Spatial Interpolation Techniques for Assessing Rainfall Spatial Distribution Based on Rain Gauge Data

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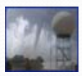
Outline

- Introduction and background
- Data and methods
- Results
- Discussion
- Conclusion

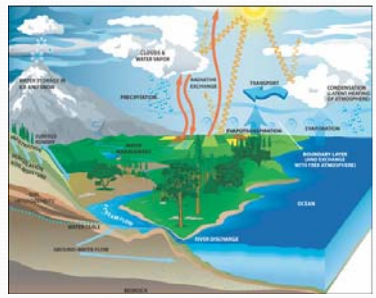
Introduction

- Precipitation is one of the most important components of water cycle
  - Flash flood prediction (Hazards Management)
  - Long term water budget estimation (Water Resource Management)
- Precipitation spatial distribution affects the hydrologic response of a watershed
- Precipitation measurement


- Rain gauge data

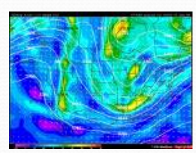

- Radar

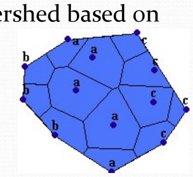

- Satellite



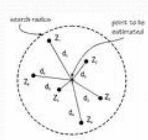
Rainfall Distribution

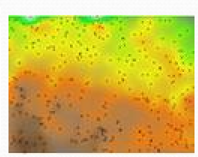
- Rainfall historical data are generally available in rain gauge stations
- Unit areal precipitation or rainfall distribution amount is needed for most hydrologic models.
- How we can obtain rainfall pattern and distribution over a watershed based on rain gauge data?

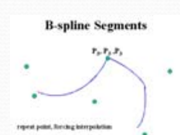




- Traditional Methods
  - Thiessen Polygons
  - Isohyet
  - etc…
- More Advanced Methods
  - Inverse Distance Weighted Average (IDW)
  - Kriging
  - Proximal
  - B-Spline
  - Fourier Series, Wavelet, etc…



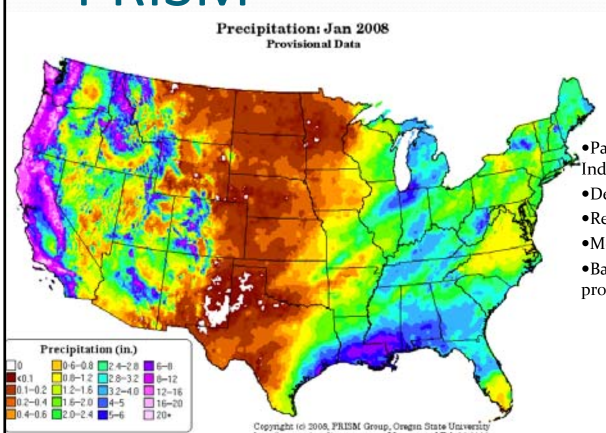




Research Question

- Is there a difference in how well three interpolation techniques (IDW, splining, kriging) estimate precipitation from rain gauges in the Willamette Valley?

PRISM



- Parameter-Elevation Regressions on Independent Slopes Model
- Developed by the PRISM Group at OSU
- Regression model
- Monthly scale, 4 km grid cells
- Based on elevation, slope, aspect, and coastal proximity

Inverse Distance Weighted Average

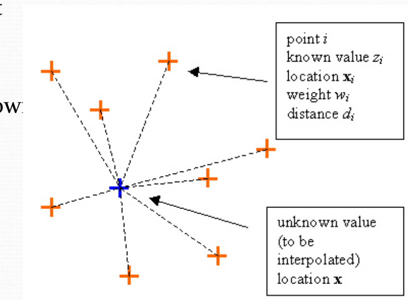
- Each input point has local influence that diminishes with distance
- Estimates are averages of values at n know points within window

$$z(\mathbf{x}) = \sum_i w_i z_i \Big/ \sum_i w_i$$

- Where W is some function of distance

$$w_i = 1 \big/ d_i^2$$

- The above formula is the simplest form of IDW which was introduced by Shepard (1968)



Splining

In the mathematical field of numerical analysis, **spline interpolation** is a form of interpolati where the interpolant is a special type of piecewise polynomial called a spline. Spline interpolation is preferred over polynomial interpolation because the interpolation error can b made even when using low degree polynomials for the spline.

Linear spline interpolation

$$S_i(x) = y_i + \frac{y_{i+1} - y_i}{x_{i+1} - x_i}(x - x_i)$$

Quadratic spline interpolation

$$S_i(x) = y_i + z_i(x - x_i) + \frac{z_{i+1} - z_i}{2(x_{i+1} - x_i)}(x - x_i)^2$$

$$z_{i+1} = -z_i + 2\frac{y_{i+1} - y_i}{x_{i+1} - x_i}$$

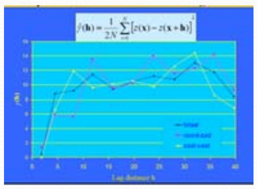
The coefficients can be found by choosing a and then using the recurrence relation

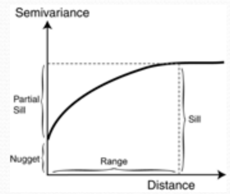
Kriging

Step 1. Estimating mean from the data

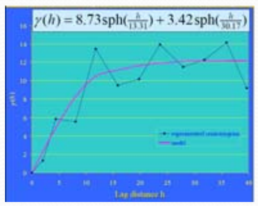
$$\hat{\mu}_z = m_z = \sum_{i=1}^{n} w_i z_i$$

Step 2. Estimating semivariogram





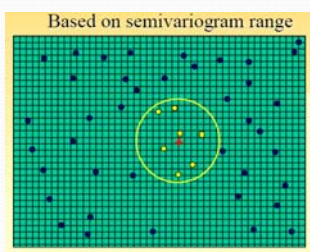
Step 3. Fit a semivariogram model



Example:

$$\gamma(h) = 22.69\left[1 - e^{-\left(\frac{h}{2}\right)}\right]$$

Step 4. Selecting observation within a search radius



Step 5. Solving kriging equation

$$\sum_{j=1}^{n} \lambda_j C(\mathbf{x}_i - \mathbf{x}_j) = C(\mathbf{x}_i - \mathbf{x}_0) \qquad i = 1,..,n$$

with $C(\mathbf{x}_i - \mathbf{x}_j) = \sigma_z^2 - \gamma(\mathbf{x}_i - \mathbf{x}_j)$

Trivial:

$$\lambda_1 C(\mathbf{x}_1 - \mathbf{x}_1) + \lambda_2 C(\mathbf{x}_1 - \mathbf{x}_2) + \lambda_3 C(\mathbf{x}_1 - \mathbf{x}_3) = C(\mathbf{x}_1 - \mathbf{x}_0)$$
$$\lambda_1 C(\mathbf{x}_2 - \mathbf{x}_1) + \lambda_2 C(\mathbf{x}_2 - \mathbf{x}_2) + \lambda_3 C(\mathbf{x}_2 - \mathbf{x}_3) = C(\mathbf{x}_2 - \mathbf{x}_0)$$
$$\lambda_1 C(\mathbf{x}_3 - \mathbf{x}_1) + \lambda_2 C(\mathbf{x}_3 - \mathbf{x}_2) + \lambda_3 C(\mathbf{x}_3 - \mathbf{x}_3) = C(\mathbf{x}_3 - \mathbf{x}_0)$$

Met $C(\mathbf{x}_i - \mathbf{x}_j) = 22.69 e^{-\frac{|\mathbf{x}_i - \mathbf{x}_j|}{2}}$

	0	1	2	3
0	0	3	1	1
1	3	0	3	4
2	1	3	0	1
3	1	4	1	0

Step 6. Solving kriging equation for lambda i

Trivial:

$$22.69\lambda_1 + 5.063\lambda_2 + 3.071\lambda_3 = 5.063$$
$$5.063\lambda_1 + 22.69\lambda_2 + 13.76\lambda_3 = 13.76$$
$$3.071\lambda_1 + 13.76\lambda_2 + 22.69\lambda_3 = 13.76$$

Solution:

$$\lambda_1 = 0.0924 \qquad \lambda_2 = 0.357 \qquad \lambda_3 = 0.378$$

Step 7. Kriging prediction

Step 6: Kriging prediction

$$\hat{Z}(\mathbf{x}) = \mu_z + \sum_{i=1}^{n} \lambda_i [Z(\mathbf{x}_i) - \mu_z]$$

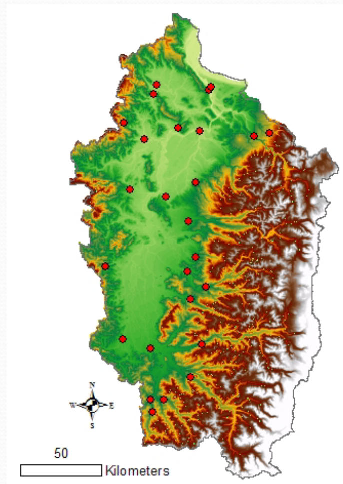
Trivial:

$$\hat{Z}(\mathbf{x}) = 7.1 + 0.0924 \cdot (-4.1) + 0.357 \cdot (-0.1) + 0.378 \cdot 6.9 = 9.29$$

Methods

Data and Study Area

- Willamette Basin



- Data have been obtained from Oregon Climate Service,

  http://www.ocs.oregonstate.edu/
index.html

- Monthly Precipitation have been gathered for three years: 2003, 2004 and 2005

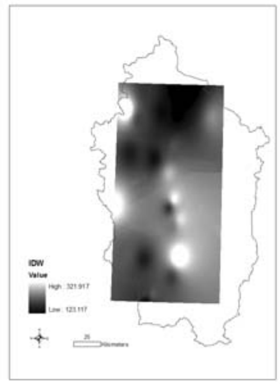
- 26 rain gauge stations have been used in our study

Interpolation Methods

- Inverse Distance Weighted Average (IDW)
- Splining
- Kriging

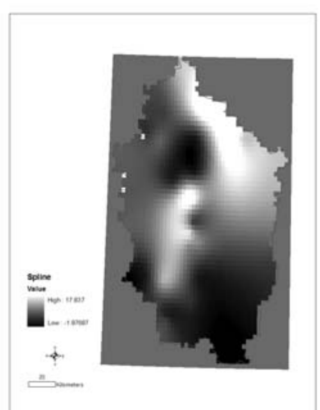
Inverse Distance Weighted

- Power of 2
- Variable Search Radius



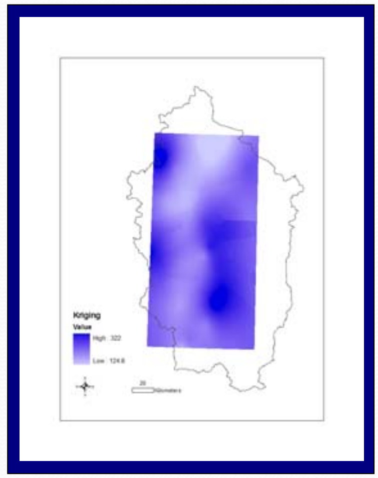
Splining

- Spline with Barriers
- Willamette Basin
- Smoothing Factor - 0



Kriging

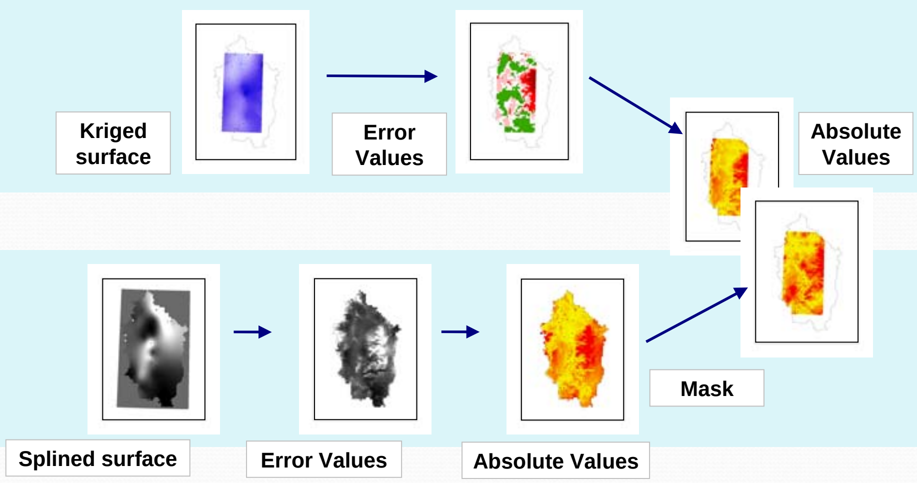
- Kriging method: ordinary
- Semivariogram model: spherical
- Search radius: variable



Additional Methods

- Standardized cell sizes and areas
- Converted to Absolute Values
- Compared ANOVA mean for each month

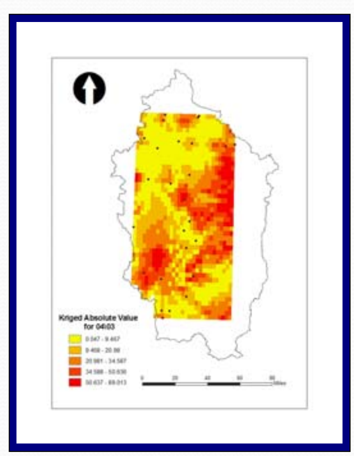
Standardization and Comparison



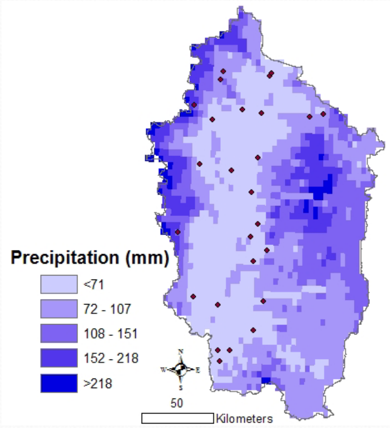
Absolute Values and Statistics

Zonal Statistics for April 03 Kriged Absolute Values

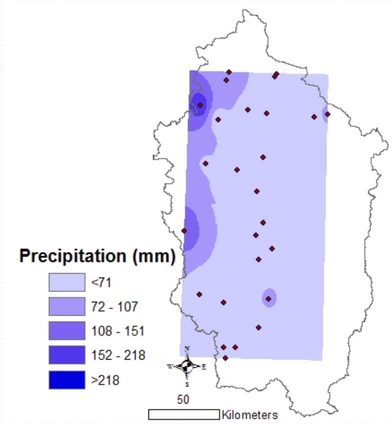
Rowid	VALUE	COUNT	AREA	MIN	MAX	RANGE	MEAN	STD
1	0	1951	3.387207	108	273.84076	165.84076	196.68974	50.923065



Results: IDW

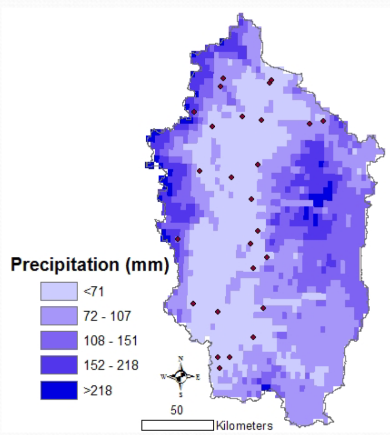


PRISM, Jan 2005

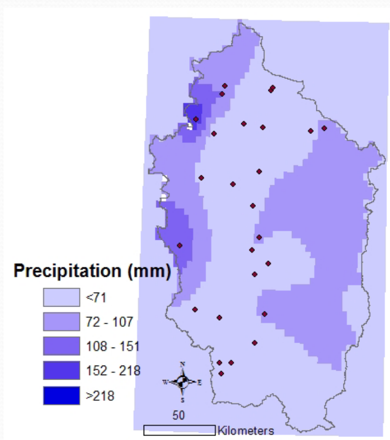


IDW, Jan 2005

Results: Splining

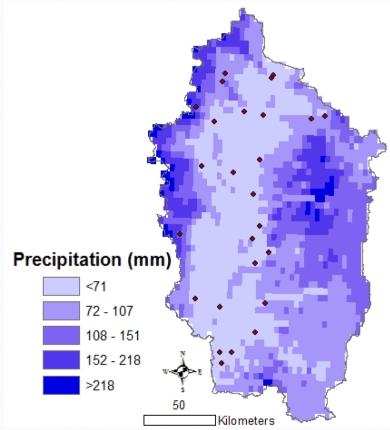


PRISM, Jan 2005

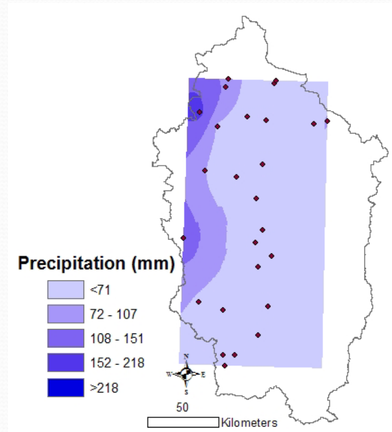


Splining, Jan 2005

Results: Kriging

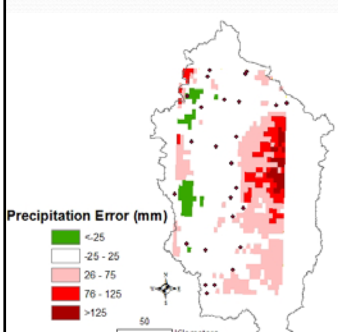


PRISM, Jan 2005



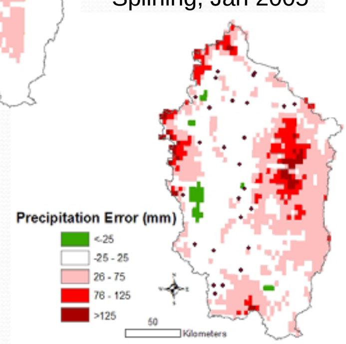
Kriging, Jan 2005

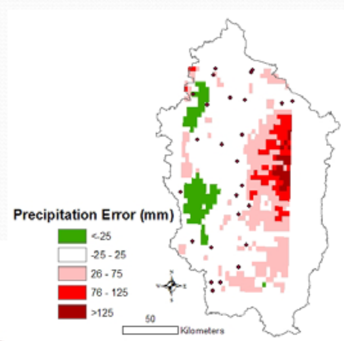
Results: Error Maps



IDW, Jan 2005

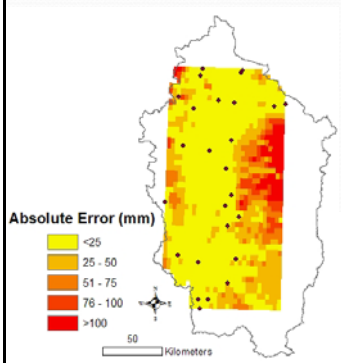
Splining, Jan 2005





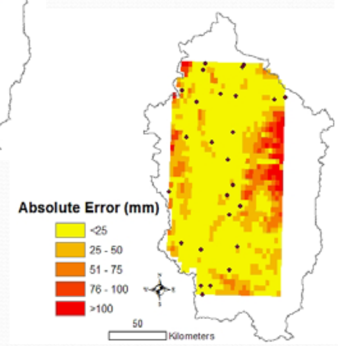
Kriging, Jan 2005

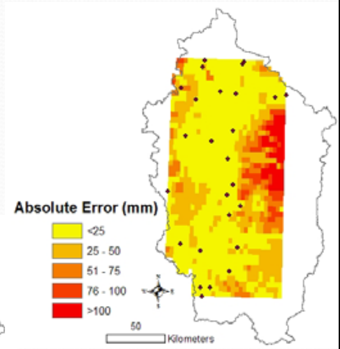
Results: Absolute Error



IDW, Jan 2005

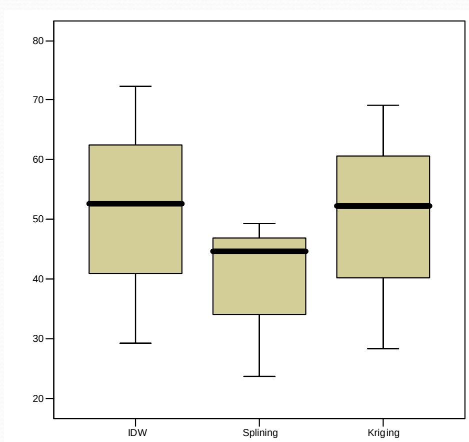
Splining, Jan 2005





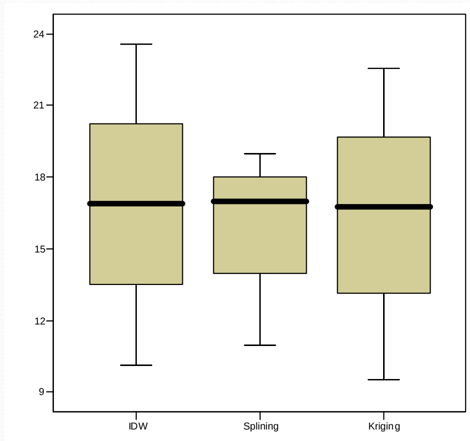
Kriging, Jan 2005

Results: Mean Absolute Error



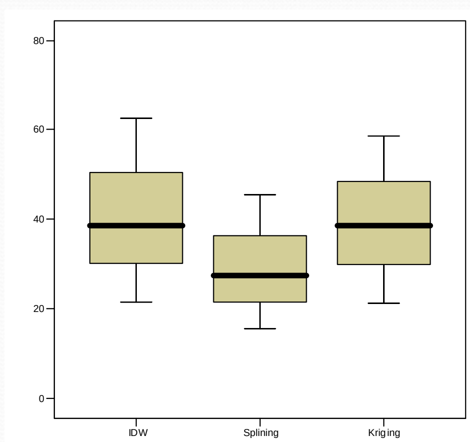
January

Results: Mean Absolute Error



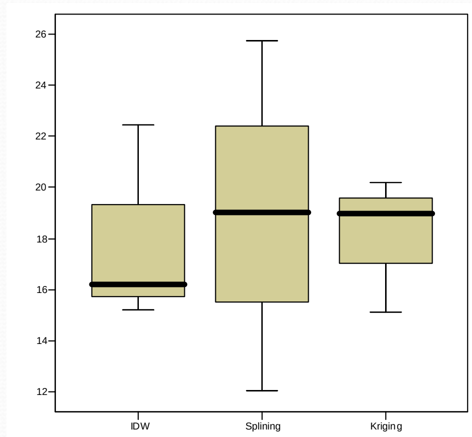
February

Results: Mean Absolute Error



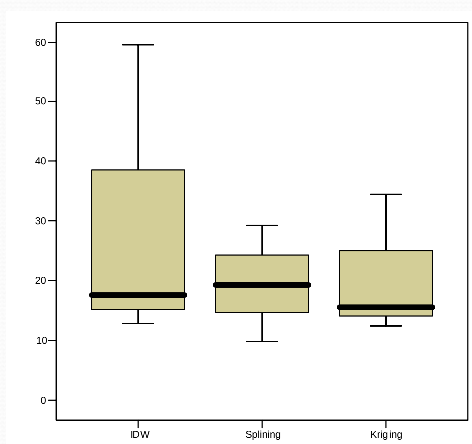
March

Results: Mean Absolute Error



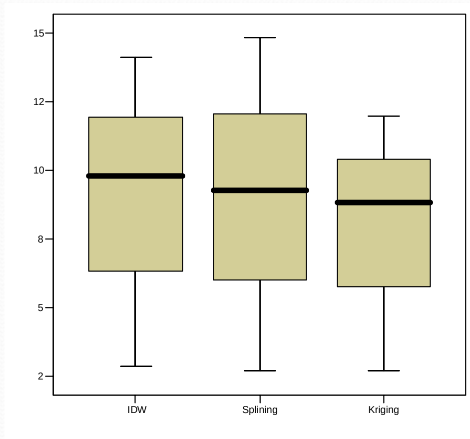
April

Results: Mean Absolute Error



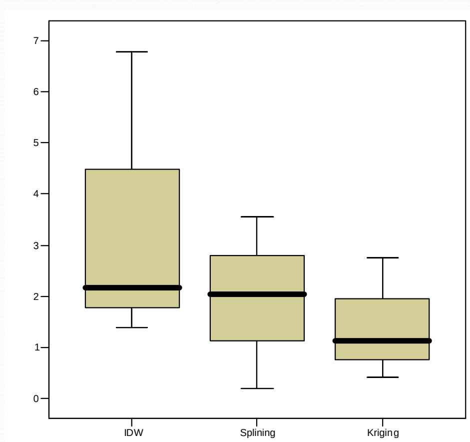
May

Results: Mean Absolute Error



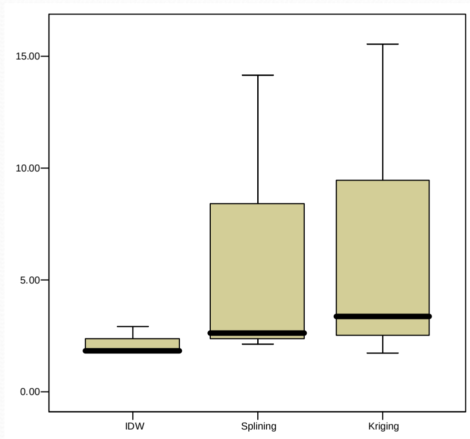
June

Results: Mean Absolute Error



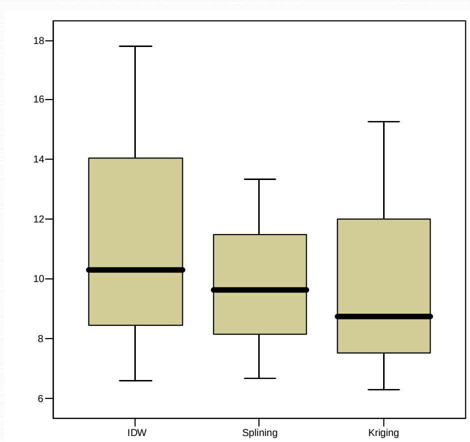
July

Results: Mean Absolute Error



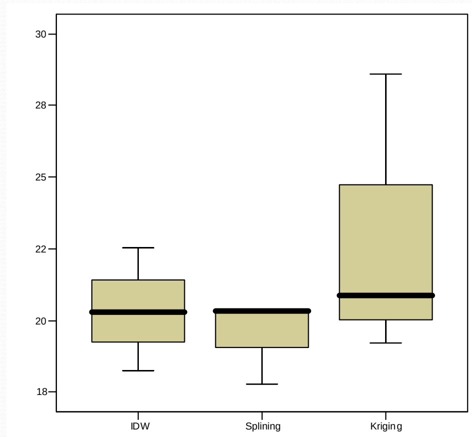
August

Results: Mean Absolute Error



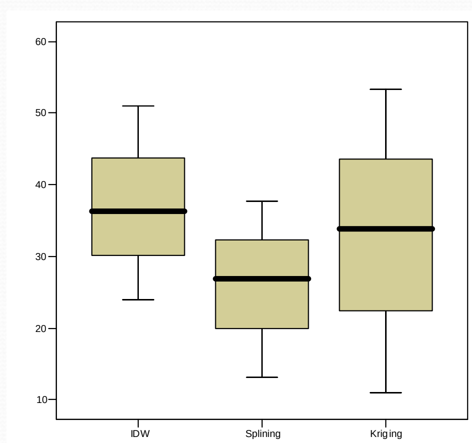
September

Results: Mean Absolute Error



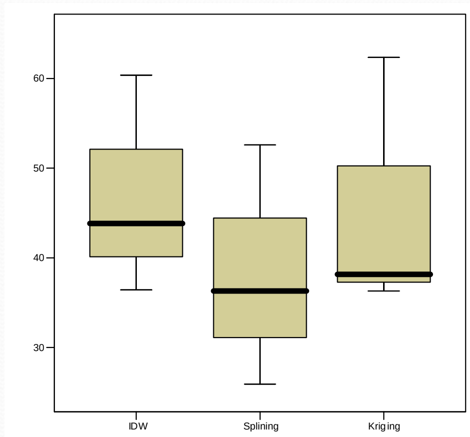
October

Results: Mean Absolute Error



November

Results: Mean Absolute Error



December

Discussion

- Limited number of stations
- Few stations in Cascades
- Assumption of homogeneity

Conclusions

- All techniques model mean precipitation well
- No significant difference in accuracy among techniques
- Error is highest in mountains
- Applications for further research

References

- Daly, C., W.P. Gibson, G.H. Taylor, G.L. Johnson, and P. Pasteris. 2002. A knowledge-based approach to the statistical mapping of climate. *Climate Research* 22:99-113.
- Earls, J., and B. Dixon. Spatial interpolation of rainfall data using ArcGIS: A comparative study. ESRI User Conference 2007 Proceedings.
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