Stat 566
5-19-15

From last time:

(1)

$$f(\vec{y}) = \prod_{i=1}^{a} \prod_{j=1}^{n} f(y_{ij})$$

$$= \prod_{i=1}^{a} \prod_{j=1}^{n} \left[\frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x_{ij}-\mu-\tau_i}{\sigma}\right)^2} \left| y_{ij}^{\lambda-1} \right| \right]$$

$$= \sigma^{-an} (2\pi)^{-\frac{1}{2}an} e^{-\frac{1}{2\sigma^2}\sum_i \sum_j (x_{ij}-\mu-\tau_i)^2} \left(\prod_{i=1}^{a} \prod_{j=1}^{n} |y_{ij}| \right)^{\lambda-1}$$

$$= L(\mu, \sigma, \tau_1, \ldots, \tau_a, \lambda)$$

Goal: find the values of the parameters that maximize L.

(2)

maximizing L is the same problem as maximizing $\ln L$.

$$l = \ln L = -an\ln\sigma - \frac{an}{2}\ln(2\pi) - \frac{1}{2\sigma^2}\sum_i\sum_j (x_{ij}-\mu-\tau_i)^2 + (\lambda-1)\ln\left(\prod_{i=1}^{a}\prod_{j=1}^{n}|y_{ij}|\right)$$

$$\frac{\partial l}{\partial \mu} = -\frac{1}{\cancel{2}\sigma^2}\sum_i\sum_j \cancel{2}(x_{ij}-\mu-\tau_i)(\cancel{-1}) \overset{\text{set}}{=} 0$$

$$x_{..} - an\mu - 0 = 0$$

$$\hat{\mu} = \frac{x_{..}}{an} = \bar{x}_{..}$$

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$$\frac{\partial l}{\partial \tau_i} = -\frac{1}{\cancel{2}\sigma^2}\sum_j \cancel{2}(x_{ij}-\mu-\tau_i)\cancel{(-1)} \overset{set}{=} 0$$

$$x_{i\cdot} - n\mu - n\tau_i = 0$$

$$\hat{\tau}_i = \frac{x_{i\cdot}}{n} - \hat{\mu}$$

$$= \bar{x}_{i\cdot} - \bar{x}_{\cdot\cdot}$$

$$\frac{\partial l}{\partial \sigma} = -\frac{an}{\sigma} - \frac{1}{2}\sum_i\sum_j (x_{ij}-\mu-\tau_i)^2(-2\sigma^{-3}) \overset{set}{=} 0$$

$$-an\sigma^2 + \sum_i\sum_j (x_{ij}-\mu-\tau_i)^2 = 0$$

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$$\hat{\sigma}^2 = \frac{\sum_i\sum_j (x_{ij}-\hat{\mu}-\hat{\tau}_i)^2}{an}$$

$$= \frac{\sum_i\sum_j (x_{ij}-\bar{x}_{\cdot\cdot}-(\bar{x}_{i\cdot}-\bar{x}_{\cdot\cdot}))^2}{an}$$

$$= \frac{\sum_i\sum_j (x_{ij}-\bar{x}_{i\cdot})^2}{an}$$

$$L(\hat{\mu}, \hat{\tau}_1, \ldots, \hat{\tau}_a, \hat{\sigma}, \lambda)$$

$$= \hat{\sigma}^{-an}(2\pi)^{-\frac{1}{2}an} e^{-\frac{1}{2\hat{\sigma}^2}\sum_i\sum_j (x_{ij}-\hat{\mu}-\hat{\tau}_i)^2}\left(\prod_i\prod_j |y_{ij}|\right)^{\lambda-1}$$

(5)

$$= \hat{\sigma}^{-an}(2\pi)^{-\frac{1}{2}an} e^{-\frac{1}{2}an} \left(\prod_i \prod_j |y_{ij}|\right)^{\lambda-1}$$

$$= C\,\hat{\sigma}^{-an} \left(\prod_i \prod_j |y_{ij}|\right)^{\lambda-1}$$

Maximizing this is equivalent to maximizing

$$\hat{\sigma}^{-an} \left(\prod_i \prod_j |y_{ij}|\right)^{\lambda-1},$$

which is equivalent to minimizing

$$an\left[\frac{\hat{\sigma}^{an}}{\left(\prod_i \prod_j |y_{ij}|\right)^{\lambda-1}}\right]^{\frac{2}{an}} = \frac{an\,\hat{\sigma}^2}{\left(\sqrt[an]{\prod_i \prod_j |y_{ij}|}\right)^{2(\lambda-1)}}$$

(6)

$$= \frac{an\hat{\sigma}^2}{\dot{y}^{2(\lambda-1)}}$$ where $\dot{y}$ represents the geometric mean of the $|y_{ij}|$'s

$$= \frac{\sum_i \sum_j (x_{ij} - \bar{x}_{i\cdot})^2}{\dot{y}^{2(\lambda-1)}}$$

$$= \sum_i \sum_j \left(\frac{x_{ij}}{\dot{y}^{\lambda-1}} - \frac{\bar{x}_{i\cdot}}{\dot{y}^{\lambda-1}}\right)^2$$ Let $w_{ij} = \frac{x_{ij}}{\dot{y}^{\lambda-1}}$

$$= \sum_i \sum_j \left(w_{ij} - \bar{w}_{i\cdot}\right)^2$$ = SSE of the w_{ij}'s

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$$w_{ij} = \frac{x_{ij}}{\dot{y}^{\lambda-1}} = \frac{y_{ij}^{\lambda} - 1}{\lambda\, \dot{y}^{\lambda-1}}$$

Choose $\lambda = \overset{\sim \log}{0}, .1, .2, .3, \ldots, .9$

Create the w_{ij}'s for each λ.

Run a 1-way ANOVA in each case.

Pick the one with the smallest SSE.

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If the violations on the error assumptions vanish after Box-Cox, then continue with the 1-way ANOVA on the x_{ij}'s.

If the violations persist, use Kruskal-Wallis.

Thurs: Example of response surface

Ridge regression
Kernel regression

Mixed 2^k & 3^k designs