Questions was: How many strata?

Stat 576
10-24-17

①

Assume that Dalenius-Hodges demarcation and Neyman allocation will be used.

$$V(\bar{y}_{str}) \approx \frac{1}{n}\left(\sum_{h=1}^{H} W_h S_h\right)^2$$

$$\approx \frac{1}{n}\left(\sum_{h=1}^{H} \frac{a_h^2}{\sqrt{12}}\right)^2 \quad \text{where } a_h = \sqrt{f_h}\,(y_h - y_{h-1})$$

$$\text{let } \bar{a} = \frac{1}{H}\sum_{h=1}^{H} a_h$$

②

If D-H demarcation is carried out correctly, all of the a_hs will equal a constant $\bar{a}$

$$\text{So } V[\bar{y}_{str}] \approx \frac{1}{n}\left(\sum_{h=1}^{H} \frac{\bar{a}^2}{\sqrt{12}}\right)^2$$

$$= \frac{1}{n}\left(H \frac{\bar{a}^2}{\sqrt{12}}\right)^2 = \frac{H^2 \bar{a}^4}{12n}$$

$$= \frac{H^2\left(\frac{1}{H}\sum_{h=1}^{H} a_h\right)^4}{12n} = \frac{1}{H^2\,12\,n}\left(\sum_{h=1}^{H}\sqrt{f_h}\,(y_h - y_{h-1})\right)^4$$

For planning purposes, suppose that the stratum widths will be constant $= l$

Now $V[\bar{y}_{st}] \approx \dfrac{1}{12nH^2} l^4 \left(\sum_{h=1}^{H} \sqrt{f_h}\right)^4$

Say that we are given a desired margin of error, E

Set $E = z_{\alpha/2} \sqrt{\dfrac{l^4}{12nH^2} \left(\sum_{h=1}^{H} \sqrt{f_h}\right)^4}$

+ solve for H

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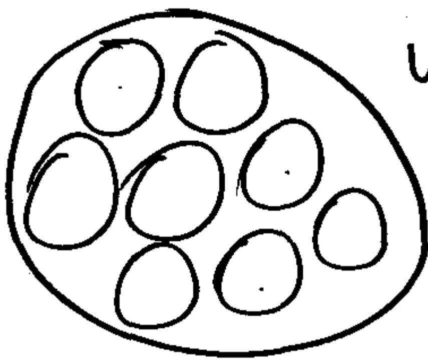
$$H = \frac{z_{\alpha/2}\, l^2 \left(\sum_{h=1}^{H} \sqrt{f_h}\right)^2}{E\sqrt{12n}}$$

Cluster Sampling



Use a SRSWOR of Clusters, and take every item in those clusters

(5)

PSU = primary sampling units = clusters

SSU = secondary sampling units = items in the clusters

N = # of PSUs in the population
= # clusters in population

n = # clusters in sample

M_i = # of items in cluster i

$K = \sum_{i=1}^{N} M_i$ = # items in population

t_i = total of the Y-values in i^{th} cluster (6)

$t = \sum_{i=1}^{N} t_i$ = population total of Y-values

$\bar{Y} = \frac{t}{K}$ = population mean , $\bar{T} = \frac{t}{N}$ = average cluster total

$\bar{Y}_i = \frac{t_i}{M_i}$ = i^{th} cluster mean

$$S^2 = \frac{\sum_{i=1}^{N}\sum_{j=1}^{M_i}(Y_{ij} - \bar{Y})^2}{K-1}$$

Say we want to estimate t. ⑦

$$\text{Try } \hat{t} = N\bar{t}$$
$$= N\frac{1}{n}\sum_{i=1}^{n} t_i$$
$$= \frac{N}{n}\sum_{i=1}^{n}\sum_{j=1}^{M_i} Y_{ij}$$

$$E[\hat{t}] = E[N\bar{t}] = NE[\bar{t}] =$$
$$= N\bar{T} = N\left(\frac{t}{N}\right) = t$$

⑧

$$V[\hat{t}] = V[N\bar{t}] = N^2 V[\bar{t}]$$
$$= N^2 \frac{S_t^2}{n}\left(1 - \frac{n}{N}\right), \text{ where}$$
$$S_t^2 = \frac{\sum_{i=1}^{N}(t_i - \bar{T})^2}{N-1}$$

$$\hat{V}[\hat{t}] = N^2 \frac{s_t^2}{n}\left(1 - \frac{n}{N}\right), \text{ where}$$
$$s_t^2 = \frac{\sum_{i=1}^{n}(t_i - \bar{t})^2}{n-1}$$

(9)

ANOVA decomposition:

$$\sum_{i=1}^{N}\sum_{j=1}^{M_i}(y_{ij}-\bar{y})^2 = \sum_{i=1}^{N}\sum_{j=1}^{M_i}(y_{ij}-\bar{y}_i+\bar{y}_i-\bar{y})^2$$

$$= \underbrace{\sum_{i=1}^{N}\sum_{j=1}^{M_i}(y_{ij}-\bar{y}_i)^2}_{SSW} + \underbrace{\sum_{i=1}^{N} M_i(\bar{y}_i-\bar{y})^2}_{SSB} + 0$$

Since $V[\hat{t}]$ involves a term similar to SSB,
we want SSW to be large & SSB to be small.

(10)

That is, we want heterogeneity within
& homogeneity between clusters

To estimate $\bar{y}$, use $\bar{y}_{clus} = \dfrac{\hat{t}}{K}$

$$E[\bar{y}_{clus}] = \frac{E[\hat{t}]}{K} = \frac{t}{K} = \bar{y}$$

$$V[\bar{y}_{clus}] = \frac{1}{K^2}V[\hat{t}] = \frac{1}{K^2}N^2\frac{S_t^2}{n}\left(1-\frac{n}{N}\right)$$

$$\hat{V}[\bar{y}_{clus}] = \frac{1}{K^2} N^2 \frac{s_t^2}{n}\left(1-\frac{n}{N}\right)$$

Special case: $M_i \equiv M$

Then $K = NM$

$$\bar{y}_{clus} = \frac{\hat{t}}{K} = \frac{N\bar{t}}{NM} = \frac{\bar{t}}{M}$$

$$V[\bar{y}_{clus}] = \frac{1}{\cancel{N^2} M^2} \cancel{N^2} \frac{S_t^2}{n}\left(1-\frac{n}{N}\right) = \frac{1}{M^2}\frac{S_t^2}{n}\left(1-\frac{n}{N}\right)$$