

Response surface example

Stat 566

5-21-15

①

We wish to maximize the variable called "Yield".

There are 2 factors: "Time" and "Temperature".

Start with a 2^k design with 5 center points.

Result: Hyp. test for curvature $p = .814$
Fail to reject.

Gradient vector, found by fitting a 1st order regression model, is $\begin{bmatrix} .775 \\ .325 \end{bmatrix}$, or $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$

1st step: let 1 "unit" in the time dir. be 5 secs

②

So add 5 secs to time, add 2° to temp.

Center: 35 155
 40 157
 ⋮
 ⋮

Find peak around 85, 175.

So start over with that center point

At the new center point, we found
Significant curvature $p = .000$

$$\vec{b} = \begin{bmatrix} .995 \\ .5152 \end{bmatrix}$$

③

$$B = \begin{bmatrix} -1.376 & .125 \\ .125 & -1.001 \end{bmatrix}$$

$$\hat{x}_m = -\frac{1}{2} B^{-1} \vec{b} = \begin{bmatrix} .389 \\ .306 \end{bmatrix}$$

$$\text{Time} \quad 85 + .389(5) = 86.945$$

$$\text{Temp} \quad 174 + .306(5) = 176.53$$

HW #7 11.1, 11.8

Most of the quantities on the EVOP calculation sheet follow directly from the analysis of the 2^k factorial design. For example, the variance of any effect, such as $\frac{1}{2}(\bar{y}_3 + \bar{y}_5 - \bar{y}_2 - \bar{y}_4)$, is simply σ^2/n where σ^2 is the variance of the observations (y). Thus, two standard deviation (corresponding to 95 percent) error limits on any effect would be $\pm 2\sigma/\sqrt{n}$. The variance of the change in mean is

$$\begin{aligned} V(\text{CIM}) &= V\left[\frac{1}{5}(\bar{y}_2 + \bar{y}_3 + \bar{y}_4 + \bar{y}_5 - 4\bar{y}_1)\right] \\ &= \frac{1}{25}(4\sigma_y^2 + 16\sigma_y^2) = \left(\frac{20}{25}\right)\frac{\sigma^2}{n} \end{aligned}$$

Thus, two standard deviation error limits on the CIM are $\pm(2\sqrt{20/25})\sigma/\sqrt{n} = \pm 1.78\sigma/\sqrt{n}$.

The standard deviation σ is estimated by the range method. Let $y_i(n)$ denote the observation at the i th design point in cycle n and $\bar{y}_i(n)$ the corresponding average of $y_i(n)$ after n cycles. The quantities in row (iv) of the EVOP calculation sheet are the differences $y_i(n) - \bar{y}_i(n-1)$. The variance of these differences is

$$V[y_i(n) - \bar{y}_i(n-1)] \equiv \sigma_D^2 = \sigma^2 \left[1 + \frac{1}{(n-1)}\right] = \sigma^2 \frac{n}{(n-1)}$$

The range of the differences, say R_D , is related to the estimate of the standard deviation of the differences by $\hat{\sigma}_D = R_D/d_2$. The factor d_2 depends on the number of observations used in computing R_D . Now $R_D/d_2 = \hat{\sigma}\sqrt{n/(n-1)}$, so


$$\hat{\sigma} = \sqrt{\frac{(n-1)}{n}} \frac{R_D}{d_2} = (f_{k,n})R_D \equiv s$$

can be used to estimate the standard deviation of the observations, where k denotes the number of points used in the design. For a 2^2 design with one center point, we have $k = 5$ and for a 2^3 design with one center point, we have $k = 9$. Values of $f_{k,n}$ are given in Table 11.27.

■ TABLE 11.27 Values of $f_{k,n}$

$n =$	2	3	4	5	6	7	8	9	10
$k = 5$	0.30	0.35	0.37	0.38	0.39	0.40	0.40	0.40	0.41
9	0.24	0.27	0.29	0.30	0.31	0.31	0.31	0.32	0.32
10	0.23	0.26	0.28	0.29	0.30	0.30	0.30	0.31	0.31

11.8 Problems

 **11.1.** A chemical plant produces oxygen by liquifying air and separating it into its component gases by fractional distillation. The purity of the oxygen is a function of the main condenser temperature and the pressure ratio between the upper and lower columns. Current operating conditions are temperature (ξ_1) = -220°C and pressure ratio (ξ_2) = 1.2. Using the following data, find the path of steepest ascent:

Temperature (ξ_1)	Pressure Ratio (ξ_2)	Purity
-225	1.1	82.8
-225	1.3	83.5
-215	1.1	84.7
-215	1.3	85.0
-220	1.2	84.1
-220	1.2	84.5
-220	1.2	83.9
-220	1.2	84.3

11.2. An industrial engineer has developed a computer simulation model of a two-item inventory system. The decision variables are the order quantity and the reorder point for each item. The response to be minimized is total inventory cost. The simulation model is used to produce the data shown in Table P11.1. Identify the experimental design. Find the path of steepest descent.

■ **TABLE P11.1**
The Inventory Experiment, Problem 11.2

Item 1		
Order Quantity (ξ_1)	Reorder Point (ξ_2)	
100	25	
140	45	
140	25	
140	25	
100	45	
100	45	
100	25	
140	45	
120	35	
120	35	
120	35	

Item 2		
Order Quantity (ξ_3)	Reorder Point (ξ_4)	Total Cost
250	40	625
250	40	670
300	40	663
250	80	654
300	40	648
250	80	634
300	80	692
300	80	686
275	60	680
275	60	674
275	60	681

11.3. Verify that the following design is a simplex. Fit the first-order model and find the path of steepest ascent.

x_1	x_2	x_3	y
0	$\sqrt{2}$	-1	18.5
$-\sqrt{2}$	0	1	19.8
0	$-\sqrt{2}$	-1	17.4
$\sqrt{2}$	0	1	22.5

11.4. For the first-order model

$$\hat{y} = 60 + 1.5x_1 - 0.8x_2 + 2.0x_3$$

find the path of steepest ascent. The variables are coded as $-1 \leq x_i \leq 1$.

11.5. The region of experimentation for three factors are time ($40 \leq T_1 \leq 80$ min), temperature ($200 \leq T_2 \leq 300^\circ\text{C}$), and pressure ($20 \leq P \leq 50$ psig). A first-order model in coded variables has been fit to yield data from a 2^3 design. The model is

$$\hat{y} = 30 + 5x_1 + 2.5x_2 + 3.5x_3$$

Is the point $T_1 = 85$, $T_2 = 325$, $P = 60$ on the path of steepest ascent?

11.6. The region of experimentation for two factors are temperature ($100 \leq T \leq 300^\circ\text{F}$) and catalyst feed rate ($10 \leq C \leq 30$ lb/in). A first-order model in the usual ± 1 coded variables has been fit to a molecular weight response, yielding the following model:

$$\hat{y} = 2000 + 125x_1 + 40x_2$$

- Find the path of steepest ascent.
- It is desired to move to a region where molecular weights are above 2500. On the basis of the information you have from experimentation in this region, about how many steps along the path of steepest ascent might be required to move to the region of interest?

11.7. The path of steepest ascent is usually computed assuming that the model is truly first order; that is, there is no interaction. However, even if there is interaction, steepest ascent ignoring the interaction still usually produces good results. To illustrate, suppose that we have fit the model

$$\hat{y} = 20 + 5x_1 - 8x_2 + 3x_1x_2$$

using coded variables ($-1 \leq x_i \leq +1$).

- Draw the path of steepest ascent that you would obtain if the interaction were ignored.
- Draw the path of steepest ascent that you would obtain with the interaction included in the model. Compare this with the path found in part (a).

11.8. The data shown in the Table P11.2 were collected in an experiment to optimize crystal growth as a function of three variables x_1 , x_2 , and x_3 . Large values of y (yield in grams) are desirable. Fit a second-order model and analyze the fitted surface. Under what set of conditions is maximum growth achieved?



■ **TABLE P11.2**
The Crystal Growth Experiment, Problem 11.8

x_1	x_2	x_3	y
-1	-1	-1	66
-1	-1	1	70
-1	1	-1	78
-1	1	1	60
1	-1	-1	80
1	-1	1	70
1	1	-1	100
1	1	1	75
-1.682	0	0	100
1.682	0	0	80
0	-1.682	0	68
0	1.682	0	63
0	0	-1.682	65
0	0	1.682	82
0	0	0	113
0	0	0	100
0	0	0	118
0	0	0	88
0	0	0	100
0	0	0	85

11.9. The data in Table P11.3 were collected by a chemical engineer. The response y is filtration time, x_1 is temperature, and x_2 is pressure. Fit a second-order model.

- What operating conditions would you recommend if the objective is to minimize the filtration time?
- What operating conditions would you recommend if the objective is to operate the process at a mean filtration rate very close to 46?

■ **TABLE P11.3**
The Experiment for Problem 11.9

x_1	x_2	y
-1	-1	54
-1	1	45
1	-1	32
1	1	47
-1.414	0	50
1.414	0	53
0	-1.414	47
0	1.414	51
0	0	41
0	0	39
0	0	44
0	0	42
0	0	40

11.10. The hexagon design in Table P11.4 is used in an experiment that has the objective of fitting a second-order model:

■ **TABLE P11.4**
A Hexagon Design

x_1	x_2	y
1	0	68
0.5	$\sqrt{0.75}$	74
-0.5	$\sqrt{0.75}$	65
-1	0	60
-0.5	$-\sqrt{0.75}$	63
0.5	$-\sqrt{0.75}$	70
0	0	58
0	0	60
0	0	57
0	0	55
0	0	69

- Fit the second-order model.
- Perform the canonical analysis. What type of surface has been found?
- What operating conditions on x_1 and x_2 lead to the stationary point?
- Where would you run this process if the objective is to obtain a response that is as close to 65 as possible?

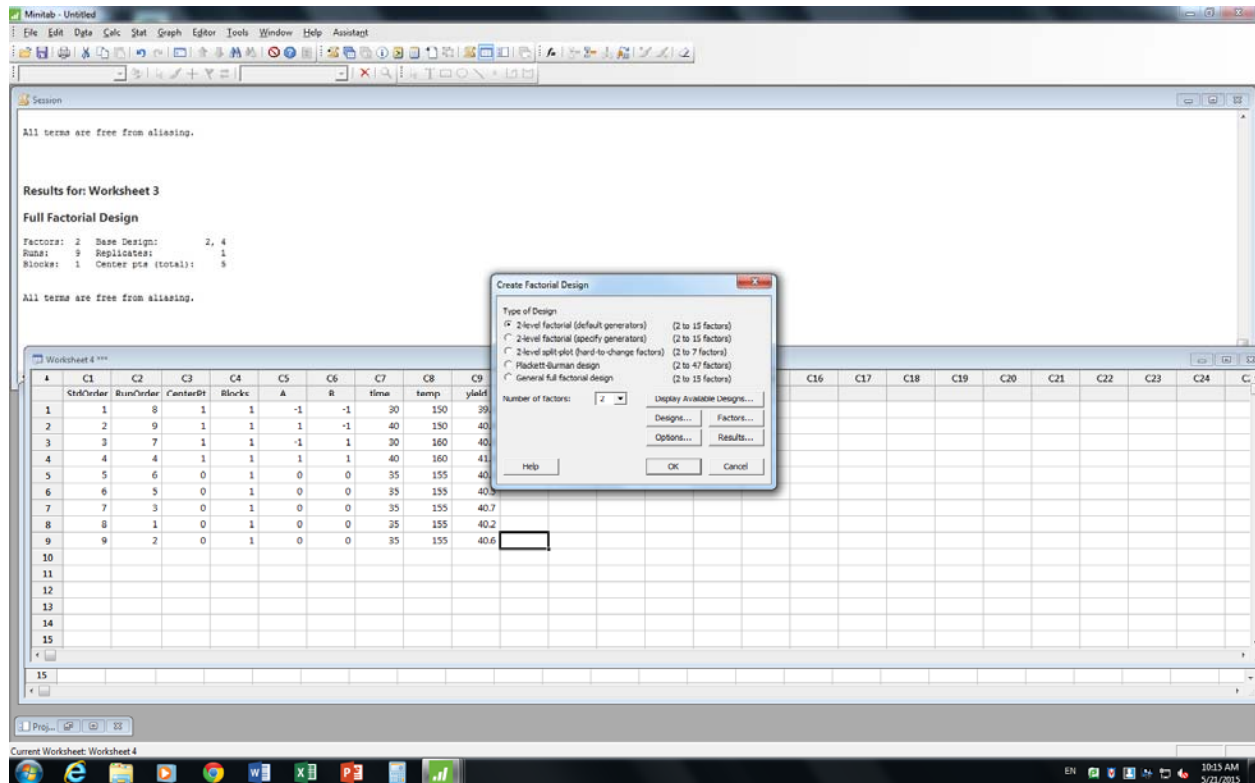
Response surfaces

STEP 1

2^2 design with center points (5 center points)

Stat →DOE→Factorial→create factorial design

(choose 2 factors)



Sort it by standard order

Response surfaces

The screenshot shows the Minitab 'Session' window with a data table. The table has 15 rows and 16 columns. The first 8 columns are: C1:StdOrder, C2:RunOrder, C3:CenterPt, C4:Blocks, C5:A, C6:B, C7:time, C8:temp. The next 8 columns are: C9:yield, C10:StdOrder_1, C11:RunOrder_1, C12:CenterPt_1, C13:Blocks_1, C14:A_1, C15:B_1, C16:temp_1. The data is as follows:

C1:StdOrder	C2:RunOrder	C3:CenterPt	C4:Blocks	C5:A	C6:B	C7:time	C8:temp	C9:yield	C10:StdOrder_1	C11:RunOrder_1	C12:CenterPt_1	C13:Blocks_1	C14:A_1	C15:B_1	C16:temp_1
1	1	8	1	1	-1	-1	30	150	1	1	1	1	1	1	1
2	2	9	1	1	1	-1	40	150	2	2	1	1	2	2	1
3	3	7	1	1	-1	1	30	160	3	3	1	1	3	3	1
4	4	4	1	1	1	1	40	160	4	4	1	1	4	4	1
5	5	6	0	1	0	0	35	155	5	5	1	1	5	5	1
6	6	5	0	1	0	0	35	155	6	6	1	1	6	6	1
7	7	3	0	1	0	0	35	155	7	7	1	1	7	7	1
8	8	1	0	1	0	0	35	155	8	8	1	1	8	8	1
9	9	2	0	1	0	0	35	155	9	9	1	1	9	9	1
10															
11															
12															
13															
14															
15															

The screenshot shows the Minitab 'Session' window with a regression model summary and a 'Sort' dialog box. The model summary is as follows:

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.171863	94.10%	92.13%	91.81%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	40.4444	0.0073	705.99	0.000	
A	0.7750	0.0089	8.52	0.000	1.00
B	0.3250	0.0059	5.70	0.000	1.00

Regression Equation

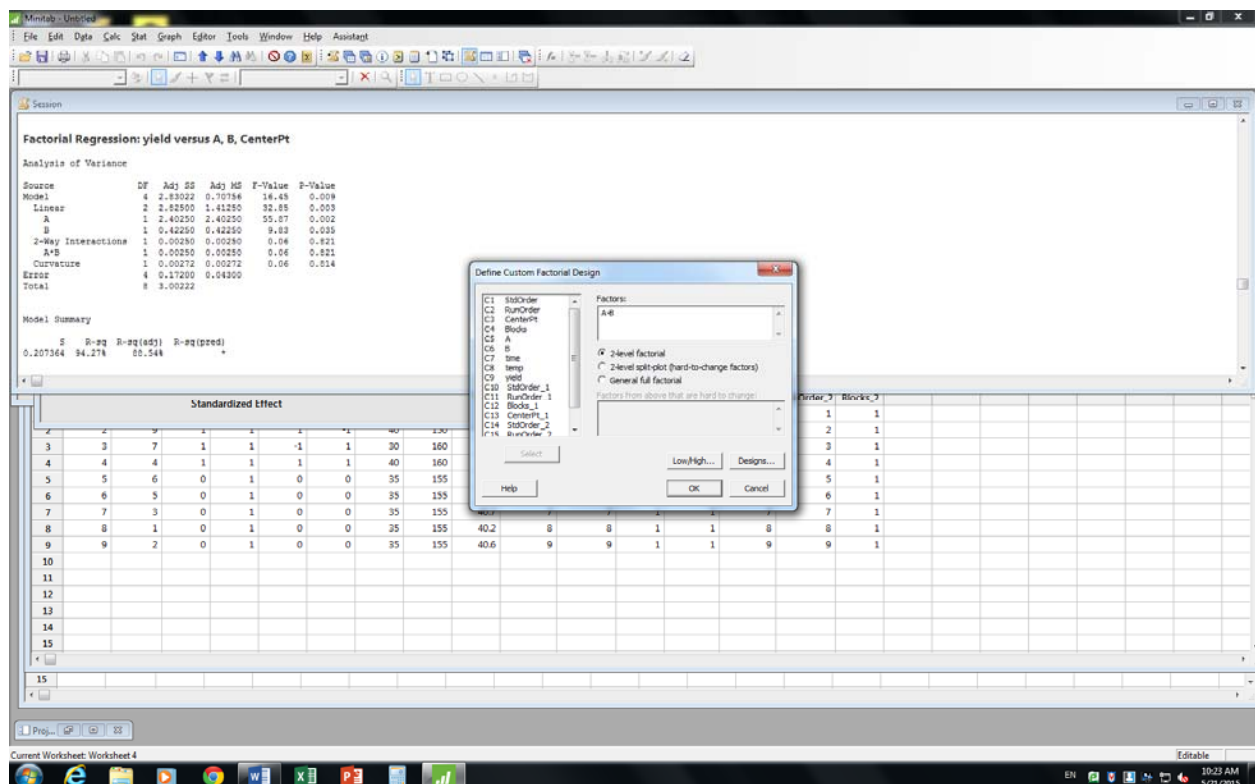
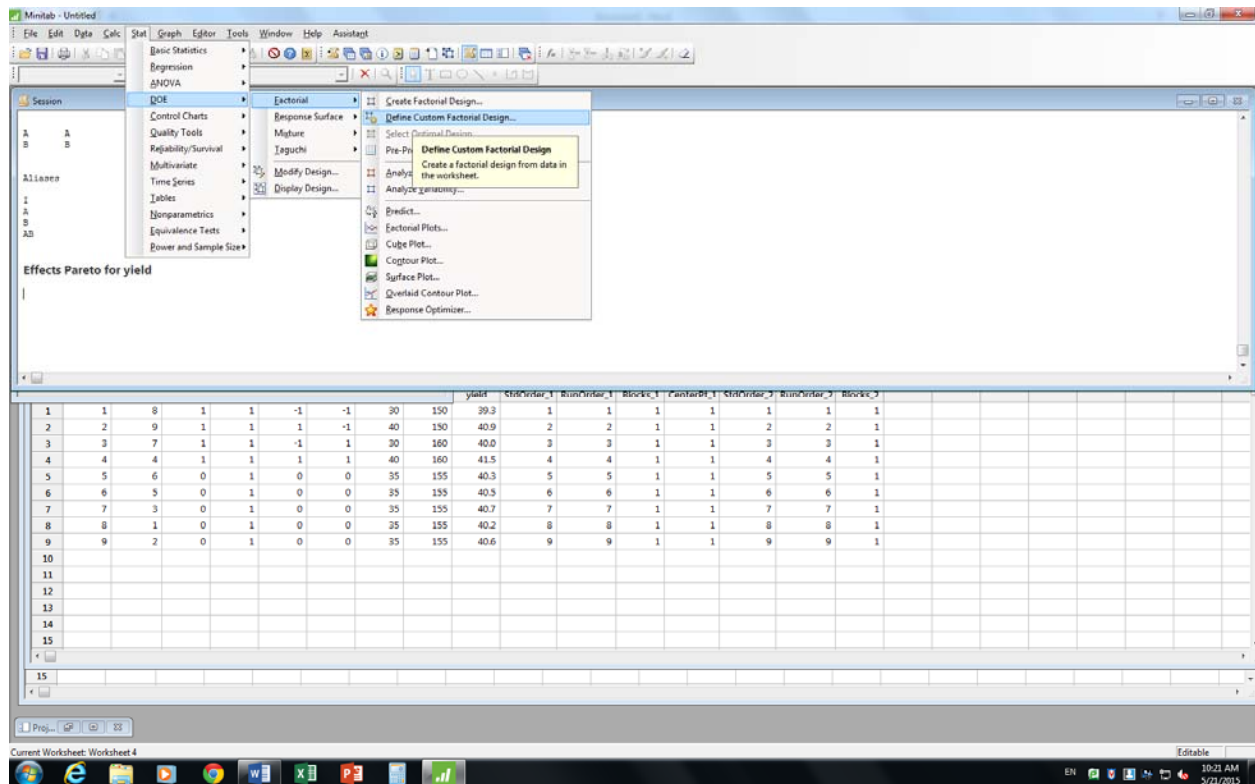
$$\text{yield} = 40.4444 + 0.7750 A + 0.3250 B$$

The 'Sort' dialog box is open, showing the following options:

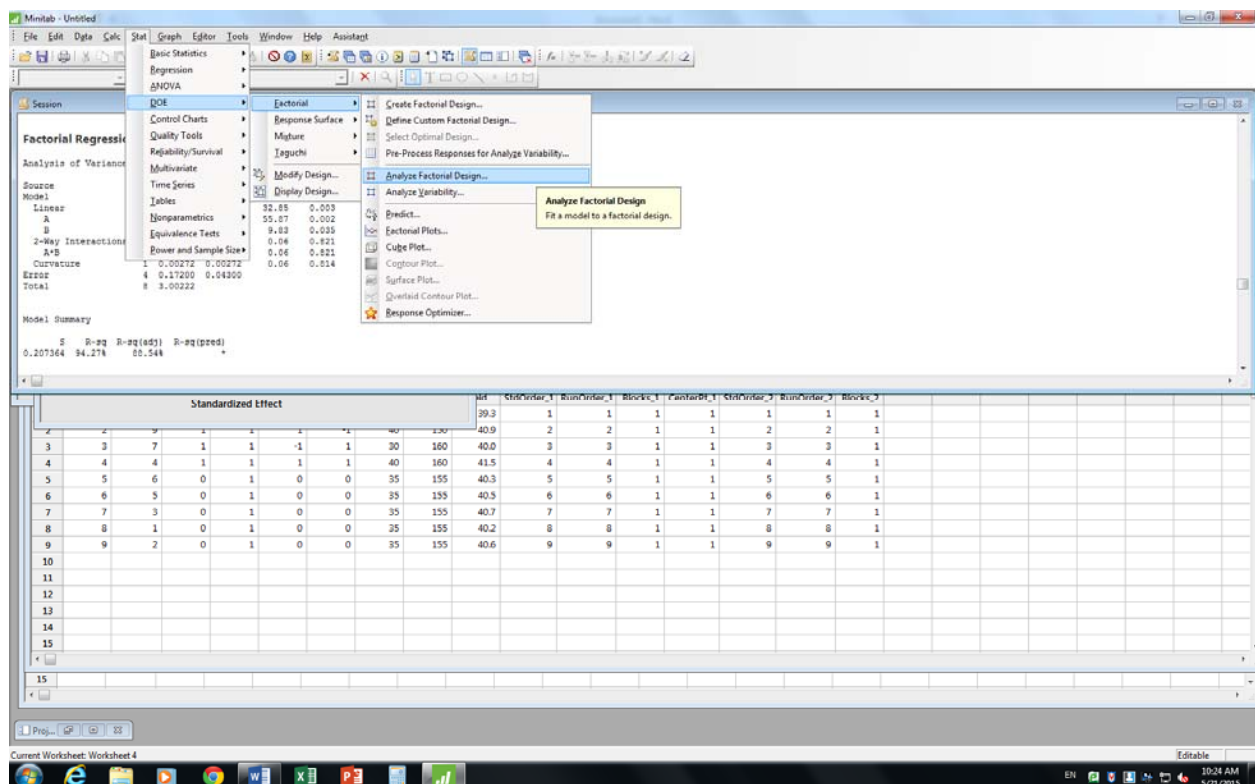
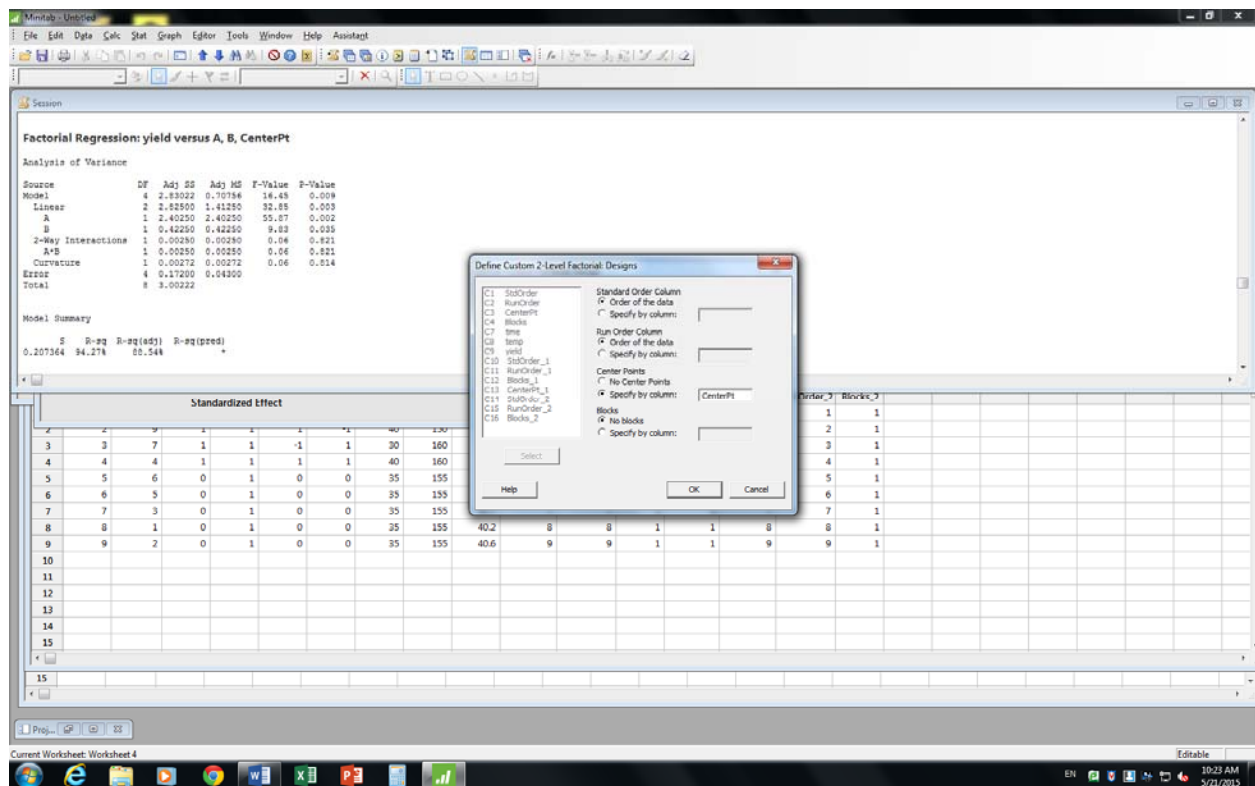
- Sort column(s): C9:yield
- By column: C1:StdOrder
- Descending: ☐
- Store sorted data in: ☐ New worksheet
- Name: (Optional)
- ☐ Original column(s)
- ☐ Column(s) of current worksheet

Response surfaces

Now analyze the data



Response surfaces



Response surfaces

The Minitab Session window displays the following ANOVA results:

Model	4	2.83022	0.70756	16.45	0.009
Linear	2	2.82500	1.41250	32.85	0.003
A	1	2.40250	2.40250	55.87	0.002
B	1	0.42250	0.42250	9.83	0.035
2-Way Interactions	1	0.00250	0.00250	0.06	0.821
A*B	1	0.00250	0.00250	0.06	0.821
Curvature	1	0.00272	0.00272	0.06	0.814
Error	8	0.17200	0.04300		
Total	8	3.00222			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.207364	94.17%	86.34%	

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VI*	
Constant		40.425	0.104	389.89	0.000		
A		1.550	0.775	0.104	7.47	0.002	1.00

Standardized Effect

Term	Effect	Coef	SE Coef	T-Value	P-Value	VI*	
Constant		40.425	0.104	389.89	0.000		
A		1.550	0.775	0.104	7.47	0.002	1.00

Analyze Factorial Design dialog box:

- Responses: yield
- Terms: C1 StdOrder, C2 RunOrder, C4 Blocks, C7 time, C8 temp, C9 yield, C10 StdOrder_1, C11 RunOrder_1, C12 Blocks_1, C13 CenterPt_1

DATA

The Minitab Worksheet window displays the following data table:

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20	C21	C22	C23	C24	C25
1	StdOrder	RunOrder	CenterPt	Blocks	A	B	time	temp	yield																
2	1	8	1	1	-1	-1	30	150	39.3																
3	2	9	1	1	1	-1	40	150	40.9																
4	3	7	1	1	-1	1	30	160	40.0																
5	4	4	1	1	1	1	40	160	41.5																
6	5	6	0	1	0	0	35	155	40.3																
7	6	5	0	1	0	0	35	155	40.5																
8	7	3	0	1	0	0	35	155	40.7																
9	8	1	0	1	0	0	35	155	40.2																
10	9	2	0	1	0	0	35	155	40.6																

Response surfaces

OUTPUT:

Factorial Regression: yield versus A, B, CenterPt

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	4	2.83022	0.70756	16.45	0.009
Linear	2	2.82500	1.41250	32.85	0.003
A	1	2.40250	2.40250	55.87	0.002
B	1	0.42250	0.42250	9.83	0.035
2-Way Interactions	1	0.00250	0.00250	0.06	0.821
A*B	1	0.00250	0.00250	0.06	0.821
Curvature	1	0.00272	0.00272	0.06	0.814
Error	4	0.17200	0.04300		
Total	8	3.00222			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.207364	94.27%	88.54%	*

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		40.425	0.104	389.89	0.000	
A	1.550	0.775	0.104	7.47	0.002	1.00
B	0.650	0.325	0.104	3.13	0.035	1.00
A*B	-0.050	-0.025	0.104	-0.24	0.821	1.00
Ct Pt		0.035	0.139	0.25	0.814	1.00

Regression Equation in Uncoded Units

yield = 40.425 + 0.775 A + 0.325 B - 0.025 A*B + 0.035 Ct Pt

Alias Structure

Factor Name

A	A
B	B

Aliases

I
A
B
AB

Fits and Diagnostics for Unusual Observations

Obs	yield	Fit	Resid	Std Resid
1	39.300	39.300	0.000	* X
2	40.900	40.900	0.000	* X

Response surfaces

3	40.000	40.000	0.000	*	X
4	41.500	41.500	0.000	*	X

X Unusual X

H_0 : no curvature

H_1 : there is curvature

STEP 2:

Fit a regression line

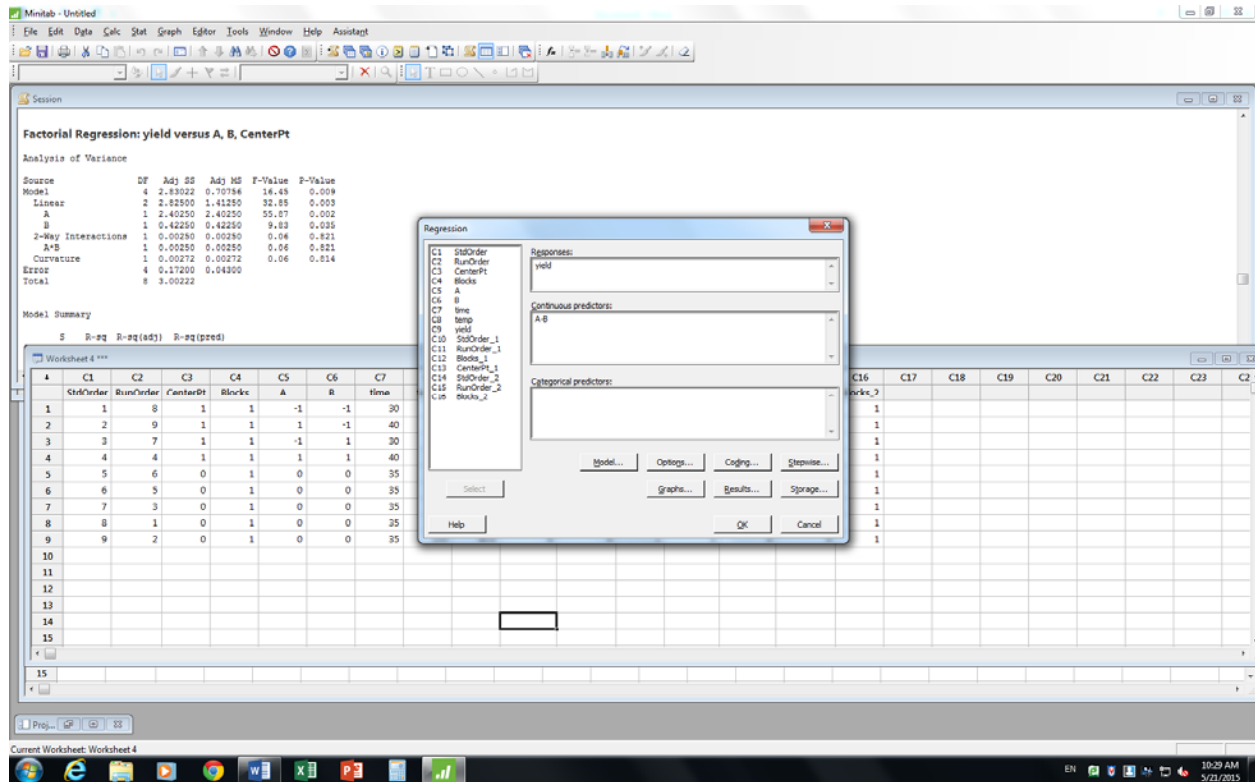
The screenshot shows the Minitab software interface. The 'Fit Regression Model' dialog box is open, displaying a list of regression models to fit. The 'Model Summary' section shows the following statistics:

Model	R-sq	R-sq (adj)	R-sq (pred)
1	0.00272	0.00272	
Error			0.17200
Total			0.04300

The worksheet 'Worksheet4' contains the following data:

RunOrder	RunOrder	CenterPt	Blocks	A	B	Time	temp	yield	StdOrder_1	RunOrder_1	Blocks_1	CenterPt_1	StdOrder_2	RunOrder_2	Blocks_2
1	1	0	1	-1	-1	30	150	39.3	1	1	1	1	1	1	1
2	2	0	1	1	-1	40	150	40.9	2	2	1	1	2	2	1
3	3	7	1	1	-1	30	160	40.0	3	3	1	1	3	3	1
4	4	4	1	1	1	40	160	41.5	4	4	1	1	4	4	1
5	5	6	0	1	0	0	35	155	40.3	5	5	1	1	5	1
6	6	3	0	1	0	0	35	155	40.5	6	6	1	1	6	1
7	7	3	0	1	0	0	35	155	40.7	7	7	1	1	7	1
8	8	1	0	1	0	0	35	155	40.2	8	8	1	1	8	1
9	9	2	0	1	0	0	35	155	40.6	9	9	1	1	9	1

Response surfaces



OUTPUT

Regression Analysis: yield versus A, B

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	2	2.82500	1.41250	47.82	0.000
A	1	2.40250	2.40250	81.34	0.000
B	1	0.42250	0.42250	14.30	0.009
Error	6	0.17722	0.02954		
Lack-of-Fit	2	0.00522	0.00261	0.06	0.942
Pure Error	4	0.17200	0.04300		
Total	8	3.00222			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.171863	94.10%	92.13%	91.81%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	40.4444	0.0573	705.99	0.000	
A	0.7750	0.0859	9.02	0.000	1.00

Response surfaces

B 0.3250 0.0859 3.78 0.009 1.00

Regression Equation

yield = 40.4444 + 0.7750 A + 0.3250 B

NEW DATA

The screenshot shows the Minitab software interface. The top window displays the 'Full Factorial Design' setup with the following parameters:

- Factors: 2 Base Designs: 2, 4
- Runs: 8 Replicates: 1
- Blocks: 1 Center pts (total): 5

All terms are free from aliasing.

The bottom window shows a worksheet with the following data table:

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20	C21	C22	C23	C24
	StdOrder	RunOrder	CenterPt	Blocks	A	B	time	temp	yield															
1	1	9	1	1	-1	-1	80	170	76.5															
2	2	4	1	1	1	-1	90	170	78.0															
3	3	8	1	1	-1	1	80	180	77.0															
4	4	1	1	1	1	1	90	180	79.5															
5	5	7	0	1	0	0	85	175	79.9															
6	6	3	0	1	0	0	85	175	80.0															
7	7	2	0	1	0	0	85	175	80.3															
8	8	5	0	1	0	0	85	175	79.7															
9	9	6	0	1	0	0	85	175	79.8															
10																								
11																								
12																								
13																								
14																								
15																								

Response surfaces

Step 1 output for new data

Factorial Regression: yield versus A, B, CenterPt

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Model	4	15.9080	3.9770	75.04	0.001
Linear	2	5.0000	2.5000	47.17	0.002
A	1	4.0000	4.0000	75.47	0.001
B	1	1.0000	1.0000	18.87	0.012
2-Way Interactions	1	0.2500	0.2500	4.72	0.096
A*B	1	0.2500	0.2500	4.72	0.096
Curvature	1	10.6580	10.6580	201.09	0.000
Error	4	0.2120	0.0530		
Total	8	16.1200			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.230217	98.68%	97.37%	*

Coded Coefficients

Term	Effect	Coef	SE Coef	T-Value	P-Value	VIF
Constant		77.750	0.115	675.45	0.000	
A	2.000	1.000	0.115	8.69	0.001	1.00
B	1.000	0.500	0.115	4.34	0.012	1.00
A*B	0.500	0.250	0.115	2.17	0.096	1.00
Ct Pt		2.190	0.154	14.18	0.000	1.00

Regression Equation in Uncoded Units

yield = 77.750 + 1.000 A + 0.500 B + 0.250 A*B + 2.190 Ct Pt

Alias Structure

Factor	Name
--------	------

A	A
B	B

Aliases

I
A
B
AB

Fits and Diagnostics for Unusual Observations

Obs	yield	Fit	Resid	Std Resid
1	76.500	76.500	0.000	* X
2	78.000	78.000	0.000	* X

Response surfaces

3	77.000	77.000	0.000	*	X
4	79.500	79.500	0.000	*	X

X Unusual X

After this, since $p\text{-value} < 0.05$ we will do the following:

The screenshot shows the Minitab software interface. The 'DOE' menu is open, and 'Create Response Surface Design...' is selected. A tooltip for 'Create Response Surface Design' is visible, stating: 'Create a central composite or Box-Behnken design.' Below the menu, the 'Worksheet' window displays a table of experimental data.

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20	C21	C22	C23	C24
	StdOrder	RunOrder	CenterPt	Blocks	A	B	time	temp	yield	StdOrder_1	RunOrder_1	Blocks_1												
1	1	9	1	1	-1	-1	80	170	76.5	1	1	1												
2	2	4	1	1	1	-1	90	170	78.0	2	2	1												
3	3	8	1	1	-1	1	80	180	77.0	3	3	1												
4	4	1	1	1	1	1	90	180	79.5	4	4	1												
5	5	7	0	1	0	0	85	175	79.9	5	5	1												
6	6	3	0	1	0	0	85	175	80.0	6	6	1												
7	7	2	0	1	0	0	85	175	80.3	7	7	1												
8	8	5	0	1	0	0	85	175	79.7	8	8	1												
9	9	6	0	1	0	0	85	175	79.8	9	9	1												

Response surfaces

The image displays two screenshots of the Minitab software interface, illustrating the steps to create a response surface design.

Top Screenshot: The 'Create Response Surface Design' dialog box is open. The 'Type of Design' is set to 'Central composite (2 to 10 continuous factors)'. The 'Number of continuous factors' is 2, and the 'Number of categorical factors' is 0. The 'Regression Equation in Uncoded Units' is displayed in the background window.

Bottom Screenshot: The 'Create Response Surface Designs' dialog box is open. The 'Designs' list shows 'Full' selected. The 'Number of Center Points' is set to 'Default'. The 'Value of Alpha' is set to 'Default'. The 'Number of replicates' is set to 1. The 'Regression Equation in Uncoded Units' is also displayed in the background window.

Regression Equation in Uncoded Units:

```

Curvature    1 10.4880 10.4880 201.09 0.000
Error         4  0.2120  0.0530
Total         8 16.1200

Model Summary
S      R-sq    R-sq(adj)    R-sq(pred)
0.230217  98.48%    97.37%

Coded Coefficients
Term      Effect    Coef    SE Coef    T-Value    P-Value    VIF
Constant  77.750    0.115    675.45    0.000
A          2.000    1.000    0.115    8.69    0.001    1.00
B          1.000    0.500    0.115    4.34    0.012    1.00
A*B        0.500    0.250    0.115    2.17    0.094    1.00
C*Pc       2.190    0.154    14.10    0.000    1.00
  
```

Response surfaces

It gives us

The screenshot displays the Minitab interface. The top window, titled 'Results for Worksheet 5', shows the design parameters for a Central Composite Design. Below this, a worksheet window titled 'Worksheet5 ***' is open, showing a table with 23 columns (C1 to C23) and 15 rows (1 to 15). The table contains the design matrix for the CCD, including standard order, run order, point type, blocks, and the two factors A and B.

Results for Worksheet 5

Central Composite Design

Factors: 2 Replicates: 1
Base runs: 12 Total runs: 13
Base blocks: 1 Total blocks: 1

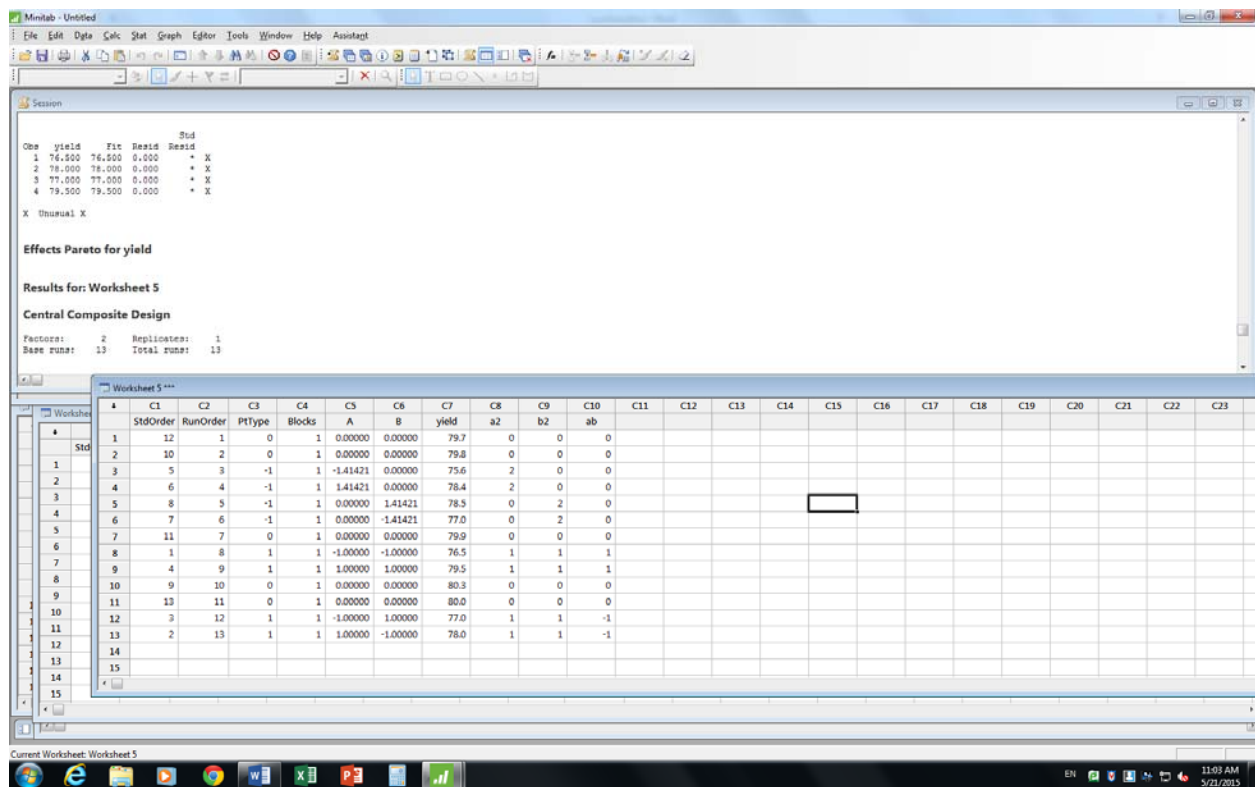
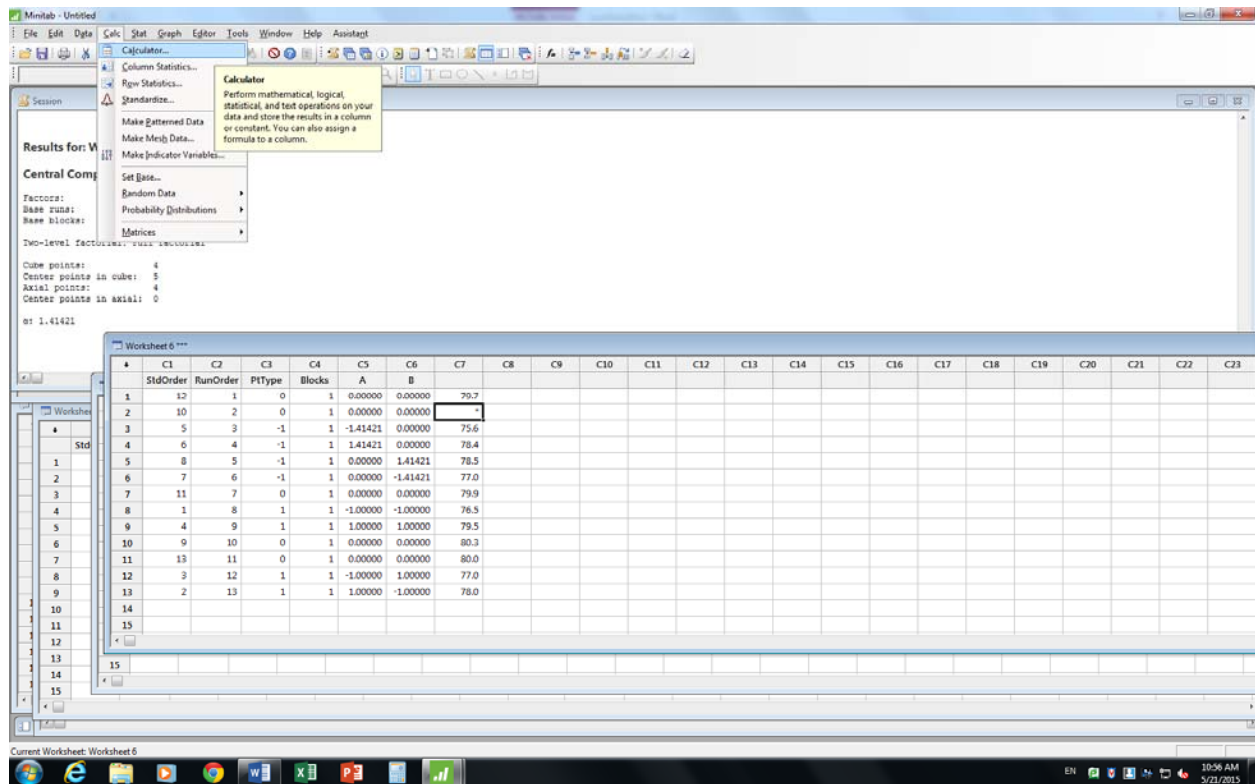
Two-level factorial: Full factorial

Cube points: 4
Center points in cube: 5
Axial points: 4
Center points in axial: 0
or 1.41421

	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11	C12	C13	C14	C15	C16	C17	C18	C19	C20	C21	C22	C23
	StdOrder	RunOrder	PTType	Blocks	A	B																	
1	12	1	0	1	0.00000	0.00000																	
2	10	2	0	1	0.00000	0.00000																	
3	5	3	-1	1	-1.41421	0.00000																	
4	6	4	-1	1	1.41421	0.00000																	
5	8	5	-1	1	0.00000	1.41421																	
6	7	6	-1	1	0.00000	-1.41421																	
7	11	7	0	1	0.00000	0.00000																	
8	1	8	1	1	-1.00000	-1.00000																	
9	4	9	1	1	1.00000	1.00000																	
10	9	10	0	1	0.00000	0.00000																	
11	13	11	0	1	0.00000	0.00000																	
12	3	12	1	1	-1.00000	1.00000																	
13	2	13	1	1	1.00000	-1.00000																	
14																							
15																							

Find A^2 , B^2 and $A*B$

Response surfaces



Response surfaces

Now run regression with all the quadratic forms that was created in last step.

Regression Analysis: yield versus A, B, a2, b2, ab

Analysis of Variance

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Regression	5	28.2478	5.6496	79.85	0.000
A	1	7.9198	7.9198	111.93	0.000
B	1	2.1232	2.1232	30.01	0.001
a2	1	13.1761	13.1761	186.22	0.000
b2	1	6.9739	6.9739	98.56	0.000
ab	1	0.2500	0.2500	3.53	0.102
Error	7	0.4953	0.0708		
Lack-of-Fit	3	0.2833	0.0944	1.78	0.290
Pure Error	4	0.2120	0.0530		
Total	12	28.7431			

Model Summary

S	R-sq	R-sq(adj)	R-sq(pred)
0.266000	98.28%	97.05%	91.84%

Coefficients

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	79.940	0.119	672.00	0.000	
A	0.9950	0.0940	10.58	0.000	1.00
B	0.5152	0.0940	5.48	0.001	1.00
a2	-1.376	0.101	-13.65	0.000	1.02
b2	-1.001	0.101	-9.93	0.000	1.02
ab	0.250	0.133	1.88	0.102	1.00

Regression Equation

yield = 79.940 + 0.9950 A + 0.5152 B - 1.376 a2 - 1.001 b2 + 0.250 ab

Response surfaces

NEW STEP

The screenshot shows the Minitab software interface. The 'Session' menu is open, and the 'Copy Columns to Matrix' option is selected. A dialog box titled 'Copy Columns to Matrix' is displayed, with the text 'Create a matrix from columns in the worksheet.' The main window shows a worksheet with the following data:

	StdOrder	RunOrder	PTType	Blocks	A	B	yield	a2	b2	ab
1	12	1	0	1	0.00000	0.00000	79.7	0	0	0
2	10	2	0	1	0.00000	0.00000	79.8	0	0	0
3	5	3	-1	1	-1.41421	0.00000	75.6	2	0	0
4	6	4	-1	1	1.41421	0.00000	78.4	2	0	0
5	8	5	-1	1	0.00000	1.41421	78.5	0	2	0
6	7	6	-1	1	0.00000	-1.41421	77.0	0	2	0
7	11	7	0	1	0.00000	0.00000	79.9	0	0	0
8	1	8	1	1	-1.00000	-1.00000	76.5	1	1	1
9	4	9	1	1	1.00000	1.00000	79.5	1	1	1
10	9	10	0	1	0.00000	0.00000	80.3	0	0	0
11	13	11	0	1	0.00000	0.00000	80.0	0	0	0
12	3	12	1	1	-1.00000	1.00000	77.0	1	1	-1
13	2	13	1	1	1.00000	-1.00000	78.0	1	1	-1

The status bar at the bottom indicates 'Current Worksheet: Worksheet 5' and 'Editable'.