Rank-based method for the RCBD
(randomized complete block design)

$$y_{ij} = \mu + \tau_i + \beta_j + \varepsilon_{ij} \qquad \begin{matrix} i = 1,\dots,a \\ j = 1,\dots,b \end{matrix} \qquad N = ab$$

within each block, rank the items from 1 to a

$$SS_{TOT} = \sum_{i=1}^{a}\sum_{j=1}^{b} y_{ij}^2 - N\bar{y}_{\cdot\cdot}^2$$

Now, $SS_{TOT} = \sum_{i=1}^{a}\sum_{j=1}^{b} R_{ij}^2 - N\bar{R}_{\cdot\cdot}^2$

(2)

$$\bar{R}_{\cdot\cdot} = \frac{1}{N}\sum_{i=1}^{a}\sum_{j=1}^{b} R_{ij} = \frac{1}{N}\sum_{j=1}^{b}\underbrace{\sum_{i=1}^{a} R_{ij}}_{\frac{a(a+1)}{2}}$$

$$= \frac{1}{N} b \frac{a(a+1)}{2} = \frac{a+1}{2}$$

$$\sum_{i=1}^{a}\sum_{j=1}^{b} R_{ij}^2 = \sum_{j=1}^{b}\underbrace{\sum_{i=1}^{a} R_{ij}^2}_{\frac{a(a+1)(2a+1)}{6}} = b\frac{a(a+1)(2a+1)}{6}$$

$$= N\frac{(a+1)(2a+1)}{6}$$

Then $SS_{TOT} = \frac{N(a+1)(2a+1)}{6} - N\left[\frac{(a+1)}{2}\right]^2$ ③

$$= \frac{N(a^2-1)}{12}$$

But $SS_{TRT} + SS_{BLK} + SS_{ERR} = SS_{TOT}$

$\therefore SS_{TRT}$ & SS_{ERR} cannot be independent

$$SS_{TRT} = b\sum_{i=1}^{a} \bar{R}_{i\cdot}^2 - N\bar{R}_{\cdot\cdot}^2$$

$$= b\sum_{i=1}^{a} \bar{R}_{i\cdot}^2 - N\left(\frac{(a+1)}{2}\right)^2$$

Under H_0, $\frac{SS_{TRT}}{\sigma^2} \sim \chi^2_{a-1}$ ④

$$\sigma^2 = Var(R_{ij}) = \frac{a^2-1}{12} \quad \left(\text{Note: } \sigma^2 = \frac{SS_{TOT}}{N}\right)$$

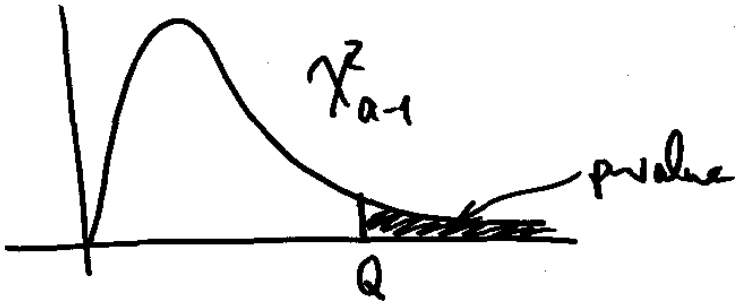
↑ discrete uniform on $\{1, \ldots, a\}$

$$\frac{SS_{TRT}}{\sigma^2} = \frac{b\sum_{i=1}^{a} \bar{R}_{i\cdot}^2 - N\frac{(a+1)^2}{4}}{\frac{a^2-1}{12}}$$

$$= \frac{12}{a^2-1}\left(b\sum_{i=1}^{a} \bar{R}_{i\cdot}^2 - \frac{N}{4}(a+1)^2\right)$$

$$= \frac{a}{a-1}\left[\underbrace{\frac{12}{a(a+1)}\left(b\sum_{i=1}^{a}\bar{R}_{i\cdot}^{2} - N\left(\frac{a+1}{2}\right)^2\right)}\right]$$

Friedman's Q



Review:

1-way ANOVA – 1 treatment factor with a levels

RCBD – 1 treatment factor with a levels, plus
1 blocking factor with b levels

Latin square – 1 treatment factor
2 blocking factors
} p levels of each

Graeco-Latin square – 2 treatment factors
2 blocking factors
} p levels of each

BIBD – 1 treatment factor with a levels
1 blocking factor with b levels

There are missing values following 3 constraints

The F test for the treatment factor must be based on SS_{TRT} (adjusted for blocks)

Factorial designs – Any number of treatment factors, any numbers of levels, plus interactions

2^k – k factors, 2 levels each

2^k with blocking

2^{k-p}

3^k – k factors, 3 levels each

3^k with blocking

3^{k-p}

Additional topics:
Multiple comparisons
missing values

Kruskal-Wallis nonparametric alternative to 1-way ANOVA

Friedman " " " RCBD

Final exam will be 100% Take-home

due Thes 3/20 by noon

HW #7 due 3/15

8.10 ab
8.19
9.17

8.10. An article by J. J. Pignatiello Jr. and J. S. Ramberg in the *Journal of Quality Technology* (Vol. 17, 1985, pp. 198–206) describes the use of a replicated fractional factorial to investigate the effect of five factors on the free height of leaf springs used in an automotive application. The factors are A = furnace temperature, B = heating time, C = transfer time, D = hold down time, and E = quench oil temperature. The data are shown in Table P8.1

■ **TABLE P8.1**

Leaf Spring Experiment

A	*B*	*C*	*D*	*E*	Free Height		
−	−	−	−	−	7.78	7.78	7.81
+	−	−	+	−	8.15	8.18	7.88
−	+	−	+	−	7.50	7.56	7.50
+	+	−	−	−	7.59	7.56	7.75
−	−	+	+	−	7.54	8.00	7.88
+	−	+	−	−	7.69	8.09	8.06
−	+	+	−	−	7.56	7.52	7.44
+	+	+	+	−	7.56	7.81	7.69
−	−	−	−	+	7.50	7.25	7.12
+	−	−	+	+	7.88	7.88	7.44
−	+	−	+	+	7.50	7.56	7.50
+	+	−	−	+	7.63	7.75	7.56
−	−	+	+	+	7.32	7.44	7.44
+	−	+	−	+	7.56	7.69	7.62
−	+	+	−	+	7.18	7.18	7.25
+	+	+	+	+	7.81	7.50	7.59

(a) Write out the alias structure for this design. What is the resolution of this design?

(b) Analyze the data. What factors influence the mean free height?

(c) Calculate the range and standard deviation of the free height for each run. Is there any indication that any of these factors affects variability in the free height?

(d) Analyze the residuals from this experiment, and comment on your findings.

(e) Is this the best possible design for five factors in 16 runs? Specifically, can you find a fractional design for five factors in 16 runs with a higher resolution than this one?

8.19. Construct a 2_{III}^{6-3} design. Determine the effects that may be estimated if a full fold over of this design is performed.

9.17. Construct a 3^{9-6} design and verify that it is a resolution III design.