Stat 571
11-5-13

①

Suppose that the variables $X_1, X_2, \ldots, X_p$ are independent and we find confidence intervals for $\mu_1, \mu_2, \ldots, \mu_p$, each with confidence $1-\alpha$.

Then the joint confidence would be $(1-\alpha)^p$

<u>Bonferroni Method</u>: Let $E_1, \ldots, E_m$ be events with probabilities $1-\alpha_1, \ldots, 1-\alpha_m$.

$$P\left(\bigcap_{i=1}^{m} E_i\right) = 1 - P\left(\overline{\bigcap_{i=1}^{m} E_i}\right) = 1 - P\left(\bigcup_{i=1}^{m} \overline{E_i}\right)$$

②

$$\geq 1 - \sum_{i=1}^{m} P(\overline{E_i}) = 1 - \sum_{i=1}^{m} \alpha_i$$

So $P\left[\bigcap_{i=1}^{m} E_i\right] \geq 1-\alpha$, where $\alpha = \sum_{i=1}^{m} \alpha_i$

If you apply the Bonferroni method to simultaneous confidence intervals for $\mu_1, \ldots, \mu_p$, the individual confidence intervals should be done at the $1-\frac{\alpha}{p}$ level.

$$\bar{X}_i \pm t_{\frac{\alpha}{2p},\, n-1\, df} \frac{S_i}{\sqrt{n}} \qquad \text{Bonf.}$$

(3)

$$\bar{X}_i \pm \sqrt{\frac{p(n-1)}{(n-p)} F_{p,n-p}(\alpha)} \; \frac{S_i}{\sqrt{n}} \qquad \text{Scheffé}$$

Two-sample tests

Special case: paired comparisons, or "matched pairs"

We measure p pairs of responses on each of n observations.

(4)

let X_{11i} = treatment 1, variable 1, obs i

X_{12i} = " 1, " 2, " i

$\vdots$

X_{1pi} = " 1, " p, " i

X_{21i} = " 2, " 1, " i

$\vdots$

X_{2pi} = " 2, " p, " i

let $D_{i1} = X_{11i} - X_{21i}$

$\vdots$

$D_{ip} = X_{1pi} - X_{2pi}$

$\underset{n\times p}{D}$ is the data matrix

(5)

Assume that the $\vec{D}_i$ are iid $N_p(\vec{\delta}, \Sigma_d)$

Test $H_0: \vec{\delta} = \vec{0}$

$H_1: \vec{\delta} \neq \vec{0}$

$$T^2 = n(\bar{d} - \vec{0})' S_d^{-1} (\bar{d} - \vec{0})$$

$$= n \bar{d}' S^{-1} \bar{d}$$

Reject H_0 if $T^2 > \frac{p(n-1)}{n-p} F_{p,(n-p)}^{(\alpha)}$

(6)

Example 6.1 from text

(bio-chem O_2 demand) BOD diff	(suspended sediment) SS diff
-19	12
-22	10
-18	42
-27	15
-4	-1
-10	11
-14	-4
17	60
9	-2
4	10
-19	-7

$n = 11$

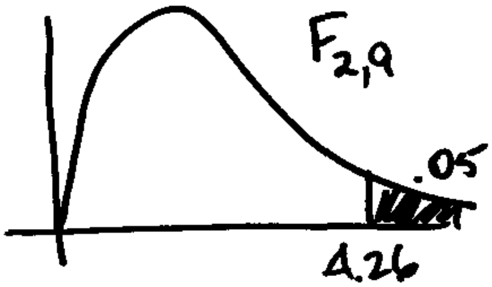
$$\bar{d} = \begin{bmatrix} -9.36 \\ 13.27 \end{bmatrix}$$

$$S_d = \begin{bmatrix} 199.26 & 88.38 \\ 88.38 & 418.61 \end{bmatrix}$$

$$T^2 = n \bar{d}' S_d^{-1} \bar{d}$$

$$= 13.6$$

(7)



$$\frac{p(n-1)}{n-p}F = \frac{2(10)}{9}4.26$$
$$= 9.47$$

Since $13.6 > 9.47$, Reject H_0

As a post hoc analysis, we might do 95% simultaneous C.I.'s for the 2 components of $\vec{\delta}$.

(8)

<u>Scheffe method</u>

$$\delta_1 : \bar{d}_1 \pm \sqrt{\frac{p(n-1)}{n-p}F}\ \frac{S_{d1}}{\sqrt{n}}$$

$$-9.36 \pm \sqrt{9.47}\ \frac{\sqrt{199.26}}{\sqrt{11}}$$

$$(-22.46, 3.74)$$

$$\delta_2 : 13.27 \pm \sqrt{9.47}\ \sqrt{\frac{418.61}{11}}$$

$$(-5.71, 32.25)$$

(9)

$$\underset{\substack{2\times 4\\ q\times p}}{\begin{bmatrix}1 & 0 & -1 & 0\\ 0 & 1 & 0 & -1\end{bmatrix}} \underset{4\times 1}{\begin{bmatrix}\mu_1\\ \mu_2\\ \mu_3\\ \mu_4\end{bmatrix}} \begin{array}{l}\text{BOD Lab1}\\ \text{SS Lab1}\\ \text{BOD Lab2}\\ \text{SS Lab2}\end{array} = \underset{2\times 1}{\begin{bmatrix}0\\ 0\end{bmatrix}}$$

$$H_0:\ A\vec{\mu} = \vec{c}$$

Use $T^2 = n(A\bar{x} - \vec{c})'(ASA')^{-1}(A\bar{x} - \vec{c})$

(10)

Reject H_0 if $T^2 > \frac{q(n-1)}{n-q} F_{q,n-q}(\alpha)$

Midterm Thursday

- 1 page of notes
- Calculator
- Computer (R, Matlab)
- F table (.05 page)