Stat 576
1-10-17

Defns:

(1)

An *experiment* is a process that leads to one of several possible outcomes.

The *sample space* S is the set of all possible outcomes of an experiment.

Example: Toss a coin. $S = \{H, T\}$

Roll a die. $S = \{1, 2, 3, 4, 5, 6\}$

(2)

An *event* is a subset of the sample space.

A *probability function* $P(E)$ must satisfy

i) $0 \le P(E) \le 1$

ii) $P(S) = 1$

iii) $P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$, provided that $E_i \cap E_j = \phi \quad \forall\, i \neq j$

③

Example: Roll a die $S = \{1,2,3,4,5,6\}$

Define $P(E) = \dfrac{\#\text{ items in } E}{6}$

$E = \{1,3,5\}$ $P(E) = \frac{3}{6} = \frac{1}{2}$

i) $0 \le P(E) \le 1$ ✓

ii) $P(S) = 1$ ✓

iii) $P(\{1,2\} \cup \{4\}) = P(\{1,2,4\}) = \frac{3}{6} = \frac{1}{2}$

$\overset{?}{=}$

$P(\{1,2\}) + P(\{4\}) = \frac{2}{6} + \frac{1}{6} = \frac{3}{6}$ ✓

④

Same example

Define $P(E) = \begin{cases} 1 & \text{if } 6 \in E \\ 0 & \text{otherwise} \end{cases}$

i) $0 \le P(E) \le 1$ ✓

ii) $P(S) = 1$ ✓

iii) $P(\{1,2\} \cup \{6\}) = P(\{1,2,6\}) = 1$

$\overset{?}{=}$

$P(\{1,2\}) + P(\{6\}) = 0 + 1 = 1$ ✓

⑤

Using (i), (ii), (iii), show

$$P(E^c) = 1 - P(E)$$

Pf: $S = E \cup E^c$ and $E \cap E^c = \phi$

By (iii), $P(E \cup E^c) = P(E) + P(E^c)$

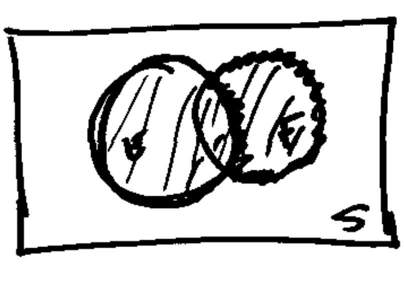
$$\overset{\shortparallel}{P(S)} = 1 \text{ by (ii)}$$

//

⑥

Show $P(E \cup F) = P(E) + P(F) - P(\underbrace{E \cap F}_{EF})$

Pf: $E \cup F = E \cup (F \cap E^c)$



$P(E \cup F) = P(E) + P(F \cap E^c)$ by (iii)

$$F = (F \cap E^c) \cup (F \cap E)$$

$P(F) = P(F \cap E^c) + P(F \cap E)$ by (iii)

$$\therefore\ P(F\cap E^c) = P(F) - P(EF)$$

$$\therefore\ P(E\cup F) = P(E) + P(F) - P(EF)$$

$$P(\underbrace{E\cup F}\cup G) = P(E\cup F) + P(G) - P((E\cup F)\cap G)$$

$$= P(E) + P(F) - P(EF) + P(G) - P\big((E\cap G)\cup(F\cap G)\big)$$

$$= P(E) + P(F) + P(G) - P(EF) - \Big[P(EG) + P(FG) - P(\underbrace{E\cap G\cap F\cap G}_{EFG})\Big]$$

$$= P(E) + P(F) + P(G) - P(EF) - P(EG) - P(FG) + P(EFG)$$

Inclusion/Exclusion rule

Conditional probability

Defn. $P(E|F) = \dfrac{P(EF)}{P(F)}$, provided $P(F)>0$

9

Example: Assume there are 2 children
- independence
- prob(male) = .5

Observe 1 son

Find the prob. that both children are male.

Let F be the event that at least one child is male

Let E be the event that both children are male.

Find $P(E|F) = \frac{P(EF)}{P(F)}$

10

$S = \{mm, mf, fm, ff\}$

$F = \{mm, mf, fm\}$ $\quad P(F) = \frac{3}{4}$

$EF = E \cap F = \{mm\} \cap F$
$= \{mm\} = E \quad P(E) = \frac{1}{4}$

$$P(E|F) = \frac{P(EF)}{P(F)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$