Stat 565
4-7-15
1

Rules for finding the expected mean squares (restricted version of model)

(1) Write ε_{ijkl} as $\varepsilon_{(ijk)l}$

(2) If a term has a subscript in parentheses, then there will be no interaction between the factors represented by terms in the ( ) and those outside

Example: $\beta_{j(i)} \Rightarrow$ no AB

2

(3) Subscripts outside the ( ) are live

" inside the ( ) are dead

" not present in the term are absent

(4) For a random effect, the variance component is of the form σ_τ^2.

For a fixed effect, the variance component is of the form $\frac{\sum_{i=1}^{a} \tau_i^2}{a-1}$

③

EMS table

Factor	F i	R j	R k	EMS
τ_i				
β_j				
$\tau\beta_{ij}$				
$\varepsilon_{(ij)k}$				

⑤ ⓐ Enter a "1" in the table for each dead subscript in that term.

④

ⓑ Enter a "0" for a live subscript in an F column, and "1" for a live subscript in an R column.

ⓒ For absent subscripts, enter the # levels of that factor

ⓓ In each row, cover all the columns corresponding to the live subscripts for that term. The EMS will contain terms corresponding to each row with <u>at least</u> the same subscripts, and the coefficient of the variance component is the product of the visible numbers.

Last Thursday's example (A fixed, B random) ⑤

Factor	F a i	R b j	R n k	EMS	F
τ_i	0	b	n	$nb \frac{\sum \tau_i^2}{a-1} + n\sigma^2_{\tau\beta} + \sigma^2$	MSA/MSAB
β_j	a	1	n	$na\,\sigma^2_\beta + \sigma^2$	MSB/MSE
$\tau\beta_{ij}$	0	1	n	$n\sigma^2_{\tau\beta} + \sigma^2$	$\frac{MSAB}{MSE}$
$\varepsilon_{(ij)k}$	1	1	1	σ^2	

⑥ For each H_0, find a term whose EMS matches the EMS of the numerator, under H_0, & use that as the denom.

3 factors A fixed B, C random ⑥

Factor	F a i	R b j	R c k	R n l	EMS
τ_i	0	b	c	n	$bcn \frac{\sum \tau_i^2}{a-1} + cn\sigma^2_{\tau\beta} + bn\sigma^2_{\tau\gamma} + n\sigma^2_{\tau\beta\gamma} + \sigma^2$
β_j	a	1	c	n	$acn\sigma^2_\beta + an\sigma^2_{\beta\gamma} + \sigma^2$
γ_k	a	b	1	n	$abn\,\sigma^2_\gamma + an\sigma^2_{\beta\gamma} + \sigma^2$
$\tau\beta_{ij}$	0	1	c	n	$cn\sigma^2_{\tau\beta} + n\sigma^2_{\tau\beta\gamma} + \sigma^2$
$\tau\gamma_{ik}$	0	b	1	n	$bn\sigma^2_{\tau\gamma} + n\sigma^2_{\tau\beta\gamma} + \sigma^2$
$\beta\gamma_{jk}$	a	1	1	n	$an\sigma^2_{\beta\gamma} + \sigma^2$
$\tau\beta\gamma_{ijk}$	0	1	1	n	$n\sigma^2_{\tau\beta\gamma} + \sigma^2$
$\varepsilon_{(ijk)l}$	1	1	1	1	σ^2

⑦

A : no exact F test ← approximate test to be introduced next time.

B : MSB/MS_{BC}

C : MSC/MS_{BC}

AB : MS_{AB}/MS_{ABC}

AC : MS_{AC}/MS_{ABC}

BC : MS_{BC}/MS_E

ABC : MS_{ABC}/MS_E

But notice: if you could assume $\sigma^2_{\tau\beta} = 0$, then MS_A/MS_{AC} would be an exact F test for A.

⑧

Rules for df and SS

df = product of (# levels − 1) for each live subscript times # levels for each dead subscript.

$df_{AB} = (a-1)(b-1)$

3 factors

$df_E = abc(n-1)$

For SS, start by symbolically multiply out the df.

SS_{AB}: $(a-1)(b-1) = ab - a - b + 1$

For each term in the expansion, put a dot for each missing index, square the term, divide by # levels of each missing index, & sum over all present indices.

$$SS_{AB} = \sum_i \sum_j \frac{y_{ij..}^2}{cn} - \sum_i \frac{y_{i...}^2}{bcn} - \sum_j \frac{y_{.j..}^2}{acn} + \frac{y_{....}^2}{abcn}$$