Nonlocal optical effects on the Goos-Hänchen shift at an interface of a composite material of metallic nanoparticles

J. H. Huang*
Department of Electronic Engineering, Oriental Institute of Technology, Taipei, Taiwan, Republic of China

P. T. Leung
Department of Physics, Portland State University, P. O. Box 751, Oregon 97207-0751, USA

Abstract

We present a theoretical study on the nonlocal optical effects on the Goos-Hänchen (GH) shift of reflected light from a composite material of metallic nanoparticles (MNP). Using different nonlocal effective medium models, it is observed that such effects can be significant for small MNP of sizes down to a few nanometers. For small metallic volume fractions, the composite behaves like dielectric and the nonlocal effects lead to significant different Brewster angles at which large negative GH shifts take place. For larger volume fractions or shorter wavelengths, the composite behaves more like metals and the nonlocal effects also lead to different Brewster angles but at values close to grazing incidence. These results will have significant implications in the application of different effective medium models for the characterization of these nano metallic composites when the MNP’s are down to a few nanometers in size.

*Corresponding author: ff021@mail.oit.edu.tw
Introduction

The intriguing lateral movement of a light beam upon reflection from a material surface was first observed by Goos and Hänchen (GH) in 1947 under the condition of total internal reflection [1]. Since then, such effect has been found to be a rather universal phenomenon due to the wave nature of light and can take place under general partial reflection [2] from all kinds of materials including homogeneous (e.g. metal, semiconductors,…etc.) and heterogeneous (e.g. composites, alloys,…etc.) materials [3]. In several recent publications [4-7], these GH shifts have been studied for metallic nanocomposite interfaces with particular interest, respectively, in the temperature effects [4], nonlinear effects [5], MNP clustering effects [6], and anisotropic effects [7]. However, in all these studies, the optical response of the composite has been assumed to be only temporal dispersive so that the dielectric constants of the metal is limited to have only frequency dependence.

It is well-known, however, when MNP’s are down to very small sizes (below ~ 10 nm), spatial dispersion can no longer be ignored which will lead to dielectric functions having dependence on both the wave vector and the frequency [8-10]. This is originated from the fact that the surface-to-volume ratio becomes significant for small particles and hence the large fluctuation of the fields across the surface becomes rather prominent, leading to a significant dependence of the dielectric function on the wave vector in the Fourier space.
These (spatial) nonlocal optical effects for small MNP’s have been studied intensively over several decades both for an isolated MNP [8-10], as well as composites of MNP’s [11, 12]. It has been justified in Ref. [11] that the commonly-used mean field theories for composite materials [13, 14] can still be applied in the presence of spatial dispersion (nonlocality) provided that one can reduce the dielectric response to an “effective local” description with all the wave vector dependence integrated over for a given frequency [9, 11]. This clarification has thus greatly simplified the account for the nonlocal optical response of a nanocomposite of ultra-small MNP’s.

It is the purpose of this work to study these nonlocal effects on the GH shifts at the interface of a nanocomposite of ultra-small MNP’s. We shall follow previously-established approaches [11, 12] and adopt the effective local models derived from Refs. [9] and [10], together with the well-established effective medium theories in the literature [13, 14] to account for the dielectric response of the composite; and shall limit ourselves to the case of random distributions of MNP’s in the insulating host without any formation of metal clusters [15]. In the literature, we are aware of only the works by Birman and coworkers [16-18] which have studied the nonlocal effects on the GH shifts. However, these previous works had focused on total internal reflection with the incident beam undergoing the GH shift inside a nonlocal (dielectric) medium which leads to two possible beam shifts correspond to the excitation of two exciton polaritons. In our present study, however, the incident beam
propagates in an ordinary (local) medium (e.g. vacuum) and is partially reflected from a metallic composite medium with the nonlocal response coming from the free electrons of the metallic MNP’s (see Fig. 1). Of course, strictly speaking, any such reflected beam will have experienced certain nonlocal response even at an interface of a homogeneous medium. However, such “interfacial spatial dispersion effects” are usually exceedingly small and will be negligible in our present study [19]. Thus the nonlocal effects in our study here are mainly from the surface plasmons excited in the MNP’s of the composite.

Model

We first give a brief summary of the effective medium models previously applied [11, 12] to the description of the optical properties of a nanocomposite of spherical MNP’s (Fig. 1). Since our main purpose here is to elucidate the qualitative features of the nonlocal effects via some model studies, for a two-phase system (with phase index \(i\), \(i = 1\) for the MNP’s and \(i = 2\) for the dielectric host), we shall simply use the two popular models - Maxwell-Garnett (MG) model [13] and Bruggeman (B) model [14] - to describe the effective dielectric properties of the composite. Since to our knowledge, the exact domain of applicability for each of the MG and B models can be controversial at times, except that they both give similar results for small inclusions of low volume fractions [20], we simply show the results from each model over a broad range of volume fractions in our numerical illustrations. In general, the MG model is accurate for low concentrations of inclusion particles and the B model is
more appropriate when the concentrations of the inclusion and the host are close to each other (i.e. ~ 50% each). Thus we have the effective dielectric functions given by, for each model:

\[ \frac{\varepsilon_{MG} - \varepsilon_i}{\varepsilon_{MG} + 2\varepsilon_i} = f_j \frac{\varepsilon_j - \varepsilon_i}{\varepsilon_j + 2\varepsilon_i}, \quad (1) \]

and

\[ \frac{(\varepsilon_h - \varepsilon_i)f_i}{2\varepsilon_h + \varepsilon_i} = \frac{(\varepsilon_j - \varepsilon_h)f_j}{2\varepsilon_h + \varepsilon_j}, \quad (2) \]

respectively, where \( f_i + f_j = 1 \) with \( f \) the volume fraction of each of the two phases. Note that \( i \) and \( j \) are symmetric in (2) but not in (1).

While the dielectric function \( (\varepsilon_z) \) for the host is taken to be a real constant, that for the MNP \( (\varepsilon_i) \) will be considered in the following two cases:

**For local response**

In this case we simply use the following Drude model for \( \varepsilon_i \) in Eqs. (1) and (2):

\[ \varepsilon_i(\omega) = \varepsilon_r - \frac{\omega_p^2}{\omega(\omega + i\gamma)} , \quad (3) \]

where the plasmon frequency and damping constant are characteristics of the free electrons and \( \varepsilon_r \) accounts for the response of the bound electrons.

**For nonlocal response**

Although the most accurate account for the nonlocal optical response of small MNP will be from a quantum mechanical approach such as time dependent density functional theory (TDDFT) and its variations [21], here we shall adopt simpler
phenomenological approaches with the implementation of the following two models which have been found effective leading to reasonable account of the nonlocal effects in the literature.

Model 1: the nonlocal free electron gas model

Here we follow the semi-classical infinite barrier (SCIB) model established by Fuchs and coworkers [9], among others, with the following “effectively-local” dielectric function for the nonlocal multipolar response of a metal sphere:

\[
\tilde{\varepsilon}_j(\omega) = \left[ \frac{2}{\pi} (2\ell + 1) a \int_0^\infty j_\ell^2(ka) \frac{\omega}{\varepsilon(k,\omega)} \, dk \right]^{-1}.
\] (4)

The above is derived by introducing a fictitious continuation of the metallic region beyond the radius of the MNP, together with an additional boundary condition which leads to the discontinuity of the radial component of the displacement vector. The electric field can thus be made continuous across the MNP surface and the nonlocal response of which can then be obtained using Fourier transform techniques [9].

Note that the dependence on the mode order and the sphere radius \(a\) is clear in Eq. (4), and \(\varepsilon(k,\omega)\) is the specific nonlocal function to be adopted, while \(j_\ell\) in is the spherical Bessel function. To apply (4) to (1) and (2), we shall only consider the \(\ell = 1\) term since both the conventional MG and B models were derived by limiting to dipolar response of the MNP’s. Hence we have in this case for the dielectric function of the MNP in Eqs. (1) and (2) in the following form:
\[ \varepsilon_i = \tilde{\varepsilon}_i(\omega) = \left[ \frac{6}{\pi} a \int_0^\infty \frac{j_i^2(ka)}{\varepsilon(k,\omega)} \, dk \right]^{-1} = [3aF(a)]^{-1}, \quad (5) \]

where the integral \( F(r) \equiv \frac{2}{\pi} \int_0^\infty \frac{j_i(kr) j_i(ka)}{\varepsilon(k,\omega)} \, dk \) is as defined in Refs. [9] and [11].

Note that the result in Eq. (5) can be further improved by imposing the additional condition to require the normal (radial) component of the polarization in the MNP to vanish at the boundary. This will lead to the following modification of (5):

\[ \varepsilon_i(\omega) = \tilde{\varepsilon}_i(\omega) = \left[ \frac{3aF(a) + K}{1 + \varepsilon_L K} \right]^{-1}, \quad (6) \]

where \( K \equiv \frac{1}{\varepsilon_L - 1} \left\{ 3a^2 \left[ \frac{dF(r)}{dr} \right]_{r=a} - 1 \right\} \) and \( \varepsilon_L(\omega) \equiv \varepsilon(k = 0, \omega) \) is the local limit of the dielectric function as defined in Refs. [9] and [11], which will simply be the one given in Eq. (3) for our application below. In practice, it has been found that the numerical difference between Eqs. (5) and (6) is only minimal for typical nonlocal models [9].

Next we have to input a model for the nonlocal dielectric function in the integral of Eq. (5). The simplest one will be that obtained from the random-phase approximation for a quantum free electron gas [22, 23], which to the lowest order correction in wave number leads to the simple hydrodynamic model [23]:

\[ \varepsilon(k, \omega) = \varepsilon_r - \frac{\omega_p^2}{\omega^2 + i\omega \gamma - \beta^2 k^2}, \quad (7) \]

where \( \beta^2 = \frac{3}{5} v_F^2 \) with \( v_F \) the Fermi velocity which reduces back to the Drude model in Eq. (3) for \( \beta = 0 \). Although the model in (7) looks over-simplified and sometimes
leads to predictions inconsistent with those from TDDFT calculations for certain metals (e.g. alkali metals); it is self-contained with no adjustable parameters, and found to give reasonable trends for noble metals as revealed from some recent studies of MNP’s made with these metals [24-26].

Model 2: the d-parameter model

To go beyond the limitations of Model 1 as elaborated above, we follow a different approach to account for the nonlocal response of the MNP established by Apell and coworkers [10], among others. Following Feibelman’s d-parameter theory [27], Apell et al had introduced the following modified dipole polarizability for a metal sphere in terms of a parameter $d$, which accounts for the deviation in the distribution of the surface electrons from the geometrical boundary (radius $a$) due to the quantum nature of these electrons (to lowest order in $1/a$):

$$\alpha = a^3 \frac{(\varepsilon_1 - \varepsilon_2)(1 - d_e/a)}{\varepsilon_1 + 2\varepsilon_2 + 2(\varepsilon_1 - \varepsilon_2)d_e/a},$$  \hspace{1cm} (8)

where the dielectric function $\varepsilon_1$ is that of the background (host) and $\varepsilon_2$ that of the metal given by the Drude model in Eq. (3). Now let us define an “effective dielectric function” for the metal according to (8) and call it $\zeta$ which is required to satisfy the following relation:

$$\frac{\zeta - \varepsilon_2}{\zeta + 2\varepsilon_2} = \frac{(\varepsilon_1 - \varepsilon_2)(1 - d_e/a)}{\varepsilon_1 + 2\varepsilon_2 + 2(\varepsilon_1 - \varepsilon_2)d_e/a} \equiv A,$$ \hspace{1cm} (10)

which can be solved to yield:
\[ \zeta = \left( \frac{1 + 2A}{1 - A} \right)^{\varepsilon_2}. \]  

(11)

Hence by using this \( \zeta \) in place of the metal dielectric function \( \varepsilon_j \) (i.e. \( \varepsilon_i \)) in both the MG and B models, we can obtain a nonlocal effective function for the composite as a function of the parameter \( d_r \). Note that this \( d_r \) parameter can in principle be obtained either from microscopic theories or from fitting experimental data. But since its value is specific to the system under study and depends on many factors such as the dielectric properties of the background and the kind of metals, here we shall treat it as an adjustable parameter to mimic different possible real systems. In practice, the magnitude of this parameter will be in the order of 0.1 nm [10]. In addition, for simplicity, we shall not account for the dispersion property of \( d_r \) in this parameterization approach.

Thus Eqs. (1), (2), (3), (6) or (11) provide a complete description of our nanocomposite in both the cases of local and nonlocal optical response, according to each of Model 1 and Model 2, respectively. Using these optical properties, we can now calculate the GH shifts for a beam incident on these composites using the stationary phase method as follows [28]:

\[ D = -\frac{1}{k} \frac{d\phi}{d\theta}, \]

(12)

where \( D \) is the parallel beam shift (which can be projected to give the lateral shift along the interface), \( \theta \) the incident angle, and \( \phi \) the phase of the complex reflection coefficient which can be obtained from the Fresnel formula. The above Artmann formula (12) is the simplest theory for GH shift which considers a broad but finite extended incident wave with a small
distribution of wave numbers about a central $k_0$, since one must go beyond the simple plane-wave incidence to introduce any shift of the beam. Leading order expansion of the amplitude and phase for the Fourier components of the reflected field then leads to the result in Eq. (12). Note that for GH shifts upon total internal reflection, it is known that (12) is not accurate when being close to the critical angle incidence. But for our present problem which involves only partial reflection, it was found that (12) has enough accuracy (and simplicity) provided that the beam width is broad enough [7].

**Numerical results**

We have studied the above nonlocal effects from silver MNP on the GH shifts for TM incident waves at a surface of a nanocomposite of these particles using the hydrodynamic nonlocal dielectric function for silver according to Model 1. In addition, we have also studied similar effects predicted from Model 2 assuming $d_i$ as an adjustable parameter. In both models, the local limit is the same and is given by the Drude model in Eq. (3) with parameters for silver taken from Ref. [7], except that we have added a surface damping term to the Drude damping constant: $\gamma \rightarrow \gamma + \frac{A v_F}{a}$, with $A \sim 1$ [29]. Without loss of generality, we just set $A = 1$ in all our numerical computations, while the Fermi velocity for silver is taken to be $1.40 \times 10^6$ m s$^{-1}$. The host dielectric is simply taken to be glass with a dielectric constant of 2.25. For each of the MG and B models, we have calculated the GH shifts as a function of the incident angle within a broad range of metallic volume fractions. Since in the
nonlocal model, the effective dielectric function depends not only on the volume fraction but also on the MNP sizes as implied from Eqs. (6) and (11), we have first to clarify this size effect on the shifts. As was indicated in the previous works [8-12, 24], one expects the nonlocal effects to be more significant only for smaller particles due to the large surface-to-volume ratio of these particles. Figure 2 illustrates this by plotting the GH shifts at a fixed wavelength for both the local and nonlocal MG and B models according to Model 1 at a fixed volume fraction of 0.1 for two MNP sizes. It is clear that while the size effect in the local case only enters via the surface damping term leading to smaller maximum negative GH shifts (with unchanged “Brewster dip angles”, see below), the nonlocal results for the composite with 5 nm MNP’s deviate more significantly (in terms of the dip angle and amplitude) from the local ones in comparison to those from the 10 nm MNP’s. Hence in the following, we shall only study composites with 5 nm MNP’s for the manifestation of the nonlocal effects.

Figure 3 shows the GH shifts according to both the local and nonlocal MG models at two wavelengths, where the nonlocal response of the MNP is characterized by Model 1. As established previously [6, 30], a distinct signature in the GH shifts at a metallic substrate surface versus that of a dielectric is that while for the latter case, large negative shifts take place at an acute Brewster angle, the metallic case will have large negative shifts at angles close to grazing incidence. One thus sees that while at a longer wavelength (1.15
the composite does behave like a dielectric according to the MG models (both local and nonlocal), the results in Fig. 3(a) show that the nonlocal effects will lead to smaller values for the Brewster angles and larger negative shifts, with such effects becoming more significant for larger volume fraction of metal. This can be understood by comparing the effective dielectric functions from both MG models [31], from which one can show that the nonlocal effects from Model 1 yield blue-shifted resonance and hence smaller (absolute) values for both the real and imaginary parts of the effective dielectric function at low frequencies. Note that the blue-shifted resonance obtained from this phenomenological approach [8, 9, 11, 12, 15, 24-26] using nonlocal models such as the hydrodynamic (or the more accurate random phase approximation [23]) has been controversial in the literature, since other approaches using density functional theories (DFT) had reported red-shifts instead [32]. However, while the DFT red-shifts are justified for alkali metals, it is known that the situation could be reversed for noble metals due to reduced s-d screening [21, 29, 33, 34]. As expected, when the volume fraction increases, greater Brewster angle shows up which is a tendency towards metallic behavior. Furthermore, from Fig. 3(b) which is for a shorter incident wavelength (0.632 \(\mu\)m), one observes the following trends: (i) smaller negative GH shifts and greater corresponding Brewster angles, (ii) relatively more prominent are the nonlocal effects, and (iii) metallic behavior to take place at high enough volume fractions (\(f \sim 0.8\)). Most of these can be understood by again referring to the effective dielectric functions (local and nonlocal)
which show more obvious metallic behavior at shorter wavelengths and larger metallic occupancy [31]. The most interesting result is that at a volume fraction \( f \) of \( \sim 0.7 \), the nonlocal effects can switch the composite to a dielectric behaving system from a metallic one as described by the local theory at this shorter wavelength. Note also that the resonance in the effective bulk dielectric functions undergoes a red shift with increase of \( f \) [12], and for the range of \( f \) considered here (0.1 – 0.8), this resonance lies within \( \sim 0.4 – 0.7 \) micron with the nonlocal effects further decrease these values by \( \sim 10\% \). Hence the two wavelengths chosen here are longer than the resonance of most cases, except perhaps for the highest \( f \sim 0.8 \). The above qualitative difference in the description of the composite from the two models (local vs nonlocal) should have some significant implication towards the understanding of the behavior of a composite using various effective medium theories.

Next we study the possibility of going beyond the predictions of Model 1 by allowing for both possible red and blue shifts in the surface plasmon resonance due to nonlocal effects. To achieve this we apply Model 2 with both positive and negative values for the real part of the parameter \( d_r \) introduced in Eq. (8). Incidentally, we find that by choosing \( \text{Re}(d_r) = +0.12 \) nm the results in Model 1 can be closely reproduced. Fig. 4 shows the results for both \( \text{Re}(d_r) = +0.12 \) nm and \( \text{Re}(d_r) = -0.12 \) nm with a zero imaginary part in comparison with the result from local theory. Note that the different signs of \( \text{Re}(d_r) \) simulate outward relaxation (-) and inward compression (+) of surface charges, respectively, which correspond
to red (-) and blue (+) shifts of the surface plasmon resonance for the MNP [10]. It is clear that while Model 2 yields very similar qualitative results compared to those from Model 1 for the GH shifts as a function of incident wavelength, incident angle, and volume fraction of the MNP; the case with a negative $\text{Re}(d_r)$ will lead to changes in the Brewster angles towards greater values along with smaller dip value for the GH shifts. Again, this behavior can be understood with reference to the nonlocal effective dielectric function in this case which exhibits a red shift in the surface plasmon resonance [31].

Figure 5 and 6 show results similar to those in Figs 3 and 4, respectively, but instead for the Bruggeman model. In contrast to the MG models, it is seen that the B model predicts the composite to behave metallic-like for volume fractions $f > 0.4$ in both the local and nonlocal cases. This is likely correlated with the well-known fact that the B model has a percolation threshold at a relatively low value of $f \sim 0.3$ for spherical particles. For a shorter wavelength (0.632 µm), the sharp “Brewster dip” starts to disappear at even lower values of $f$, (see Figs. 5 (b) and 6(b)). Moreover, the negative GH shifts at the Brewster dip for lower volume fractions are broader and not as large in (absolute) magnitude compared to those predicted from the MG model. In addition, the nonlocal effects here can shift the Brewster resonance to either smaller angles in case of small $f$ at long wavelength or larger angles in case with shorter wavelength. This occurs for both Model 1 and Model 2, with Model 2 predicting again opposite behaviors with different signs for $\text{Re}(d_r)$. But in all cases with a
blue-shift in the resonance, nonlocal effects will lead to more negative GH shifts at the
Brewster dips. Again, these can be understood from plotting the effective dielectric functions
(not shown) and compared the local and nonlocal Bruggeman models [31]. It turns out that
unlike the MG model, the effective dielectric function from the B model in the local case
does not exhibit any resonance over a relatively large range of frequencies. In the nonlocal
case, however, the B model shows a resonance which is rather insensitive to the value of the
volume fraction. As a result, in the B model, the nonlocal dielectric functions turn out to
have the real part either smaller (for very small $f$'s), or greater (for larger $f$'s) than that in
the local case. Moreover, the imaginary part in the nonlocal case is still mostly smaller than
that in the local case leading to the more negative GH shifts at the Brewster dip angles.
Finally, we have studied the effect of introducing an imaginary part the parameter $d_r$ in
Model 2. Fig. 7 shows the GH shifts for both positive and negative values of $\text{Im}(d_r)$. Since
the surface absorption in this model is proportional to the negative of $\text{Im}(d_r)$, we see that
with a more negative $\text{Im}(d_r)$, the GH dip becomes less deep and broadened [Fig. 7(a)]. With
a positive $\text{Im}(d_r)$, however, a reversed behavior can be seen with the negative GH shifts
[Fig. 7(b)] until the positive $\text{Im}(d_r)$ becomes a little too large (implying a significantly-
decreased absorption), the GH shifts turn positive similar to the case for a nearly-transparent
dielectric.
Discussion and Conclusion

Several recent works [4-7] have studied the GH shifts at a nanocomposite interface focusing on various physical effects, but the optical response has been restricted exclusively to be only frequency dependent (i.e. temporal dispersive). In this work, we have studied the effects due to the nonlocal optical response of the MNP’s of relatively small sizes (< 10 nm), thus accounting for spatial dispersion effects. Based on previously-established rigorous formulation of various nonlocal effective medium theories [11], we have observed some significant different results compared to those from the local response theories [4-7], to the extent that the nonlocal effects can under certain conditions, “flip” between the metallic and dielectric behaviors for a composite of MNP’s.

It is of interest to note that although such nonlocal effects have been studied for almost half a century, it is only recently that significant focus on them has been brought up due to the advent in plasmonics and nanoparticle science [25, 26]. However, for most nano-systems or systems of planar interfaces, these nonlocal effects are usually found to be rather small and only barely observable for systems of ultra small sizes (e.g. ultra small particles, ultra thin films,…etc.). Our study has thus shown that a composite made of such nanoparticles can be an ideal system for such nonlocal effects to be manifested and be observed since both the fabrication and experimentation with such systems are relatively accessible to modern technology. In doing so, one can also test various effective medium
models and see, for example, how the effects compared between those predicted by the nonlocal Maxwell-Garnett and Bruggeman models.

To conclude, we would like to point out that there is still room for improvements of our present theoretical prediction since the theory in [11] has not considered the nonlocality due to *inter*-particle interaction [35]. Future theoretical work in this direction should lead to better description of these nanocomposite and a better understanding of the optical response of these systems.

**Acknowledgments**

Discussion with Prof. Railing Chang is gratefully acknowledged.
References


[31] Note that in order to limit the number of figures in our paper, the plots of the dielectric functions are not included but are available upon request.


Figure captions:

1. Geometric configuration of the problem.

2. Nonlocal effects on the GH shifts from MNP’s of different sizes for both of (a) MG and (b) B models. Solid lines are for MNP’s of 5 nm in radius while dashed lines are for 10 nm MNP’s. The dashed-dotted and dotted lines are results from local (Drude) model for the 5 nm and 10 nm MNP’s, respectively. The incident wavelength is fixed at 1.55 µm and the volume fraction at 0.1. Only Model 1 is applied in this calculation.

3. Nonlocal effects for the MG model according to Model 1 (see text) over a broad range of MNP volume fractions at a wavelength of (a) 1.15 µm and (b) 0.632 µm. Solid lines are for nonlocal results and dotted lines are for local ones. The MNP size is fixed at 5 nm.

4. Similar to Fig. 3, but for results from Model 2 (see text) at a wavelength of (a) 1.15 µm and (b) 0.632 µm. The $d_r$ parameter is chosen as $d_r = 0.12$ (solid line), $d_r = 0$ (i.e. local result, dashed line), and $d_r = -0.12$ (dotted line), respectively, with the imaginary part of $d_r$ set zero.

5. Similar to Fig. 3, but for the B model.

6. Similar to Fig. 4, but for the B model.

7. Study of the effect due to the imaginary part of the parameter $d_r$; this ranges from $-0.05$ to $+0.05$ with the real part fixed at 0.12. Volume fraction is fixed at 0.1.
Fig. 2
Fig. 3

(a) Angle of incidence (deg.)

(b) Angle of incidence (deg.)

DTM (m)

f = 0.1
f = 0.4
f = 0.7
f = 0.8
Fig. 4
Fig. 5
Fig. 6

(a) Angle of incidence (deg.)

(b) Angle of incidence (deg.)

Fig. 6
Fig. 7