High resolution angular measurement using surface-plasmon-resonance heterodyne interferometry at optimal incident wavelengths

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ABSTRACT

We have recently demonstrated that ultra high resolution of angular measurement down to $10^6$ degree can be achieved via surface-plasmon-resonance heterodyne interferometry, in which the phase difference between p- and s-polarized reflected waves is monitored as a function of the incident angle. Here we give a brief summary of this technique and the rationale based on which such a measurement is possible. As a further study, we have also investigated, via simulation, how the change in environmental temperature will affect the resolution limit of this very versatile technique.

Keywords: Interferometry, Surface plasmons, Phase measurement

1. INTRODUCTION

It has been well-known that the optical excitation of the surface plasmon resonance (SPR) at a metal-dielectric interface\cite{1} can be utilized to achieve sensing (or monitoring) of various interfacial phenomena with ultrahigh sensitivity. These include, for example, biosensing\cite{2}, film-thickness sensing\cite{3}, laser-ablation monitoring\cite{4}, and temperature sensing\cite{5,6}. In addition, it has also been demonstrated that this SPR monitoring technique can be applied to achieve very accurate angular measurements, down to $2 \times 10^{-5}$ deg\cite{7}.

In this approach, it is most often to adopt the attenuated total reflection (ATR) method to couple the incident light into the collective oscillation of the free electrons at the interface, leading to a dip in the reflection spectrum which is then monitored to follow any changes that take place in the proximity\cite{8}. In common practice, there are at least four choices of parameters that one can monitor to accomplish the SPR sensing process, these are: (i) the change of the resonant angle ("angular interrogation"); (ii) the change of reflectance at fixed incident angle ("intensity interrogation"); (iii) the change...
of resonant wavelength at fixed incident angle ("wavelength interrogation"), and (iv) the phase difference between $p$- and $s$-polarized light in the reflection spectrum ("phase interrogation").

Among these various monitoring techniques, it has been quite well-established that the "phase interrogation" technique is by far the more sensitive one in many applications. In fact, the above-cited very fine angular measurement was achieved using this technique. Moreover, in this last work (and in most of the previous works utilizing this phase method), all the measurements were done at one fixed incident wavelength. Very recently, we have extended this technique by carrying out the phase measurements at different incident wavelengths, and have discovered that even higher accuracy of angular measurements can be achieved by optimizing the incident wavelengths. In the present work, we shall present a simulation study of the effect on the sensitivity of this method due to the fluctuation of environmental temperatures. Let us start by giving a brief summary on our previous experimental work.

2. RESULTS

The experimental setup is similar to that used in Ref. 7, which is slightly different from the one used previously in Ref. 8, wherein several laser wavelengths were used in our experiment [see Ref. 9 for details]. The light from the source is introduced through a polarizer into an electro-optic modulator (ConOptics) with fast axis in the horizontal direction, and then reflected from a quartz prism which is coated with a thin metal (Ag) film. The laser light then goes through an analyzer. Both the transmission axis of the polarizer and analyzer are at $45^\circ$ relative to the horizontal direction. The light is then detected by a photodetector and the converted electric signal from the photodetector is phase-locked by a lock-in amplifier. A sawtooth signal is applied to the electro-optic modulator and the sawtooth voltage is used as the phase-locking reference. By measuring the signal intensity and by assuming almost unity reflectivity for the $s$ wave, the relative phase difference between the $p$ and the $s$ waves arriving at the photodetector can finally be calculated using Jones calculus.

Theoretically, the relative phase ($\phi_p - \phi_s$) can also be computed using the standard Fresnel formulas. Figure 1 re-displays the previous theoretical results as a function of incident angle for four different wavelengths of light. The Ag film thickness is fixed at 50 nm and the optical constants are obtained from the Drude model. We also show in Fig. 1b the enhancement factor $\gamma$ of the corresponding curves in Fig. 1a. This enhancement factor $\gamma$ is defined as the absolute value of slopes: $\gamma = |\frac{\partial \phi}{\partial \theta}|$, where $\phi$ is the relative phase and $\theta$ is the angle of incidence. The resolution of angle measurement can be determined as $\Delta \theta = \Delta \phi / \gamma$, where $\Delta \theta$ and $\Delta \phi$ are the measurement resolutions for the angle of incidence $\theta$ and for the lock-in amplifier, respectively. The larger the enhancement factor $\gamma$ we can obtain, the better resolution of angular measurement we can achieve. It is clear from these plots that a certain "critical wavelength" ($\lambda_c \sim 900$ nm) exists with respect to which $\gamma$ can be optimized. From the simulation results, two phase change behaviors are observed with a "critical wavelength" $\lambda_c \sim 900$ nm: Case (i): for wavelengths below $\lambda_c$, the overall phase change is about $360^\circ$ across the SPR resonance angle, and Case (ii): for those above $\lambda_c$, the corresponding overall phase change is close to $0^\circ$. This phenomenon has been well-established in the literature, and was previously explained in details via a plot of the...
imaginary versus the real part of the p-wave reflection amplitude, in which the two cases mentioned above were clearly demonstrated for incident wavelengths both below and above the critical value.

We have also demonstrated the above optimization mechanism through some measurements in our previous experiment. Figure 2 displays the well-known SPR reflectance curves from a 50-nm Ag film for each of the four wavelengths in Fig. 1, from which the dip angles are clearly seen for each case. Figure 3 shows the corresponding measurements of the relative phase and the enhancement factor for the four wavelengths with two above and two below \( \lambda_c \), from which we were able to achieve an angular measurement down to \( 1.9 \times 10^{-6} \) deg.

Next we study the effect on the sensitivity of the above measurements due to the fluctuation of environmental temperature via a simple theoretical model. This temperature fluctuation takes place very often in many real applications of the SPR technique. Briefly, we start with the following dielectric function:

\[
e(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega + i\omega_c)},
\]

where \( \omega_c \) is the collision frequency and \( \omega_p \) the plasma frequency given by:

\[
\omega_p = \sqrt{\frac{4\pi Ne^2}{m^*}},
\]

with \( N \) and \( m^* \) the density and effective mass of the electrons, respectively. The collision frequency will have contributions from both phonon-electron and electron-electron scattering:

\[
\omega_c = \omega_{cp} + \omega_{ce}.
\]

As reported previously, for a resonance phenomenon like SPR, one must account for the temperature (T) variation of \( \omega_p \) besides that for \( \omega_c \). As before, \( \omega_p \) will depend on T via volumetric effects as follows:

\[
\omega_p = \omega_{p0} \left[ 1 + \gamma(T-T_0) \right]^{1/2},
\]

where \( \gamma \) is the expansion coefficient of the metal, and \( T_0 \) is a reference temperature taken to be the room temperature. \( \omega_c \) can then be modeled using the phonon-electron scattering model and the electron-electron scattering model, respectively. We thus obtain:

\[
\omega_{cp}(T) = \omega_0 \left[ \frac{2}{5} + 4 \frac{\Gamma T^3}{6} \right]^{1/2},
\]

where \( \theta \) is the Debye temperature and \( \omega_0 \) is a constant to be determined from the static limit of the above expression together with the knowledge of the d.c. conductivity. In addition, we have:

\[
\omega_{ce}(T) = \frac{1}{12} \pi^3 \frac{\Gamma \Delta}{\hbar E_F} \left[ (k_B T)^2 + (\hbar \omega / 2\pi)^2 \right],
\]

where \( \Gamma \) and \( \Delta \) are defined in Ref. 7. Thus Eqs. (1) – (6) together with the well-known Fresnel equations provide a complete model for the simulation of the SPR sensor response to the variation of the temperature of the environment. The reflectance and phase are then calculated. Fig. 4a shows the simulated maximum enhancement factor \( \gamma \) as a function of incident wavelength at room and several elevated temperatures, from which the role of the critical wavelength can be
clearly seen. We have also plotted the four data obtained in our experiment\(^9\) at room temperature in Fig. 4a. It is seen that just like the comparison between the theoretical and experimental results in Figures (2) and (3), only qualitative agreement was obtained in our present work between theory and experiment. This is likely due to the uncertainty of many factors which include: the exact value of the Ag film thickness; possible oxidation of the film; the accuracy of the optical constants\(^7\) used in the simulation; and the limitation in the resolution of the measurements close to resonance. Furthermore, it is observed that, as the temperature increases, the critical wavelength becomes shorter in value while \(\gamma_{\text{max}}\) varies from about 5000 to 8000 for a large range temperature from room to almost the melting point of the metal. However, when these values are used into the definition \(\Delta\theta = \Delta\phi / \gamma\), it is found that the maximum resolution is relatively insensitive to temperature as shown in Fig. 4b. One only loses a small amount in angular resolution at highly elevated temperatures. Fig. 5 shows the critical wavelength as a function of temperature, from which the blue-shift in critical wavelength for higher temperatures can clearly be seen.

3. CONCLUSIONS

In this work, we have demonstrated, both theoretically and experimentally\(^9\), that the recent finely-resolved angular measurement\(^8\) via SPR heterodyne interferometry can be further improved, by optimizing the incident wavelengths with respect to a certain critical value for the optical excitation of the surface plasmon. The physical origin of this critical wavelength is complicated\(^7\), it depends crucially on two damping rates of the surface plasmon, namely, the internal damping and the radiative damping as defined in Ref. 1.\(^7\) It can be argued that in the case with smaller internal damping, longer incident wavelength will definitely lead to finer resolution in the angular measurement\(^7\). We have further studied the effect of temperature fluctuation on this technique via simulation, and have come to the conclusion that while the value of the critical wavelength is blue-shifted as temperature rises, the maximum possible resolution remains relatively constant (in order of magnitude) and is obtained to be about 1.9x10\(^-4\) deg in our recent experiment\(^9\).

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Fig. 1. Theoretical results of (a) relative phase ($\phi_p - \phi_n$) and (b) corresponding enhancement factor $\gamma$ as a function of incident angle for four different wavelengths of light.
Fig. 2. Experimental results for the SPR reflectance curves corresponding to the four wavelengths used in Fig. 1.
Fig. 3. Experimental results of (a) relative phase ($\phi_r - \phi_i$) and (b) corresponding enhancement factor $\gamma$ as a function of incident angle for the four wavelengths used in Fig. 2.
Fig. 4. (a) Theoretical and experimental results of the maximum enhancement factor $\gamma$ as a function of incident wavelength. Note that while the experimental data were obtained at room temperature, the theoretical results are shown at several room and elevated temperatures. (b) Values of maximum angular resolution calculated from $\gamma_{\text{max}}$ at the optimal (critical) wavelength as a function of temperature.
Fig. 5. Critical wavelength as a function of temperature