Swapping

Criteria and Constraints

- No vertex lies within the interior of any of the circumcircles of the triangles in the network.
- The minimum interior angle of all triangles is maximized, and the maximum minimized. Triangles are as equi-angular as possible.

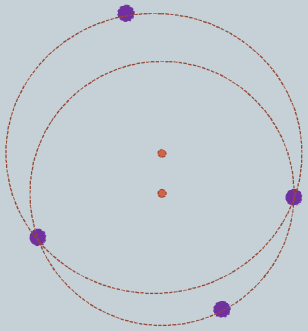
The result is that long, thin triangles are avoided as much as possible.





Circumcircles

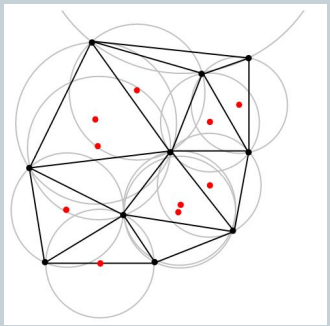
Circumcircle is the center of a polygon that passes through all vertices of a polygon.



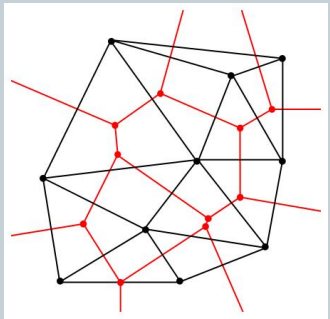
For a set P of points in the plane is a triangulation DT(P) so that no point in P is inside the circumcircle of an triangle in DT(P)

-mathworks.com

Delaunay and Veronoi

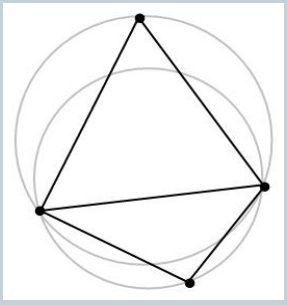


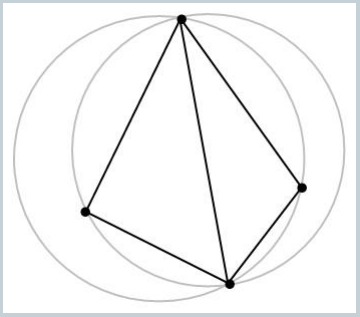
Connected vertices



Connected circumcircles

Angles





The minimum interior angle of all triangles is maximized.
The maximum interior angle of all triangles in minimized.

Which image satisfies the Delaunay Crietria?

Advantages To TIN

- An advantage of Delaunay triangulation is independent of the order the points are processed.

  Allows for editing

- Because nodes can be placed irregularly over a surface, TINs can have a higher resolution in areas where a surface is highly variable or where more detail is desired and a lower resolution in areas that are less variable.

  High Resolution

- The input features used to create a TIN remain in the same position as the nodes or edges in the TIN. This allows a TIN to preserve all the precision of the input data while simultaneously modeling the values between known points. You can include precisely located features on a surface—such as mountain peaks, roads, and streams—by using them as input features to the TIN nodes.

  -ESRI Support

  Precise Location

Disadvantages and Application

- TIN models are less widely available than raster surface models and are more time consuming to construct and process- it is a highly complex data structure.

- Typically used for high precision modeling of small areas

TIN Aspect- Derives aspect from a TIN
TIN Contour- Creates Isolines
TIN Slope- Derives slope
TIN Difference- Calculates volumetric differences in a TIN
TIN Polygon Volume- Calculates the volumetric and surface area between polygons of an input feature class and TIN surface

References

- About 3D surfaceshttp://webhelp.esri.com/
- Delaunay triangulation, www.mathworks.com
- Li, Z., Zhu, Q, and Gold, C. 2004. *Digital Terrain Modeling: Principles and Methodology*. CRC Press.