Fuzzy Classification

Hard- versus soft-classifiers

Why use soft-classifiers?
- Sub-pixel classification
- Uncertainty of classification/scheme
- Incorporating ancillary data (hardeners)

Fuzzy Classification Steps

- Classification scheme
- (Fuzzy) signatures
- Fuzzy classifiers
- Hardener (defuzzification)
- Classification uncertainty
- Classification accuracy
Fuzzy Classification Scheme

(Fuzzy) Signatures

- Training sites (homogeneous vs. fuzzy)

<table>
<thead>
<tr>
<th></th>
<th>Water</th>
<th>Forested Wetland</th>
<th>Upland Forest</th>
<th>Sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Site#1</td>
<td>0.7</td>
<td>0.2</td>
<td>0.1</td>
<td>1</td>
</tr>
<tr>
<td>Site#2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.6</td>
<td>1</td>
</tr>
<tr>
<td>Site#3</td>
<td>0.5</td>
<td>0.1</td>
<td>0.4</td>
<td>1</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Fuzzy Classifiers

- Bayesian Probability Theory (BAYCLASS)
- Dempster-Shafer Theory (BELCLASS)
- Fuzzy Set Theory (FUZCLASS)
- Linear Mixture Model (UNMIX)

BAYCLASS

- Based on the Bayesian prob. of the hypothesis being true given the evidence.
- Hard classification scheme and hard signatures

BAYCLASS and MAXLIKE:
- Maximum likelihood classifier (hard-classifier)
- BAYCLASS classifier (soft-classifier)
- MAXLIKE = Hardened BAYCLASS
BELCLASS & Dempster-Shafer Theory

- Dempster-Shafer Theory: Dealing with decisions made under partial information (i.e., the presence of unknown classes).
- Belief: the degree to which evidence provides concrete support for an hypothesis.
- Plausibility: the degree to which the evidence does not refute that hypothesis.
  \[ \text{PL}(A) = 1 - \text{BEL}(\bar{A}); \text{ where } (\bar{A}) \text{ is not } (A) \]
- Belief interval = ABS( Belief – Plausibility)
  - High uncertainty: Belief Interval -> 1
  - Low uncertainty: Belief Interval -> 0

Fuzzy Set Theory Classifier

An image with \( m \) classes and \( n \) pixels, its fuzzy partition matrix is:

\[
\begin{bmatrix}
    f_{F_1}(x_1) & f_{F_1}(x_2) & \ldots & f_{F_1}(x_n) \\
f_{F_2}(x_1) & f_{F_2}(x_2) & \ldots & f_{F_2}(x_n) \\
    \vdots & \vdots & \ddots & \vdots \\
f_{F_m}(x_1) & f_{F_m}(x_2) & \ldots & f_{F_m}(x_n)
\end{bmatrix}
\]

\( 0 \leq f_{F_i}(x) \leq 1 \)

\[ \sum_{x \in X} f_{F_i}(x) > 0 \]

\[ \sum_{i=1}^{n} f_{F_i}(x) = 1 \]

Fuzzy signature components: mean and covariance matrix

\[ \mu^{*} = \frac{\sum_{i=1}^{n} f_{x_i}(x_i) x_i}{\sum_{i=1}^{n} f_{x_i}(x_i)} \]

\[ V^{*} = \frac{\sum_{i=1}^{n} f_{x_i}(x_i - \mu^{*})(x_i - \mu^{*})^T}{\sum_{i=1}^{n} f_{x_i}(x_i)} \]

\[ f_{x}(x) = \frac{P_{x}^{*}(x)}{\sum_{i=1}^{n} P_{x}^{*}(x)} \]

where

\[ P_{x}^{*}(x) = \frac{1}{(2\pi)^{n/2} |V^{*}|^{1/2}} \times \]

\[ \exp \{-0.5(x - \mu^{*})^T V^{*}^{-1} (x - \mu^{*}) \} \]
Classification Uncertainty

\[
\text{Classification Uncertainty} = 1 - \frac{\text{max - sum}}{n} \frac{n}{1 - \frac{1}{n}}
\]

where

- \text{max} = the maximum set membership value for that pixel
- \text{sum} = the sum of the set membership values for that pixel
- \(n\) = the number of classes (signatures) considered

(0.0 0.0 0.0) Classification Uncertainty = 1.00
(0.0 0.0 0.1) Classification Uncertainty = 0.90
(0.1 0.1 0.1) Classification Uncertainty = 1.00
(0.3 0.3 0.3) Classification Uncertainty = 1.00
(0.6 0.3 0.0) Classification Uncertainty = 0.55
(0.6 0.3 0.1) Classification Uncertainty = 0.60
(0.9 0.1 0.0) Classification Uncertainty = 0.15
(0.9 0.05 0.05) Classification Uncertainty = 0.15
(1.0 0.0 0.0) Classification Uncertainty = 0.00
Fuzzy Accuracy Assessment (Green & Congalton 2003)

![Fuzzy Accuracy Assessment Diagram]

Fuzzy Error Matrix (Binaghi et al. 1999)

Table 1
Error matrix with \( p_{mn} \) representing the cardinality of the intersection classification data (rows) and reference data (columns)

<table>
<thead>
<tr>
<th>Reference data</th>
<th>1</th>
<th>2</th>
<th>...</th>
<th>Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>Classification data</td>
<td>( p_{11} )</td>
<td>( p_{12} )</td>
<td>...</td>
<td>( p_{1Q} )</td>
</tr>
<tr>
<td>...</td>
<td>( p_{m1} )</td>
<td>( p_{m2} )</td>
<td>...</td>
<td>( p_{mQ} )</td>
</tr>
<tr>
<td>Total assignments</td>
<td>( p_{+1} )</td>
<td>( p_{+2} )</td>
<td>...</td>
<td>( p_{+Q} )</td>
</tr>
</tbody>
</table>

Hard Error Matrix

\[
M(m,n) = |C_m \cap R_n| = \sum_{x \in X} \mu_{C_m \cap R_n}(x)
\]

Fuzzy Error Matrix

\[
\tilde{M}(m,n) = |\tilde{C}_m \cap \tilde{R}_n| = \sum_{x \in X} \tilde{\mu}_{\tilde{C}_m \cap \tilde{R}_n}(x)
\]

\[
\mu_{C_m \cap R_n}(x) = \begin{cases} 
1 & \text{if } x \in C_m \land x \in R_n \\
0 & \text{otherwise}.
\end{cases}
\]

\[
\mu_{\tilde{C}_m \cap \tilde{R}_n}(x) = \min(\mu_{\tilde{C}_m}(x), \mu_{\tilde{R}_n}(x))
\]