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# The economics of pricing parking 

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#### Abstract

We treat parking as a common property resource and examine the benefits of pricing it. Without pricing, parking close to the destination will be excessive, and will fall off more rapidly than is socially optimal. The optimal pattern is attained under private ownership if each parking owner prices in a monopolistically competitive manner. When cruising for parking congests both parkers and through traffic, the benefits from pricing are substantially reduced. © 2004 Elsevier Inc. All rights reserved. JEL classification: R41; H23; D62


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## 1. Introduction

Search for parking represents a major source of congestion in urban areas. A significant fraction of the trip time in a congested urban area may be spent searching for a parking space. Arnott and Rowse [6] report the claim that over half the cars driving downtown in cities with serious parking problems (like Boston and major European cities) are cruising to find a parking space. A smaller number is cited by Allen [2], who reports an estimate that inefficient parking traffic accounts for up to $30 \%$ of total traffic in city centers (see also Young [22]).

Much has been written about the use of road pricing in alleviating congestion problems. The formal economics literature is quite developed in this area and the problem has

[^0]received attention from many prominent economists. Seminal work in this area starts with Dupuit [13], and continues through Knight [17], Boîteux [8], and Vickrey [20], Road pricing schemes are currently used to ease congestion in Singapore, Hong Kong, France, and parts of the USA and Canada, among others. ${ }^{1}$ Recently, the London scheme has been implemented and looks to be a great success. The technology for determining sophisticated road pricing tolls that depend on time of day and intensity of road usage is quite inexpensive. Yet there remains significant political resistance to pricing of road usage in many jurisdictions, and consequently road pricing remains scarcely used in the urban context.

Conversely, there is little formal economic analysis of parking, although technology for pricing parking is very simple (a parking meter!) and there is little social opprobrium for paying for parking. Arguably, inefficient search for parking may be at least as distortionary as excessive road use. Clearly optimal policy should account for both sources of congestion. In the absence of road pricing, efficient pricing of parking may be an effective policy toll for combatting congestion on the road and in parking.

Our objective in this paper is to study the economics of parking by setting up a simple and tractable model of parking congestion. Previous theoretical work on parking is scarce. Arnott and Rowse [6] make a valiant attempt to model the stochasticity of the parking process, but the model rapidly gets complex. Nevertheless, they are still able to make some important normative points. In particular, since their model exhibits multiple equilibria, the optimal tax (which equals the marginal externality) does not necessarily decentralize the optimum. ${ }^{2}$ The Arnott-Rowse framework though is quite different from ours. We are interested in how parking is allocated away from a common desirable location (the CBD). They have a model in which people cruise around a ring-road and have uniformly distributed desired stopping points. In that context they find some intriguing results. Specifically, "the planner chooses a shorter expected parking time and a longer expected driving time, implying a shorter cruising distance." Parkers in their model follow an algorithm regarding when to start looking for a parking spot and in equilibrium they start searching too early. This logic might suggest that the equilibrium parking span in the city model is too spread out. Instead, the opposite happens here. ${ }^{3}$

But, as Arnott and Rowse point out, there is little in the way of other formal literature, although there is quite a lot of descriptive work, and some empirical work on mode choice that includes choice of where to park. There is a bit more modelling of parking in the Transportation Science literature. The paper that is closest to this one is Verhoef, Nijkamp, and Rietveld [19]. These authors show that the alternative transport mode (to cars) ought to be priced at marginal cost, and that appropriate pricing can decentralize the optimum by pricing congestion externalities.

[^1]We are interested here in parking that is unassigned, parking that drivers must search to find. Most car commuters have long-term parking contracts to avoid daily search for parking. We are instead interested in the demand for parking from drivers whose trips are less regular, typically less predictable, and for shorter time periods. The natural examples are shoppers and tourists. We shall suppose that there is a fixed and exogenous capacity at each location. ${ }^{4}$ One application of the model is to on-street parking. In this case, private ownership is not usually viable, although this is a useful theoretical benchmark to shed light on the distortion inherent in not pricing. ${ }^{5}$ An alternative application of the model is to off-street parking and we then compare private ownership with public provision of unpriced parking lots.

If parking is unpriced, then unassigned parking lots are a common property resource that one would expect to be over-exploited in a free entry equilibrium. Parking spaces closest to the most desirable destination (the CBD) are the most coveted and will therefore be the most "overfished." Less desirable parking lots, further away, may end-up being under-used because parkers overcrowd the prime locations. The optimal pricing for parking involves charging for the congestion externality and flattens out the parking gradient by charging more at the parking meters for closer locations.

This leads us to investigate how private ownership of parking lots in a market system can decentralize the optimal configuration. We find that private ownership of parking lots can do the job if private ownership is diverse and monopolistically competitive in the following sense. Each owner must insure that potential parkers find his location at least as attractive as any other location at equilibrium. This still leaves the owner with a tradeoff over the price charged per parking spot and the number of drivers who wish to park there: the higher the price charged, the lower the equilibrium congestion level. Owners therefore face downward-sloping demands for parking. These demands are higher for more desirable parking locations, in contrast to the standard Chamberlinian [12] symmetry assumption. While the parking lot owner is not a traditional price taker since he has latitude in choosing his price, he is still akin to a competitive agent because he faces an overall utility constraint. We show that the social optimum parking pattern is obtained under this market structure.

Cruising for parking slows down through traffic. It does so by necessitating traffic lights at cross thoroughfares and by increasing traffic on the main highway because drivers circle to look for a vacant lot. Taking this feature into account introduces an externality that is not internalized by parking lot owners. This is because this externality is not localized but rather impacts all travellers passing through a location. In this case, private ownership does not render the market solution optimal but instead encourages too little parking at the prime locations and too much parking too far away.

The organization of the rest of the paper is follows. In Section 2 we present the basic parking model assuming there is no externality imposed by those who cruise for parking on those who will park closer to the CBD. We first compare the equilibrium

[^2]with unpriced parking to the optimum and then we derive the parking toll that delivers the optimum arrangement. We then show that this parking fee is exactly that chosen in the monopolistically competitive equilibrium with private ownership. Hence, private ownership and optimum are shown to yield the same solution. In Section 3 we introduce the externality from cruising and through traffic. We establish the manner in which the market solution diverges from the optimum. Section 4 provides a discussion of the practicability of the private ownership solution. It also discusses more general settings (alternate market structures in the parking market, and elastic demand for trips) and whether road pricing may enable an optimal outcome. Section 5 summarizes and concludes.

## 2. The basic model

We are interested in parking that is not assigned to individuals in advance. Shoppers typically do not have long-term parking contracts. Similarly, tourists in the neighborhood of attractions need to park but do not reserve parking in advance. For concreteness, we shall treat all parkers as shoppers. ${ }^{6}$ Most shoppers who drive come to a shopping district from a distant location and so we shall assume that shoppers reside far away.

Formally, suppose that there is a common destination located at $x=0$ (that we shall term the CBD), and there are $N$ shoppers located far away. The CBD is at the end of a long, narrow city, and is served by parallel access roads. Perpendicular to these access roads are side-streets that are used for on-street parking. ${ }^{7}$ Cars can park on street at any free location. Each shopper first drives towards downtown (at speed $v_{d}$ ), then starts to look for an empty parking spot on a side street according to the search process described below. Once an available parking spot is found, he walks (at speed $v_{w}$ ) to the CBD.

An alternative interpretation of the model is that parking is off-street in lots, with a fixed amount of parking at any location. What is crucial for the model is that it takes more time to find a vacant spot in a lot the more other parkers there are.

### 2.1. Search for a parking spot

Assuming parking is not assigned, drivers must search for a vacant parking spot. The expected time to find one at a distance $x$ from the CBD depends on the number of vacant and occupied parking spots at this location. We assume that if a driver stops at location $x$ he will search at this location till he finds a vacant parking spot. The driver then walks to the city. The walking cost is proportional to $x / v_{w}$. The total number of parking places available

[^3]on the interval $[x, x+\mathrm{d} x]$ from the CBD is denoted by $K(x) \mathrm{d} x$, and for simplicity we set $K(x)=k$. Hence, the city has width $k$, with the CBD located at the end. The number of occupied parking places over the interval $[x, x+\mathrm{d} x]$ is denoted by $n(x) \mathrm{d} x$ with $n(x) \leqslant k$.

Consider the last driver to arrive at location $x$. The probability that a randomly sampled spot is free is given by $q(x)=[k-n(x)] / k$. Suppose that search for a parking spot can be described by a stochastic process with replacement. ${ }^{8}$

We can now derive the expected dollar cost $S(x)$ of searching for and finding a vacant spot at location $x$. The probability that any spot is free is $q(x)$ and hence the expected number of spots searched before finding a vacant one is $1 / q(x) .{ }^{9}$ Assume that a driver who checks whether or not a spot is occupied incurs a dollar search cost of $\gamma$. Therefore the expected cost to an individual who chooses to search for parking at location $x$ is $S(x)=\gamma / q(x)$ or ${ }^{10}$

$$
\begin{equation*}
S(x)=\frac{\gamma k}{k-n(x)} \tag{1}
\end{equation*}
$$

This is increasing in the number of cars parked, $n(x)$, and at an increasing rate. It goes to infinity as that number approaches the parking capacity, $k$. As well as exhibiting these intuitive properties, this closed form expression is convenient in what follows to give a closed form to the equilibrium and optimum solutions of the model.

This expected cost refers to the last driver arriving at location $x$. Drivers who arrive earlier face lower search costs because fewer places are already taken. This means that there is a benefit from early arrival: the earlier the arrival time, the lower the expected search cost. So suppose that drivers choose arrival times at $x$. Earlier arrival entails a cost in terms of sub-optimally early arrival at the destination, but it carries the benefit of lower expected search costs. An equilibrium in arrival terms at any $x$ is such that each driver faces the same inclusive cost (early arrival cost plus search cost on arrival) and therefore fully dissipates the rents due to an earlier position in the search process. Formally, such an equilibrium can be derived as one in arrival times, whereby each arriving driver arrives at the time that just makes him indifferent between arriving then and taking the corresponding place in the search sequence, and arriving last of all (and incurring no early arrival delay). Any driver who delays from arriving at the specified time will find her place in the sequence usurped by one of the remaining drivers. This equilibrium exhibits full dissipation of the

[^4]rents accruing to being an early arriver, as in Fudenberg and Tirole [14] and Anderson and Engers [4].

At this equilibrium, all of the drivers face the same total inclusive search cost as the last arriver, whose costs are given above. ${ }^{11}$

Hence (1) is the search cost for all individuals who choose to park at location $x$. The search cost for the last driver arriving at $x$ follows a sequential search process with a cost $\gamma$ per lot inspected. In summary, schedule delay costs are zero for the last arrival and parking congestion costs are highest, while earlier arrivals faces lower parking search costs that are compensated for by schedule delay costs. In a (pure strategy) equilibrium to the arrival time game, all those who park at $x$ face the same inclusive parking cost (1). Alternatively, the equilibrium model could be thought of as representing a stationary regime provided that the time spent parked is independent of location. Then the flow of cars is proportional to the number of occupied spots.

The form of the expected cost of finding a parking spot is purposefully simplistic. Arnott and Rowse [4] construct an elaborate microeconomic model of cruising for parking, and conclude that it is intrinsically a very difficult problem. By using a simple formulation that captures the key trade-offs, we are able to pursue the analysis of equilibrium and optimum parking.

The expected cost from parking at location $x$ is the inclusive parking cost plus the (differential) cost of walking downtown from $x$ :

$$
\begin{equation*}
C(x)=\frac{\gamma k}{k-n(x)}+t x \tag{2}
\end{equation*}
$$

where $t$ is the net dollar cost per mile of walking instead of driving, which we have assumed the same for all individuals. ${ }^{12}$

### 2.2. Equilibrium with unpriced parking

In equilibrium, all parking locations must entail the same expected cost, and unused locations should entail a (weakly) higher cost. Denote this common cost by $c$, and note that $c \geqslant \gamma$ since $\gamma$ is the minimum possible parking cost borne by a drivers located right at the CBD and with one spot to check (which costs $\gamma$ ). From (2), the equilibrium number of cars parked at $x$ solves $\gamma k /[k-n(x)]+t x=c$, or:

$$
\begin{equation*}
n(x)=k\left(1-\frac{\gamma}{c-t x}\right) \tag{3}
\end{equation*}
$$

which must hold whenever $n(x) \geqslant 0$. Let $\hat{x}$ denote the farthest distance parked or parking span. Since $n(\hat{x})=0$, the car that is parked the farthest away incurs the minimum search

[^5]cost of $\gamma$, so that $\hat{x}$ satisfies $\gamma+t \hat{x}=c$, or:
\[

$$
\begin{equation*}
\hat{x}=\frac{c-\gamma}{t} \geqslant 0 . \tag{4}
\end{equation*}
$$

\]

We can now solve for the equilibrium expected cost by equating the supply and demand for parking. This requires that

$$
\begin{equation*}
\int_{0}^{\hat{x}} n(x) \mathrm{d} x=N \tag{5}
\end{equation*}
$$

Integrating (5) with $n(x)$ given by (3) gives:

$$
k\left[\hat{x}-\frac{\gamma}{t} \ln \left(\frac{c}{c-t \hat{x}}\right)\right]=N
$$

where we note that $k \hat{x}$ is the maximum number of cars that could park over the interval $[0, \hat{x}]$. Substituting $\hat{x}$ from (4) leads to:

$$
\begin{equation*}
c-\gamma \ln c=\frac{t N}{k}+\gamma-\gamma \ln \gamma \tag{6}
\end{equation*}
$$

which yields the equilibrium expected cost $c$ in implicit form. The RHS of this expression is independent of $c$ while the LHS is increasing in $c$ in the relevant range where $c>\gamma$ (which is required for $\hat{x}>0$ ). The LHS is below the RHS for $c=\gamma$, and goes to infinity with $c$, so that there always exists a unique solution for $c$. The other endogenous variables are then also uniquely determined. We defer a discussion of the comparative static properties of the unpriced equilibrium until we introduce cruising congestion in the next section. The next step is to derive the optimal parking pattern.

### 2.3. Social optimum

Suppose the planner chooses the optimal number of cars admissible at each location. However, once a car arrives at a parking location, it still must search for a parking place given the search cost functions $S(x)$ described above. Clearly, the optimal allocation of parking involves parking over an interval $\left[0, x_{o}\right]$, with $n(x)>0$, for $x<x_{o}$ and $n\left(x_{o}\right)=0$. The social planner faces the following problem of minimizing the social cost of getting the shoppers to the CBD, or

$$
\begin{aligned}
\min _{\{n(x)\}} S C & =\int_{0}^{x_{0}}\left(\frac{\gamma k}{k-n(x)}+t x\right) n(x) \mathrm{d} x \\
\text { s.t. } & \int_{0}^{x_{0}} n(x) \mathrm{d} x=N .
\end{aligned}
$$

The solution $\left\{n(x), x_{o}\right\}$ to this optimal control problem involves equating marginal social cost (with respect to $n(x)$ ) for all locations with positive parking. ${ }^{13}$ Call the marginal

[^6]social cost $\lambda$, which is given by differentiating the integrand above, so that
\[

$$
\begin{equation*}
\frac{\gamma k^{2}}{[k-n(x)]^{2}}+t x=\lambda, \quad x \in\left[0, x_{o}\right] . \tag{7}
\end{equation*}
$$

\]

Since $n\left(x_{o}\right)=0$, then

$$
\begin{equation*}
\gamma+t x_{o}=\lambda \tag{8}
\end{equation*}
$$

This value can now be used in (7) to determine the optimal number of shoppers parking at $x$ as

$$
\begin{equation*}
n_{o}(x)=k\left(1-\sqrt{\frac{\gamma}{\gamma+t\left(x_{o}-x\right)}}\right) . \tag{9}
\end{equation*}
$$

Clearly $n(x)>0$, for $x<x_{o}$ and $n\left(x_{o}\right)=0$. The population constraint can then be written as

$$
\int_{0}^{x_{o}} k\left(1-\sqrt{\frac{\gamma}{\gamma+t\left(x_{o}-x\right)}}\right) \mathrm{d} x=N
$$

Integrating the LHS, we get

$$
\frac{k}{t}\left(2 \gamma+t x_{o}-2 \sqrt{\gamma\left(\gamma+t x_{o}\right)}\right)=N
$$

which implicitly determines $x_{o}$ and hence the other endogenous variables. To find an explicit expression for $x_{o}$, it helps to write this last equation using the value of $\lambda$ in (8) as $k / t(\sqrt{\lambda}-\sqrt{\gamma})^{2}=N$.

The optimized marginal social cost is therefore equal to $\lambda=(\sqrt{\gamma}+\sqrt{N t / k})^{2}$. This tells us the optimal value of the location of the last parking place. Substituting this value of $\lambda$ in Eq. (8) leads to

$$
\begin{equation*}
x_{o}=\frac{N}{k}+2 \sqrt{\frac{N}{k} \frac{\gamma}{t}} . \tag{10}
\end{equation*}
$$

This expression is now to be compared with the (implicit) location $\hat{x}$ from the equilibrium problem (see (4) and (6)).

### 2.4. Comparison with the unpriced solution

The equilibrium parking arrangement has individual cost equalized at all parking locations, while the optimum arrangement has marginal social cost equalized. The differences in these arrangements are described in the following proposition.

Proposition 1. The parking span at the equilibrium with unpriced parking is smaller than the optimal one. Moreover, there exists a location $\tilde{x}<x_{o}$ such that more cars are parked at $x$ in the equilibrium than in the optimum if and only if $x<\tilde{x}$.

Proof. Recall that $n(x)=k\left(1-\gamma /(c-t x)\right.$ ) (from Eq. (3)), while from (9), $n_{o}(x)=$ $k\left(1-\sqrt{\gamma /\left(\gamma+t\left(x_{o}-x\right)\right)}\right)$. We shall show that these schedules cross once, at $\tilde{x}$. Indeed,
there must be at least one crossing because both schedules are continuous, with positive density at $x=0$ and the integral of each of them over its support is $N$. At any such crossing, $\tilde{x}, n(\tilde{x})=n_{o}(\tilde{x})$, and hence $\gamma /(c-t \tilde{x})=\sqrt{\gamma /\left(\gamma+t\left(x_{o}-\tilde{x}\right)\right)}=\phi$ (i.e., we have called the common value $\phi$ ). Denote derivatives with primes. As we will now show, $n^{\prime}(\tilde{x})<n_{0}^{\prime}(\tilde{x})$, or

$$
n^{\prime}(\tilde{x})=-\frac{k t}{\gamma} \phi^{2}<-\frac{k t}{2 \gamma} \phi^{3}=n_{0}^{\prime}(\tilde{x}) .
$$

This is true since $\phi<2$ which is satisfied because (recall $\left.\tilde{x}<x_{o}\right) \gamma<4\left(\gamma+t\left(x_{o}-\tilde{x}\right)\right)$. The fact that $n^{\prime}(\tilde{x})<n_{0}^{\prime}(\tilde{x})$ is true at any crossing implies that there can be only one crossing. The fact that the equilibrium density slopes down more steeply at the crossing means that the equilibrium density is above the optimum one for $x<\tilde{x}$ and the converse is true for $x>\tilde{x}$.

The equilibrium involves tighter parking than is optimal because of the uninternalized externality associated with parking. At equilibrium, there is more crowding, and hence excessive search cost, because drivers do not take into account that selecting a parking spot close to the CBD increases the search cost of a large number of other drivers trying to park there. This is a variant of the classic common property resource over-grazing or over-fishing problem; see Gordon [16] for a seminal treatment. Here there is the extra twist since the resources are ranked by quality - spots closer to the CBD are intrinsically more desirable. The analogy is that fishermen fish too close to the shore (see Weitzman [21] for a similar idea). One might also surmise that the lowest apples are overpicked at a common property orchard.

The optimum involves unequal treatment of equals in the sense that different individuals get different utilities at the optimum. Those who are allocated to park closer to the CBD get higher utility than those who park further away. The optimum can be decentralized via pricing of parking. Since parking is more desirable closer to the center, the optimum parking tariff increases with closeness to the CBD in order to counteract this effect and reduce the over-congestion that is most pronounced closest to the CBD. We next derive the optimum parking tariff.

### 2.5. Optimal parking tariffs

We can use the analysis above to determine the optimum price of a parking lot as a function of distance from the city. One can think of this as the rate paid at a parking meter. As we now show, the parking meter rates that decentralize the optimum decrease with distance from the city center.

The optimal price $\tau(x)$ is equal to the difference between the marginal social cost and the private cost. Hence, from (2) and (7),

$$
\begin{equation*}
\tau(x)=\left[\frac{\gamma k^{2}}{\left[k-n_{o}(x)\right]^{2}}+t x\right]-\left[\frac{\gamma k}{\left[k-n_{o}(x)\right]}+t x\right] \tag{11}
\end{equation*}
$$

where $n_{o}(x)$ is the optimal number of cars parked at location $x$. Rearranging (11) gives the optimal (positive) price as

$$
\begin{equation*}
\tau(x)=\frac{\gamma k n_{o}(x)}{\left[k-n_{o}(x)\right]^{2}} . \tag{12}
\end{equation*}
$$

Inserting the expression for the optimal parking density $n_{o}(x)$ given by (9) means that the optimal parking tariff can also be written as

$$
\begin{equation*}
\tau(x)=\left(\gamma+t x_{o}\right)-\left(\sqrt{\gamma\left[\gamma+t\left(x_{o}-x\right)\right]}+t x\right) \tag{13}
\end{equation*}
$$

with $x_{o}$ given by (10).
The first term of this expression is the marginal social cost of an additional shopper parking at (any) $x$ given the optimal occupancy (see Eq. (8)). Hence, the second term is the private cost (as can readily be checked from Eqs. (2) and (9)). The optimal parking tariff is a decreasing and concave function of distance from the CBD. It is maximum at $x=0$, and zero at the furthest parking place at $x_{o}$. The shape of the optimal price schedule reflects the property that the congestion externality diminishes with distance and it does so at an increasing rate.

### 2.6. Parking lot operators

We have just seen that the market equilibrium with unpriced parking lots is socially inefficient. This raises the question as to whether the market mechanism can deliver the optimal allocation when prices are determined through market forces. The answer is a qualified affirmative and here we explore the conditions under which this can be the case.

We consider a market where parking lots are managed by parking lot operators. To approximate a competitive market setting we assume that the parking lots at $x$ are priced by a single operator who set his price competitively, i.e. by taken the other prices as given. However, each operator has a degree of market power since he controls all the lots at $x$. This leads to a situation that might be termed monopolistically competitive, since there are many operators and each has local market power. The set-up differs from the standard monopolistic competition framework because operators are not symmetric in terms of the market conditions they face. Those located closer to downtown have a competitive advantage by dint of their more desirable location. ${ }^{14}$

With parking lot operators, the cost $C(x)$ is augmented by a location specific price, $p(x)$. The equilibrium condition is that this augmented cost, denoted by $C_{m}(x)=C(x)+$ $p(x)$ for the monopolistic competition case, is equal at all locations $x$ at which there is parking and is no lower at any other location.

To determine the equilibrium level of $p(x)$, we consider the decision problem faced by the individual parking lot operator. The profit of the operator located at $x$ is $\pi(x)=$ $p(x) n_{m}(x)$, with $C_{m}(x)=c_{m}$. Here the expression $C(x)$ is given by (2), where $n(x)$ is replaced by $n_{m}(x)$, the number of parkers given monopolistically competitive pricing.

[^7]The operator nonetheless faces a downward sloping demand curve because a lower price will attract more shoppers, whose presence raises the search cost. It is most convenient to substitute out for price and determine the operator's choice of number of parkers to attract. Doing so yields $\pi(x)=\left[c_{m}-C(x)\right] n_{m}(x)$. The first-order condition is

$$
\begin{equation*}
\frac{\partial \pi(x)}{\partial n_{m}(x)}=p(x)-n_{m}(x)\left[\frac{\gamma k}{\left[k-n_{m}(x)\right]^{2}}\right]=0 . \tag{14}
\end{equation*}
$$

Clearly, the price charged by the operator is therefore equal to

$$
\begin{equation*}
p(x)=n_{m}(x) \frac{\gamma k}{\left[k-n_{m}(x)\right]^{2}} \tag{15}
\end{equation*}
$$

This shows:
Proposition 2. Private ownership of parking lots in a monopolistically competitive market decentralizes the social optimum.

Proof. This result is a direct consequence from the fact that the price $p(x)$ charged by the parking lot operators (15), coincides with the optimal price $\tau(x)$ given by (12).

The intuition underlying this decentralization result is that it is a case in which the Coase theorem works. We know that assigning property rights can alleviate and even solve market failure in the presence of externalities. However, property rights need to be assigned in such a way as to preserve a competitive market. Private ownership must be disparate enough as to preclude market power. Here this means that private owners should be small enough that they take as given the cost level attainable at all other locations, and have no discernible effect on that level. What is interesting here is that the individual owners retain the power to choose the level of parking and its price (given the constraint).

### 2.7. Numerical results

For comparison purposes we now give a numerical example of the solutions described in the rest of this section. We use the following parameter values: $\gamma=10$ cents, $t=\$ 4 / \mathrm{km}$, $N=20000$ drivers, and $k=40000$ parking spots per kilometer.

The first benchmark is a technology without search congestion. Suppose that each driver is assigned a space and can find it costlessly. Then all locations at which there is parking are fully occupied. The parking span is half a kilometer, and the driver at the furthest away location pays $\$ 2$ in transport costs (plus the 10 cents to manoeuver the car into the lot) and nothing in parking. The other closer locations provide the same level of cost in equilibrium, so that the parking cost is $\$ 2$ at the CBD and declines linearly away from there. Equilibrium values are readily computed for the other scenarios from the equations above.

We first compare to the equilibrium with unpriced parking. The equilibrium cost is $c=\$ 2.42$. The parking span is $x=0.58 \mathrm{~km}$ and the average occupancy rate is $86 \%$. The maximum packing of parking (at $x=0$ ) is $96 \%$.

At the social optimum, the average cost (per driver) is $S C / N=\$ 1.70$. The optimal parking span of $x_{o}=0.72 \mathrm{~km}$ is considerably larger than at the equilibrium and as a consequence the average occupancy rate is smaller (69\%).

With monopolistically competitive parking operators, the user cost of $c^{m}=\$ 2.99$ is higher than at the unpriced equilibrium. However, this cost is reduced to $\$ 1.70$ (which verifies the value of the socially optimum cost given above) if operator revenue is lump-sum redistributed. Then the average profit per parking operator is $(\$ 2.99-\$ 1.70) 69 \%=\$ 0.90$.

## 3. Road congestion from cruising

There are many aspects of parking that have been assumed away in this analysis. These include dynamic pricing, multiple destinations, and driver heterogeneity in the value of time, parking duration and desired arrival time. This analysis has focused on the specific externality that a parker increases the search time of subsequent parkers. However, cars that are cruising and searching for a parking place slow down other cars passing by. Cruising cars slow down traffic directly on the main arteries into town if they are searching on those streets (either for on-street parking or searching for and turning into an off-street parking lot). They may also slow down traffic on the main arteries if the cruisers are searching in side-streets: after an unsuccessful search, the car searching for parking must either re-enter the main stream or cross it to get to another side-street. The more cruising traffic there is, the greater the slow-down in terms of traffic light delays, traffic flow interference, etc.

Thus the speed at which through traffic can travel is lower the more cars are looking for somewhere to park. This is a second type of externality germane to the cruising problem.

### 3.1. A model of cruising for parking

Without congestion from cruising, the user cost is $C(x)=S(x)+t x$ (see (2)). We now introduce road congestion by assuming that the driving time taken to traverse the stretch of road $[x, x+\Delta x]$ is higher the more people are searching for parking in this interval.

The simplest formulation is to stipulate that a cruising car at $x$ induces an extra delay for all cars crossing $[x, x+\Delta x]$. We assume that the total delay induced by the cruising car at location $x$ is an increasing function of the number of drivers cruising for parking in this interval. For simplicity, we assume this function is linear, i.e. the additional congestion is $\alpha n(x) \Delta x$, where $\alpha$ is the extra delay cost per cruiser (previously, $\alpha=0$ ). Alternatively, this amounts to assuming that travel speed is proportional to $1 / n(u)$.

This means that the expected cost incurred by a shopper who parks at $x$ is

$$
\begin{equation*}
C(x)=\frac{\gamma k}{k-n(x)}+t x+\alpha \int_{x}^{\bar{x}} n(u) \mathrm{d} u \tag{16}
\end{equation*}
$$

where $\bar{x}$ is the location of the car parked the furthest away. ${ }^{15}$ The third term in (16) is a non-localized externality since drivers at $x$ impact all drivers parking closer downtown.

[^8]One advantage of our formulation is that aggregate congestion is independent of the distribution of those parking. To see this, note that the total cruising congestion cost is $\alpha \int_{0}^{\bar{x}} n(x) \int_{x}^{\bar{x}} n(u) \mathrm{d} u \mathrm{~d} x$. Letting $G(x)$ denote $\int_{x}^{\bar{x}} n(u) \mathrm{d} u$, we can write this cost as $-\alpha \int_{0}^{\bar{x}} G^{\prime}(x) G(x) \mathrm{d} x=-\alpha\left[G^{2}(x) / 2\right]_{0}^{\bar{x}}=\alpha N^{2} / 2$, where the last step follows from the fact that $G(0)=N$ and $G(\bar{x})=0$. Thus the total cruising congestion cost is constant, as claimed. This property implies that the solution to the social problem is the same as in Section 2, where $\alpha=0$. This argument establishes the next result.

Proposition 3. The socially optimal parking configuration is independent of the strength of the externality on through traffic, $\alpha$.

This formulation of the externality therefore gives a clean benchmark case for comparison of the other market structures with the optimum arrangement.

### 3.2. Equilibrium with unpriced parking and cruising congestion

The equilibrium condition is that $C(x)$ (given by (16)) is constant over all locations $x$ at which there is parking and that it is no lower at any other location. Since the last location $\bar{x}$ involves $n(\bar{x})=0$, the equilibrium relation between the common equilibrium $\operatorname{cost} c$ and $\bar{x}$ is ${ }^{16}$

$$
\begin{equation*}
c=\gamma+t \bar{x} . \tag{17}
\end{equation*}
$$

Clearly, the equilibrium involves a band of on-street parking contiguous to the CBD. Over this interval, expected cost is constant, so that

$$
\begin{equation*}
\frac{\mathrm{d} C(x)}{\mathrm{d} x}=\frac{\gamma k n^{\prime}(x)}{(k-n(x))^{2}}+t-\alpha n(x)=0, \quad x \in[0, \bar{x}] . \tag{18}
\end{equation*}
$$

The unique solution of this first-order differential equation is denoted by $n_{e}(x)$. Note that the number of parkers may rise with distance from the CBD, $x$, if $\alpha$ is large enough: since drivers wish to avoid congestion from cruisers, they tend to park far away and then walk. This induces other drivers to act in a similar manner and as a consequence all drivers end up parking far away. The induced excessive walking time is a source of additional deadweight loss since locating far away is individually efficient but collectively inefficient (the total cruising time stays the same). In the sequel, we assume that $\alpha k<t$; in the present case, this ensures that $n_{e}^{\prime}(x)<0 .{ }^{17}$ We now turn to the qualitative properties of the solution.

### 3.3. Comparative static properties of the equilibrium solution

The equilibrium solution has the following comparative static properties that are proved from Eq. (18) using techniques similar to those used to prove Proposition 1.

[^9]Proposition 4. The equilibrium parking span with unpriced parking is smaller when:
(a) the parking search cost, $\gamma$, is lower;
(b) the travel cost differential, $t$, is higher;
(c) the cruising congestion cost, $\alpha$, is lower.

Moreover, there exists a location $\tilde{x}_{i}, i=a, b, c$ such that more cars are parked at $x$ in the equilibrium before the change than after the change if and only if $x<\tilde{x}_{i}$.

The proof of this and of the next proposition are found in the discussion paper version at http://www.virginia.edu/economics/papers/anderson/parksubjue3.pdf.

The intuition behind the result is as follows. A lower $\gamma$ means that drivers are less sensitive to parking congestion. The new equilibrium therefore involves more congested parking at each location close to the city and the total area devoted to parking falls. If $t$ rises, walking becomes more of a nuisance relative to driving and parking lots closer in are more packed. If $\alpha$ falls, there is less annoyance from those cruising for parking so shoppers will tend to drive further in. Again, this creates more intense usage of parking lots further in. ${ }^{18}$

### 3.4. Equilibrium with private parking lot operators

In the absence of cruising for parking affecting travel time, an operator of a private parking lot has full control over the level of congestion at $x$ and therefore the market outcome can decentralize the social optimum (Proposition 2). By contrast, with the cruising externality, there is an additional cost that shoppers who cruise for parking impose on other drivers who park at other locations (and therefore who are not generating profit at $x$ ). This second externality is not fully internalized by the parking operators since it is not a localized externality: this cost affects all the cars which park downstream (nearer to the CBD).

With parking lot operators, $C(x)$ as given by (16) is augmented by a location specific price $p(x)$. In equilibrium, the resulting cost is equal at all locations at which there is parking and operators maximize profits. The social planner minimizes the social cost $S C$, which is now amended by the additional congestion cost term. As noted in the Proposition 3, the optimal number of parkers at any location is independent of the externality $\alpha$.

The optimum tariff that decentralizes the social optimum is $\tau(x)=\lambda-C(x)$, or

$$
\tau(x)=\frac{\gamma k n_{o}(x)}{\left[k-n_{o}(x)\right]^{2}}+\alpha \int_{0}^{x} n_{o}(u) \mathrm{d} u
$$

[^10]Therefore the price schedule is the same as in the case $\alpha=0$, plus the term reflecting the cruising externality that a driver located at $x$ imposes on all the drivers located downstream.

We can now compare with the monopolistically competitive equilibrium.
Proposition 5. The first-best optimal parking span is smaller than the monopolistically competitive one for $\alpha>0$ and the equilibrium span without pricing is always smaller. Moreover, there exists a location $\tilde{x}_{i}, i=(o, e)$ such that more cars are parked at the optimum $(i=o)$ or at the equilibrium without pricing $(i=e)$ than at the monopolistically competitive regime if and only if $x<\tilde{x}_{i} ; i=o, e$.

These results can be understood from inspection of the condition that characterizes the social optimum:

$$
\left\{\frac{\gamma k n_{o}^{\prime}(x)}{\left[k-n_{o}(x)\right]^{2}}+t-\alpha n_{o}(x)\right\}+\left\{\frac{\gamma k n_{o}^{\prime}(x)}{\left[k-n_{o}(x)\right]^{3}}\left[k+n_{o}(x)\right]\right\}+\alpha n_{o}(x)=0 .
$$

The first term in curly brackets stems from the unpriced equilibrium problem is identical to the condition (18) that characterizes equilibrium there. The monopolistically competitive case adds the second term in curly brackets, so that setting the first two terms to zero characterizes equilibrium for monopolistic competition. This second term reflects the parking congestion effect that is internalized under monopolistic competition. Since this term is negative, the slope is flatter with monopolistic competition. The final term reflects the cruising externality not accounted for under monopolistic competition. Since this term is positive, the gradient at the optimum is steeper than under monopolistic competition. The fact that the two externalities play in different directions means that either the monopolistically competitive or the unpriced equilibrium may be closer to the optimum.

### 3.5. Numerical results with cruising

We use, as before, the following parameter values: $\gamma=10$ cents, $t=\$ 4 / \mathrm{km}, N=$ 20000 drivers, and $k=40000$ parking spots per kilometer. We assume that $\alpha=0.01$ cents.

The different solutions are displayed in Fig. 1. ${ }^{19}$ The four curves represent the equilibrium solution with $\alpha>0$ (thick line); the optimum solution for any value of $\alpha \geqslant 0$ or the monopolistic competition solution when $\alpha=0$ (diamond); the monopolistic competition with $\alpha>0$ (cross); and the unpriced equilibrium with $\alpha=0$ (thin line).

The average cruising cost is $\alpha N / 2=\$ 1$. Recall that without cruising externalities, the equilibrium cost in the unpriced equilibrium was $\$ 2.42$. With the cruising externality, this rises to $\$ 2.73$, which is significantly less than the direct congestion externality and reflects that the unpriced is now closer to the optimum (as can be seen from the figure). Indeed, the average cost at the social optimum is $\$ 2.70$ which is just the original $\$ 2.70$ plus the dollar for the congestion externality. The unpriced solution is remarkably close to this one. However, the monopolistic competitive solution now moves away from the optimum to become too spread out (in contrast to the equilibrium which is still too tight). The social

[^11]

Fig. 1. Location (vertical axis) versus parkers (horizontal axis).
cost rises from $\$ 1.70$ to $\$ 2.75$ and so overshoots the extra dollar of congestion cost. In this example, the congestion externality is sufficiently severe that it renders the unpriced solution more efficient than the price solution.

## 4. Alternative market structures and cost-sensitive demand for shopping

The monopolistically competitive market structure enables the optimal solution to be attained when $\alpha=0$, but it causes insufficient occupancy close to the CBD when $a>0$. As we have shown, the monopolistic competitor internalizes the local externality from search for parking at the operator's location, but does not take into account the externality from cruising that hampers traffic flows for those parking closer to the CBD. In that sense the operator sets a local price that is too low. The price distortion is greatest at locations far away (through which many drivers must pass) because the externality is greatest at locations the furthest out. The price gradient in monopolistic competition is too shallow, and the optimal arrangement has more packed parking close to the CBD.

If, instead, all parking were owned by a single monopoly firm then this firm would internalize all of the externalities, local as well as global. It would then reach the full socially optimal arrangement of parking. However, monopolies cause distortions when demands are not perfectly inelastic. This leads us to consider the optimality of the alternative arrangements when the demand for shopping is no longer fixed at $N$.

Suppose instead that the number of shoppers depends in a decreasing fashion on the full cost paid (see also Bacon [7]). Without too much violence to the notation, let $N(c)$ be the number of shoppers as a function of the full price, $c$, so this is the downward-sloping (aggregate) demand for shopping trips. Consider first the optimum problem. The way we have set this up is to derive the marginal social cost, $\lambda$, associated to an allocation. Since $\lambda$ is the marginal social cost of an extra shopper, then the optimal number of shoppers is simply given by $N(\lambda)$ from the demand curve for shopping. As argued above, this optimum can be attained with appropriate parking meter fees (and the same is true for $\alpha>0$ ) so that all individuals pay the marginal social cost when the optimal fees $\tau(x)$ are in place. When
$\alpha=0$, we already know that the monopolistically competitive price exactly tracks the fee $\tau$. This means that the full cost is the same and the full social optimum is attained even with cost-sensitive demand.

By contrast (still for the case $\alpha=0$ ), the unpriced market equilibrium involves a lower cost level paid per shopper at the equilibrium we found with fixed $N$. This translates into too many shoppers when $N$ is variable. At the other extreme, a monopolist of all parking lots will use its market power to raise the price too high, leading to too little shopping. This effect will still be present when $\alpha$ is positive; so, despite the property that the monopolist will set the right allocation across locations for any given level of $N$, it will price too high in aggregate and deter shopping excessively.

Finally, one could consider intermediate degrees of market power between monopolistic competition and monopoly. Firms might be envisaged as each controlling a tranche of locations. Although we do not formally consider this case, it seems straightforward to conjecture that the resulting allocation lies between the monopoly and monopolistic cases, drawing the benefits and disadvantages from each. That is, prices tend to be too high from the market power side, but allocations are improved the larger the tranche controlled by a firm because then the cruising externality is internalized to a greater degree.

## 5. Conclusion

The economics of unassigned parking pose a common-property resource problem. If parking is unpriced (or priced independently of location) then the market equilibrium will overused the resource where it is most valuable (at the CBD) and the parking gradient will tail off too fast. If there is no pricing at all, too many shoppers will be attracted to the CBD. The socially optimal configuration can be attained under private ownership of the parking lots. This is not a great surprise from the Coase theorem, although the form of market structure (which we can think of as linked to the way in which the private ownership must be structured) needed to get the optimum is more intricate. First, agents need to be small enough (i.e., own few enough parking lots) to act competitively by taking as given the utility level that can be attained by drivers. Second, they need to be large enough to be able to internalize the local externality in parking congestion. This means they must control the lots at a given location. Each parking lot operator therefore acts in a monopolistically competitive manner, taking as given the overall utility constraint but with the power to set price given that occupancy will adjust. If lot owners are larger, and have more market power, they will tend to set prices too high and so there will be too little shopping activity. However, a full monopoly owner will fully internalize the problem of cruising activity that adversely impacts the road speed of drivers parking elsewhere. The small-scale owners will not do this. This gives a trade-off between efficiency across locations (that the full monopoly gets right) versus overall too high prices (that the monopoly encourages).

There are other reasons for not using a market solution for on-street parking. One is the transaction cost involved if there are many small-scale operators-think for example of the enforcement problem of parking without paying. Arguably too, local governments have wider objectives in their parking policy than simple efficiency. For example, many parking meters have a (short) time limit and regulations against coming back to "feed" them. Even
if someone-a commuter, say, who wants to park from dawn till dusk-is willing to pay more than what individual short-term users would pay over the day, he is barred from doing so. Arguably the local government is encouraging shopping at downtown establishments; and subsidies are needed because of shopping externalities and other implicit subsidies and market failures in other sectors.

However, Internet markets may provide a workable solution to the common property problem. To fix ideas, suppose there were a planner who could costlessly allocate a specific parking spot to each shopper. Then there would be no search costs (apart from finding one's designated place!). Such a planner would allocate $k$ cars to each location. This is the full first-best optimum, without the search technology constraint. This first-best optimal solution might be decentralized in a market system if property rights are properly defined and transaction costs are zero (or, hopefully, sufficiently low). Defining property rights means having owners for the parking spaces, and, as we have discussed, we need a large number of owners for this to be done efficiently in the private domain (although one could also imagine that public ownership can still be viable with an auction system). ${ }^{20}$ For market transactions to be low-cost, we need to have a fluid spot market for parking lots. One might envisage such a market developing over the Internet, with parking lots auctioned off to the highest bidder. There are some obvious benefits from such a system. Clearly eliminating cruising reduces aggregate walking time, road congestion and practically eliminates search time. However, for the Internet solution to be practical would require mass access by shoppers. It might also need substantial monitoring and enforcement costs, especially in the early stages.

The model of this paper has taken the number of parkers and parking spots as exogenous. In a companion paper (Anderson and de Palma [3]) we embed the current parking technology in a mono-centric city model with commuters and endogenous land use. This gives in equilibrium an inner ring next to the CBD comprising parking lots; next out is a ring of residential housing whose residents walk to work. Finally, there is an outer ring of residential housing for those who drive to a parking lot and then walk in to the CBD. Note in this model that land rents falls off away from the CBD and adjust so that the parking lot operators earn zero profit. Furthermore, we show that the equivalence between the social optimum and the monopolistically competitive solution continues to hold when there is no cruising congestion.

The model can be usefully expanded to deal with roads and road pricing. Road congestion has only been addressed obliquely via the cruising cost externality. But road congestion also typically depends on the number of vehicles on the road at a particular point (and downstream, in case of bottlenecks). We have assumed that the amount of parking space is exogenous. However, on-road congestion is eased if parking places are converted to road lanes. Road pricing should also optimally be used in conjunction with parking pricing. This direction leads us to consider how road pricing might be used if parking is unpriced (or is constant over locations). Parking externalities can be directly

[^12]affected by the road price. For example, in our model, the optimal price charged for using the road up to location $x$ would simply be the parking tax we have derived (see (12)). Conversely, in the absence of road pricing, parking tariffs can affect road congestion. This needs to be investigated more fully in a model with both local and through traffic (see Glazer and Niskanen [15]), and with route choices endogenous (see also Carrese et al. [9]).

Driver length of stay has no role in our model, and we focus on distance from the CBD as the key determinant of parking externalities. By contrast, Arnott and Rowse [5] concentrate on the time dimension by suppressing distance from a common destination. Likewise, time plays a key role in Calthrop, Proost and Dender [10] and Calthrop and Proost [11]. Both dimensions ought to be jointly considered. This would allow the analysis of time restrictions (as with many parking meters) and prices that are non-linear over time, and how these instruments can be used to complement the distance component of optimal pricing.

Other extensions include a detailed treatment of the interaction between on-street and off-street parking (in particular, to consider that raising on-street parking rates will induce some drivers to drive directly to a parking garage without cruising for parking, which reduces congestion), parking policy in the presence of heterogeneity of individuals (in terms of origin, destination or time costs: see also Adler [1]), and optimal parking violation policy.

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## Further reading

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[^1]:    ${ }^{1}$ See the discussion in Lindsey and Verhoef [18] for more elaboration.
    ${ }^{2}$ They suggest that the optimum in their dynamic model can be attained by a dynamic fee.
    ${ }^{3}$ In the Arnott and Rowse model, in choosing when to start cruising for parking, drivers trade off expected driving time against expected walking time. Since in their model parking entails unpriced congestion while driving docs not, drivers choose a higher-than-optimal expected parking time which corresponds to higher-thanoptimal expected walking time and hence lower-than-optimal expected driving time.

[^2]:    ${ }^{4}$ This perhaps better describe France than the US (where authorities choose to prohibit parking on many major streets). In Anderson and de Palma [3] we model endogenous parking lots.

    5 The private ownership solution we study is also valid if there are many local jurisdictions and each prices parking so as to maximize local revenues from parkers.

[^3]:    ${ }^{6}$ We shall also treat shoppers as male. Lucy Nicholson pointed out that a previous version was sexist for treating shoppers as female.

    7 The long-narrow city suffers from the drawback that it is not symmetric in terms of how drivers should search for parking spots along side streets: drivers arriving on the access roads at the city edges will search inward from there. We do not treat this asymmetry in what follows. A disc city (with radial access roads carrying traffic uniformly from outside) avoids such asymmetry, and the formal analysis would be similar to that below, apart from the additional complexity from the two-dimensional setup. We have chosen to present a long and narrow city for simplicity of exposition. Note that Arnott and Rowse [6] model a one-dimensional city, the circumference of a circle (an anulus).

[^4]:    ${ }^{8}$ This means that an individual who is cruising for parking "forgets" whether she has previously checked on a spot. With a large number of spots to check, the expected cost with replacement or without replacement is not very different.
    ${ }^{9}$ Recall the riddle about how many children on average are needed in order to have a girl. Applying this formula with $q=0.5$, we get the desired answer of 2 . More formally, the probability that the first spot is free is $q$. The probability that the first spot is occupied and the second spot is free is $(1-q) q$, because two searches are involved. The expected number of spots searched is $\Omega=q\left(1+2[1-q]+3[1-q]^{2}+\cdots\right)$. Hence $(1-q) \Omega=q\left([1-q]+2[1-q]^{2}+3[1-q]^{3}+\cdots\right)$. Subtracting the second from the first equation gives $\Omega=1+[1-q]+[1-q]^{2} \cdots=1 / q$.
    10 The astute reader familiar with the economics of congestion externalities will already be able to derive the optimal Pigouvian tax from this formula and considering the expression for $\mathrm{d}[n(x) S(x)] / \mathrm{d} n(x)$. The answer is given below in Eq. (12).

[^5]:    ${ }^{11}$ In a similar vein, the idea of schedule delay as a way to equalize total costs across drivers in equilibrium is to be found in Arnott et al. [5].
    ${ }^{12}$ More precisely, the total cost paid for a shopper whose point of origin is $\bar{X}$ is $S(x)+t_{d}(\bar{X}-x)+t_{w} x$. We can suppress $t_{d} \bar{X}$ since it is constant. In (2) we have also set $t=t_{w}-t_{d}$ where $t_{w}=\beta_{w} / v_{w}$ and $t_{d}=\beta_{d} / v_{d}$. Here $\beta_{w}$ is the value of time for pedestrians while $\beta_{d}$ is the value of time for drivers. Empirically, we have $v_{w}<v_{d}$ and $\beta_{w}>\beta_{d}$ so that $t>0$.

[^6]:    13 We verify that the solution satisfies the non-negativity constraint $n(x) \geqslant 0$.

[^7]:    14 In equilibrium, they will also earn more revenue: if operators bid for lots then profits will be driven to zero and operating revenues will be incorporated in land rents: see Anderson and de Palma [3] for a discussion of the monocentric city model with parking.

[^8]:    15 Just as $t$ is interpreted as the net cost of walking over driving, so can $\alpha$ be interpreted as the net burden from cruising interference of driving over walking. This would allow for $\alpha$ to be negative if many traffic lights slowed down pedestrians more than drivers. If the shopper gets a subway downtown after parking there is no interference to pedestrians and so $\alpha$ is positive. For concreteness we treat positive $\alpha$ in the sequel.

[^9]:    ${ }^{16}$ Similarly, at $x=0, c=\gamma k /[k-n(0)]+\alpha N$.
    ${ }^{17}$ Since $n_{e}^{\prime}(x)$ has the same sign as $\alpha n(x)-t$, the condition $\alpha k-t<0$ is sufficient, but not necessary to have $n_{e}^{\prime}(x)<0$.

[^10]:    ${ }^{18}$ If an improvement in parking information technology allows drivers to improve their search for parking, the overall social gains may be larger than the simple reduction in the search cost. To see this, recall that $c=\gamma+t \bar{x}$ (by (17)), then the total cost incurred per driver falls by more than the decrease in the search cost, $\gamma$, because $\bar{x}$ also falls (average walking distance falls).

[^11]:    ${ }^{19}$ To find the various solutions requires solving first-order differential equations; the constants of integration are then obtained by numerically solving the constraint that all $N$ shoppers park.

[^12]:    ${ }^{20}$ We showed in Section 3 that the monopolistically competitive (diverse ownership) solution diverges from the social optimum when $\alpha>0$ (congestion from cruising). However, if parking lots are perfectly assigned, there would be no cruising. So then there would be no congestion from cruising and the diverse ownership equilibrium would coincide with the first-best optimum.

