Stat 526
6-2-15
①

Kernel regression (model-free)

In ordinary LS, $\hat{Y} = X\hat{\beta} = \underbrace{X(X'X)^{-1}X'}Y = HY$

②

$$\begin{bmatrix} \hat{y}_1 \\ \vdots \\ \hat{y}_n \end{bmatrix} = \begin{bmatrix} h_{11} & \cdots & h_{1n} \\ \vdots & & \\ h_{n1} & \cdots & h_{nn} \end{bmatrix} \begin{bmatrix} y_1 \\ \vdots \\ y_n \end{bmatrix}$$

$$\hat{y}_i = \sum_{j=1}^{n} h_{ij} y_j$$

Define $\tilde{y}_i = \sum_{j=1}^{n} w_{ij} y_j \quad \left(\sum_{j=1}^{n} w_{ij} = 1\right), \quad \tilde{Y} = WY$

③

How to construct the weights?

Use a kernel function $K(t)$

such that ① $K(t) \geq 0 \quad \forall t$

② $\int_{-\infty}^{\infty} K(t)\,dt = 1$

③ $K(-t) = K(t)$

Examples of Kernel functions

Box: $K(t) = \begin{cases} 1 & |t| \leq .5 \\ 0 & \text{o.w.} \end{cases}$

Triangle: $K(t) = \begin{cases} 1 - \frac{|t|}{c} & |t| \leq \frac{1}{c} \\ 0 & \text{o.w.} \end{cases}$ ④

Normal: $K(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\frac{t^2}{\sigma^2}}$

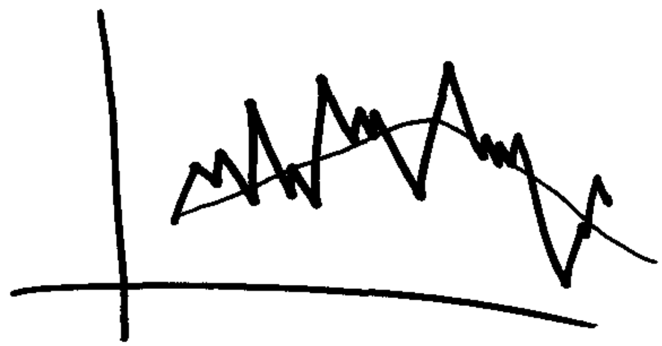
Then define $w_{ij} = \dfrac{K\left(\frac{x_i - x_j}{b}\right)}{\sum_{k=1}^{n} K\left(\frac{x_i - x_k}{b}\right)}$

b = "bandwidth"

⑤

Experiment with different kernels and bandwidths

Hope to see something like this:



⑥

LOESS regression

(Locally-weighted regression)

First, define a neighborhood by a <u>span</u>.

Span = .3 $\Rightarrow$ use the closest 30% of the data to the point in question

Let x_i be a particular point, + let

$\Delta(x_i)$ = distance from x_i to the farthest point in the neighborhood.

Let $t_{ij} = \dfrac{|x_i - x_j|}{D(x_i)}$

Let $w_{ij} = \begin{cases} (1-t^3)^3 & 0 \le t < 1 \\ 0 & \text{o.w.} \end{cases}$

Then do a weighted least squares to predict y_i.

Repeat for every x_i