Stat 567
1-19-17

(1)

Recall the defn. of conditional probability: $P(E|F) = \dfrac{P(EF)}{P(F)}$, provided $P(F)>0$

$P(EF) = P(F)\,P(E|F) \leftarrow$ multiplication rule for intersections

Example: Deal 2 card from a shuffled deck & find the prob. that both are aces.

Let F: 1^{st} card is ace
E: 2^{nd} card is ace

(2)

Find $P(EF)$ directly

$$P(EF) = \frac{\binom{4}{2}}{\binom{52}{2}} = \frac{\frac{4!}{2!\,2!}}{\frac{52!}{2!\,50!}}$$

$$= \frac{4\cdot 3\cdot \cancel{2\cdot 1}}{\cancel{2\cdot 1}} \cdot \frac{\cancel{50\cdots}}{52\cdot 51\cdot \cancel{50\cdots}}$$

$$= \frac{4\cdot 3}{52\cdot 51}$$

Find $P(EF)$ using the mult. rule. $\frac{4}{52}\cdot\frac{3}{51}$

$P(F) = \dfrac{4}{52}$ $\quad P(E|F) = \dfrac{3}{51}$

Law of Total Probability

③

Let $F_1, F_2, \ldots, F_n$ be a partition of S.

That is, $F_i \cap F_j = \emptyset \quad \forall\, i \neq j$

and $\bigcup_{i=1}^{n} F_i = S$

Then $E = \bigcup_{i=1}^{n} EF_i$ and $P(E) = \sum_{i=1}^{n} P(EF_i)$

④

Bayes' Rule

Let $F_1, \ldots, F_n$ be a partition of S.

Then $$P(F_j \mid E) = \frac{P(F_j)\,P(E \mid F_j)}{\sum_{i=1}^{n} P(F_i)\,P(E \mid F_i)}$$

Pf: $$P(F_j \mid E) = \frac{P(EF_j)}{P(E)} = \frac{P(F_j)\,P(E \mid F_j)}{\sum_{i=1}^{n} P(EF_i)}$$

$$= \frac{P(F_j)\,P(E \mid F_j)}{\sum_{i=1}^{n} P(F_i)\,P(E \mid F_i)}.$$

(5)

Example 1 Assume that .005 of a population has a particular disease.

If a person has the disease, the test will be positive 95% of the time.

If a person does not have the disease, the test will be negative 99% of the time.

Given that a person tests positive, find the probability that they have the disease.

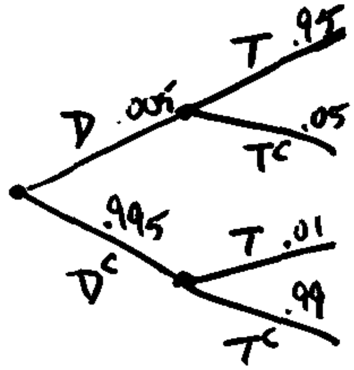
(6)

Let D: person has the disease

T: tests positive

$P(D) = .005 \qquad P(T^c \mid D^c) = .99$

$P(T \mid D) = .95 \qquad$ Find $P(D \mid T)$



$.00475 = P(D)P(T \mid D) = P(DT)$

$.00025 = P(DT^c)$

$.00995 = P(D^cT)$

$.98505 = P(D^cT^c)$

	T	T^c	
D	.00475	.00025	.005 $= P(D) = P(DT) + P(DT^c)$
D^c	.00995	.98505	.995 $= P(D^c)$
	.0147 $= P(T)$	.9853 $= P(T^c)$	1

$$P(D \mid T) = \frac{P(DT)}{P(T)} = \frac{.00475}{.0147} = .3231$$

Using Bayes' Rule: $P(D \mid T) = \dfrac{P(D)\,P(T \mid D)}{P(D)\,P(T \mid D) + P(D^c)\,P(T \mid D^c)}$

$$= \frac{.005(.95)}{.005(.95) + (.995)(.01)} = \frac{.00475}{.00475 + .00995}$$

(8)

<u>Example 2</u>: Find the prob. that a student actually knows the answer, given that they made the ~~ch~~ correct choice on a multiple-choice question.

Let K: student knows the answer

C: student answers correctly.

Assume $P(K) = p$ and $P(C \mid K^c) = \frac{1}{m}$

Find $P(K \mid C)$

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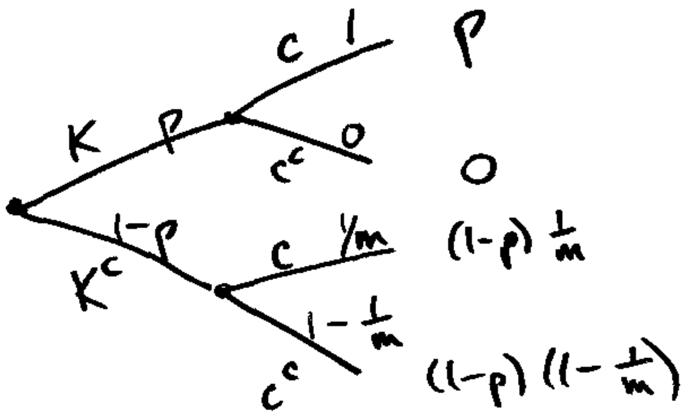

	C	C^c	
K	p	0	p
K^c	$(1-p)\frac{1}{m}$	$(1-p)(1-\frac{1}{m})$	$(1-p)$
	$p+(1-p)\frac{1}{m}$	$(1-p)(1-\frac{1}{m})$	1

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$$P(K|C) = \frac{P(KC)}{P(C)} = \frac{p}{p+(1-p)\frac{1}{m}}$$

$$= \frac{mp}{mp+1-p} = \frac{mp}{1+(m-1)p}$$

Random variables

Defn: A random variable X is a function from S into $\mathbb{R}$.

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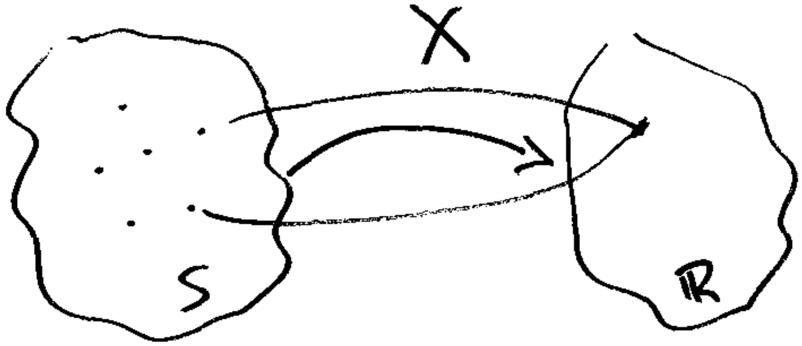

If the range of X is $\underbrace{\text{finite or countably infinite}}_{\text{countable}}$

then X is a **discrete** random variable.

If the range of X is an interval,

then X is a **continuous** random variable.

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Example: Roll 2 dice

$$S = \left\{ \begin{array}{l} (1,1), (1,2), \ldots, (1,6) \\ (2,1), (2,2), \ldots, (2,6) \\ \vdots \\ (6,1), (6,2), \ldots, (6,6) \end{array} \right\}$$

Let X = sum of the 2 dice

The range of X is the set of integers from 2 through 12.

Let Y = maximum of the 2 dice

The range of Y is the set of integers from 1 through 6.

If X is a random variable, then

the <u>cumulative distribution function</u> (cdf)

is $F(b) = P(X \leq b)$ for any $b \in \mathbb{R}$,

provided i) $F(b)$ is nondecreasing

ii) $\lim_{b \to \infty} F(b) = 1$

iii) $\lim_{b \to -\infty} F(b) = 0$

HW assignment follows this page, due Thurs 1/26

13. The dice game craps is played as follows. The player throws two dice, and if the sum is seven or eleven, then she wins. If the sum is two, three, or twelve, then she loses. If the sum is anything else, then she continues throwing until she either throws that number again (in which case she wins) or she throws a seven (in which case she loses). Calculate the probability that the player wins.

19. Two dice are rolled. What is the probability that at least one is a six? If the two faces are different, what is the probability that at least one is a six?

21. Suppose that 5 percent of men and 0.25 percent of women are color-blind. A color-blind person is chosen at random. What is the probability of this person being male? Assume that there are an equal number of males and females.

30. Bill and George go target shooting together. Both shoot at a target at the same time. Suppose Bill hits the target with probability 0.7, whereas George, independently, hits the target with probability 0.4.
    (a) Given that exactly one shot hit the target, what is the probability that it was George's shot?
    (b) Given that the target is hit, what is the probability that George hit it?

36. Consider two boxes, one containing one black and one white marble, the other, two black and one white marble. A box is selected at random and a marble is drawn at random from the selected box. What is the probability that the marble is black?