Spatial Interpolation

• What is spatial interpolation?
  – Estimate values
  – Converting point data to surface data
  – Converting line data to surface data (contours to DEM)
  – Converting area data to surface data (areal interpolation)

• Observations (control points) and interpolator

• Interpolators
  – Global / Local
  – Exact / Approximate
  – Stochastic / Deterministic
  – Geostatistical

Global/Local Methods

• Global methods
  – Trend surface analysis (Global polynomial interpolation)

• Local methods
  – IDW
  – Local polynomial interpolation
Local Method

- Neighbors
  - Distribution of control points
  - Extent of spatial autocorrelation

(a) find the closest points to the point to be estimated, (b) find points within a radius, and (c) find points within each of the four quadrants.

**IDW**

\[ z = \frac{\sum_{i=1}^{s} \frac{z_i}{d_i^k}}{\sum_{i=1}^{s} \frac{1}{d_i^k}} \]

<table>
<thead>
<tr>
<th>Point</th>
<th>Z</th>
<th>D</th>
<th>Z*1/D^k</th>
<th>1/D^k</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>10</td>
<td>8</td>
<td>1.25</td>
<td>0.125</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>2</td>
<td>2.5</td>
<td>0.5</td>
</tr>
<tr>
<td></td>
<td>Sum=3.75, 0.625</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>X</td>
<td>1</td>
<td></td>
<td>Z=6</td>
<td></td>
</tr>
</tbody>
</table>

A (10)       X (?)       B (5)
8
2
Geostatistical / Simulation Interpolation

• Geostatistical estimation (Kriging)

\[ \hat{Z}(s_0) = \sum_{i=1}^{N} \lambda_i Z(s_i) \]

• Stochastic simulation, conditional to:
  1. Observed data values at their locations
  2. The histogram of observed data set
  3. The semivariance model of observed data set

(SOURCE: Goovaerts 1997)

Spline

Produces a continuous surface with minimum curvature.
Steps of Geostatistical Interpolation

1. Calculating the empirical semivariogram
2. Fitting a model (modeled semivariogram)
3. Creating the (inverse) gamma matrix
4. Making a prediction
5. Repeat steps 3, 4 for each location to create a surface

Quiz

In the semivariogram above,
1. A is referred to as: a) cookie, b) sill, c) nugget, d) range.
2. B is referred to as: a) cookie, b) sill, c) nugget, d) range.
3. C is referred to as: a) cookie, b) sill, c) nugget, d) range.
Kriging

$$\hat{Z}(s_i) = \sum_{i=1}^{N} \lambda_i Z_i$$

- **Semivariance**
- **Sill**
- **Nugget**
- **Range**
- **Empirical Semivariance**
- **Modeled Semivariance**

**Empirical Semivariogram**

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

The empirical semivariance is

$$0.5 \times \text{average}((\text{value at location } i - \text{value at location } j)^2)$$

**Values:**

- (1, 5) = 100
- (3, 4) = 105
- (1, 3) = 105
- (4, 5) = 100
- (5, 1) = 115

**Binning the Empirical Semivariogram**

<table>
<thead>
<tr>
<th>Lag Distance</th>
<th>Pairs Distance</th>
<th>Av. Distance</th>
<th>Semivariance</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>1+2</td>
<td>2.236, 2.236</td>
<td>2.707</td>
<td>12.5</td>
<td>12.5</td>
</tr>
<tr>
<td>2+3</td>
<td>2.236, 2.236</td>
<td>2.491</td>
<td>12.5, 0, 5</td>
<td>4.167</td>
</tr>
<tr>
<td>3+4</td>
<td>3.666, 3.666</td>
<td>3.666</td>
<td>50, 12.5</td>
<td>31.25</td>
</tr>
<tr>
<td>4+5</td>
<td>4.472, 4.123</td>
<td>4.296</td>
<td>50, 112.5</td>
<td>81.25</td>
</tr>
<tr>
<td>5+</td>
<td>5.667</td>
<td>5.667</td>
<td>112.5</td>
<td>112.5</td>
</tr>
</tbody>
</table>
Fit a Model

Some mathematical models for fitting semivariograms:
Gaussian, linear, spherical, circular, and exponential.
Combining Variogram Models

Modeled Semivariogram

Spherical model

\[ \gamma(h) = \begin{cases} \theta_s \left[ \frac{3}{2} \frac{h}{\theta_r} - \frac{1}{2} \left( \frac{h}{\theta_r} \right)^3 \right] & \text{for } 0 \leq h \leq \theta_r \\ \theta_s & \text{for } \theta_r < h \end{cases} \]

where

- \( \theta_s \) is the sill value,
- \( h \) is the lag vector, and \( h \) is the length of \( h \) (distance between 2 locations),
- \( \theta_r \) is the range of the model.
Making a Prediction

Semivariance = 13.5 * h

Kriging Weights = g * Inverse of Distance Matrix

Kriging Variance

<table>
<thead>
<tr>
<th>G Vector</th>
<th>Weights (λ)</th>
<th>g Vector Times Weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>13.5</td>
<td>0.46757</td>
<td>6.312195</td>
</tr>
<tr>
<td>27.0</td>
<td>0.09834</td>
<td>2.65518</td>
</tr>
<tr>
<td>13.5</td>
<td>0.46982</td>
<td>6.34257</td>
</tr>
<tr>
<td>42.69</td>
<td>-0.02113</td>
<td>-0.90204</td>
</tr>
<tr>
<td>67.5</td>
<td>-0.0146</td>
<td>-0.9855</td>
</tr>
<tr>
<td>1</td>
<td>-0.18281</td>
<td>-0.18281</td>
</tr>
</tbody>
</table>

Kriging Variance = 13.2396
Kriging Std Error = 3.6386
Cross-Validation

For all points, cross-validation sequentially omits a point, predicts its value using the rest of the data, and then compares the measured and predicted values.

Kriging Methods

- Simple Kriging (surface with a constant mean)
- Ordinary Kriging (surface with local means)
- Universal Kriging (surface with a trend)
- Indicator Kriging (categorical surface)
- Co-Kriging (Kriging with a secondary variable)
Directional Semivariogram

Anisotropy and Directional Semivariograms
Semivariogram Surface

Spatial Interpolation with Sparse Sample Points

- Convert contours to DEM
- Generate DEM from transects
Contours to DEM

Densification of Sample Points
Densification of Sample Points

3D Kriging

- 3D data sources (x, y, z and value)
- Multiple semivariograms are needed
- Anisotropy: azimuth and dip
- Different data resolutions (z usually has a higher resolution)
- Visualization of results (e.g., slicing)
- GSLib (http://www.gslib.com/)

[Diagram showing 3D data sources and visualization of results]