# Geometric Transformation Based on Ground Control 

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## Introduction

- Several methods to transform satellite image to projected map for analysis
- Using ground control points (GCPs) one way
- Empirical or nonparametric: do not need orbital data, etc.


## Limitations

- "Easy" but labor intensive
- Must have "enough" GCPs for statistically reasonable transformation
- Limited GCPs in "boring" topographies: desert, ocean
- Map must "match" scale \& area covered
- Transformation can be complicated by heterogenous terrain


## Requirements

- Suitable vector map with sufficient GCPs
- Mapping software (GIS) to display \& transform image
- Good hand-eye coordination
- Digitizing tablet or on-screen marking



## Least Squares Regression

Regress or plot ( $\mathrm{r}, \mathrm{c}$ ) values versus ( $\mathrm{x}, \mathrm{y}$ ) values

Generic equations:
$x=f(c, r) ; y=f(c, r) ; c=f(x, y) ; r=f(x, y)$
Want to minimize sum of square of residuals i.e. difference in real values versus estimated values

Software solves all 4 equations simultaneously

## Least Squares Regression

Simplest is linear regression:

$$
X=a_{0}+a_{1} R
$$



## Least Squares Regression

Most common geometric transformation is bivariate, affine or first-order least squares function:

$$
\begin{aligned}
& X=a_{0}+a_{1} R+a_{2} C \\
& Y=b_{0}+b_{1} R+b_{2} C \\
& R=d_{0}+d_{1} X+d_{2} Y \\
& C=f_{0}+f_{1} X+f_{2} Y
\end{aligned}
$$



## Least Squares Regression

- First-order usually OK for modest resolution on relatively flat area
- Can accomplish scaling, rotation, shearing \& reflection
- May need higher-order functions for oblique angles and/or rough terrain


## GCPs

- Best are pinpoint, permanent features
- Need 10-15 for first-order fit, and image area up to $1024 \times 1024$ pixels
- Need more for relief or wide areas that induce distortion from nadir
- Need to be spread out to cover all of area
- Keep some in reserve to validate transformation


## Geometric Transformation

- Once equations known:
- Calculate X,Y coordinates of 4 corners to form bounding rectangle of transformed image
- Then calculate X,Y coordinate of the center of each pixel
- To get pixel values image has to be resampled (later)


Root-Mean-Square Error

| GCP | $R$ | $C$ | $X$ | $Y$ | Residual |
| :---: | :--- | :--- | ---: | :--- | :--- |
| 1 | 134 | 230 | 3098 | 12 | -18.9 |
| 2 | 1304 | 304 | 4449 | 23 | 20.9 |
| 3 | 120 | 3245 | 2345 | 213 | 302.3 |
| 4 | 534 | 645 | 1235 | 324 | 15.5 |
| 5 | 756 | 1287 | 3456 | 250 | -12.3 |

## RMSE

- Check each GCP for outlier
- Can try different models to minimize total RMSE
- Use other GCPs to validate transformation




## Questions ?



