Stat 526
5-14-15
(1)

$$\hat{x}_M = -\frac{1}{2} B^{-1} \vec{b}$$

But is this a max, min, or saddle point?

Evaluate y at $\hat{x}_M$

$$\hat{y}_M = b_0 + \hat{x}_M' \vec{b} + \hat{x}_M' B \hat{x}_M$$

$$= b_0 + \left(-\frac{1}{2}\right)\vec{b}' B^{-1} \vec{b} + \left(-\frac{1}{2}\right)\vec{b}' B^{-1} B \left(-\frac{1}{2}\right) B^{-1} \vec{b}$$

$$= b_0 - \frac{1}{2}\vec{b}' B^{-1} \vec{b} + \frac{1}{4}\vec{b}' B^{-1} \vec{b}$$

$$= b_0 - \frac{1}{4}\vec{b}' B^{-1} \vec{b}$$

(2)

Let x be a value close to $\hat{x}_M$

and let $\hat{y}$ be the corresponding y-value.

Write $\hat{y} = \hat{y}_M + (\hat{y} - \hat{y}_M)$

$$= \hat{y}_M + \left(\overbrace{\cancel{b_0} + x'\vec{b} + x' B x - \left(\cancel{b_0} - \frac{1}{4}\vec{b}' B^{-1} \vec{b}\right)}\right)$$

$$= \hat{y}_M + \left(x'\vec{b} + \underbrace{\frac{1}{4}\vec{b}' B^{-1} \vec{b}}_{-\frac{1}{2}\vec{b}'\hat{x}_M} + x' B x \right)$$

$$= \hat{y}_M + \left(x' - \frac{1}{2}\hat{x}_M' \right)\vec{b} + x' B x$$

3

$$= \hat{y}_M + (x - \tfrac{1}{2}\hat{x}_M)'\vec{b} + (x-\hat{x}_M)'B(x-\hat{x}_M)$$
$$+ x'B\hat{x}_M + \hat{x}_M'Bx - \hat{x}_M'B\hat{x}_M$$

$$= \hat{y}_M + (x - \tfrac{1}{2}\hat{x}_M)'\vec{b} + (x-\hat{x}_M)'B(x-\hat{x}_M)$$
$$+ x'B(-\tfrac{1}{2}B^{-1}\vec{b}) + (-\tfrac{1}{2}B^{-1}\vec{b})'Bx$$
$$- (-\tfrac{1}{2}B^{-1}\vec{b})'B(-\tfrac{1}{2}B^{-1}\vec{b})$$

$$= \hat{y}_M + (\cancel{x} - \tfrac{1}{2}\hat{x}_M)'\vec{b} + (x-\hat{x}_M)'B(x-\hat{x}_M)$$
$$\cancel{-\tfrac{1}{2}x'\vec{b}} \cancel{- \tfrac{1}{2}\vec{b}'x} - \tfrac{1}{4}\vec{b}'B^{-1}\vec{b}$$

4

$$= \hat{y}_M - \tfrac{1}{2}\underbrace{\hat{x}_M'}_{(-\frac{1}{2}\cancel{B^{-1}\vec{b}})'}\vec{b} + (x-\hat{x}_M)'B(x-\hat{x}_M)$$
$$\cancel{-\tfrac{1}{4}\vec{b}'B^{-1}\vec{b}}$$

$$\hat{y} = \hat{y}_M + (x-\hat{x}_M)'B(x-\hat{x}_M)$$

Write $B = P\Lambda P^{-1}$ Spectral decomposition

$= P\Lambda P'$ since B is symmetric

diagonal matrix of eigenvalues

its columns are the eigenvectors of B

⑤

$$\hat{y} = \hat{y}_M + \underbrace{(x - \hat{x}_M)' P}_{w'} \Lambda \underbrace{P'(x - \hat{x}_M)}_{w}$$

$$= \hat{y}_M + w' \Lambda w$$

$$= \hat{y}_M + [w_1 \; w_2 \; \cdots \; w_k] \begin{bmatrix} \lambda_1 & & & 0 \\ & \lambda_2 & & \\ & & \ddots & \\ 0 & & & \lambda_k \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_k \end{bmatrix}$$

$$\hat{y} = \hat{y}_M + \lambda_1 w_1^2 + \lambda_2 w_2^2 + \ldots + \lambda_k w_k^2$$

$\Rightarrow$ If B is pos. def, $\lambda_i > 0 \; \forall i$ so $\hat{y} > \hat{y}_M$ + $\hat{y}_M$ is a minimum

⑥

$\Rightarrow$ If B is neg def, $\lambda_i < 0 \; \forall i$, so $\hat{y} < \hat{y}_M$ & $\hat{y}_M$ a maximum

$\Rightarrow$ If B is neither pos or neg def, then some λ_i's > 0 + other λ_i's < 0 + $\hat{y}_M$ is a saddle point

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$E(f(X)) \neq f(E(X))$ unless f is linear

⑦

In an ANOVA, if you are willing to test

$$H_0: E_1(f(y)) = E_2(f(y)) = \ldots = E_k(f(y))$$

instead of $H_0: E_1(y) = E_2(y) = \ldots = E_k(y)$

then you can do transformations.

One-way ANOVA $\quad y_{ij} = \mu + \tau_i + \varepsilon_{ij},$

$$\varepsilon_{ij} \sim iid(N(0,\sigma^2))$$

⑧

Box-Cox transformation:

$$x_{ij} = \frac{y_{ij}^{\lambda} - 1}{\lambda}$$

New model $\quad x_{ij} = \mu + \tau_i + \varepsilon_{ij}$ (completely new $\mu, \tau_i, \varepsilon_{ij}$)

$$f(y_{ij}) = g(x_{ij}) \left| \frac{\partial x_{ij}}{\partial y_{ij}} \right|$$

↑ prob. density function

Our goal is to choose λ so that the x_{ij}'s are as normal as possible.

Then

$$f(y_{ij}) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x_{ij} - \mu - \tau_i}{\sigma}\right)^2} \underbrace{\left| \frac{\partial}{\partial y_{ij}} \frac{y_{ij}^{\lambda} - 1}{\lambda} \right|}_{\frac{1}{\lambda}\lambda y_{ij}^{\lambda-1}}$$

$$f(\vec{y}) = \prod_{i=1}^{a} \prod_{j=1}^{n} f(y_{ij})$$

HW #6 15.21 a) wine fixed, b) wine random — judges random

$$\left[MS_E\left(\frac{1}{n}+\frac{(\bar{x}_{i.}-\bar{x}_{..})^2}{E_{xx}}\right)\right]^{1/2}$$

Using this formula, calculate a 95 percent confidence interval on the adjusted mean of machine 1 in Example 15.5.

15.19. Show that in a single-factor analysis of covariance with a single covariate, the standard error of the difference between any two adjusted treatment means is

$$S_{\mathrm{Adj}\bar{y}_{i.}-\mathrm{Adj}\bar{y}_{j.}}=\left[MS_E\left(\frac{2}{n}+\frac{(\bar{x}_{i.}-\bar{x}_{j.})^2}{E_{xx}}\right)\right]^{1/2}$$

15.20. Discuss how the operating characteristic curves for the analysis of variance can be used in the analysis of covariance.

15.21. Three different Pinot Noir wines were evaluated by a panel of eight judges. The judges are considered a random panel of all possible judges. The wines are evaluated on a 100-point scale. The wines were presented in random order to each judge, and the following results obtained.

	Wine		
Judge	**1**	**2**	**3**
1	85	88	93
2	90	89	94
3	88	90	98
4	91	93	96
5	92	92	95
6	89	90	95
7	90	91	97
8	91	89	98

Analyze the data from this experiment. Is there a difference in wine quality? Analyze the residuals and comment on model adequacy.