2^k designs, continued

Stat 565
2-8-18
①

Consider the 2^2 design with n replicates

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \varepsilon_{ijk}$$

$$i = 1,2$$
$$j = 1,2$$
$$k = 1,\dots,n$$

We know $\hat{\mu} = \bar{y}_{\cdot\cdot\cdot}$

$$\hat{\tau}_i = \bar{y}_{i\cdot\cdot} - \bar{y}_{\cdot\cdot\cdot}$$

②

Estimate $\tau_2 - \tau_1$:

$$\hat{\tau}_2 - \hat{\tau}_1 = (\bar{y}_{2\cdot\cdot} - \bar{y}_{\cdot\cdot\cdot}) - (\bar{y}_{1\cdot\cdot} - \bar{y}_{\cdot\cdot\cdot})$$

$$= \bar{y}_{2\cdot\cdot} - \bar{y}_{1\cdot\cdot}$$

$$= \frac{a + ab}{2n} - \frac{(1) + b}{2n}$$

$$= \frac{1}{2n}\underbrace{\left[-(1) + a - b + ab\right]}_{\text{This is a }\underline{\text{Contrast}}} = \frac{C}{2n}$$

(3)

[See 1/16/18, p9]

The formula for the sum of squares due to a contrast is

$$SS_c = \frac{C^2}{n\sum c_i^2} = \frac{C^2}{n4}$$

note that today, we have a contrast of sums, where before, we had means

(4)

In general, for a 2^k design,

the estimate of an effect due to a factor will be $\frac{C}{2^{k-1}n}$ and

$$SS_c = \frac{C^2}{2^k n}$$

Degrees of freedom: 2^k with n replicates

$$df_{tot} = N-1 = 2^k n - 1$$

Each main effect has 2 levels, so 1 df (5)

Each interaction has 1 df

$$df_E = 2^k n - 1 - \Big(\underbrace{\binom{k}{1} + \binom{k}{2} + \dots + \binom{k}{k}}_{2^k - 1} \Big)$$

$$= 2^k n - 2^k$$

$$= 2^k (n-1)$$

Note: If $n=1$, $df_E = 0$; no F tests are possible

What if no replication is allowed? (6)

(1) Use the "sparsity of effects" principle:

Treat the highest order interaction as negligible.

Gain: 1 df for error

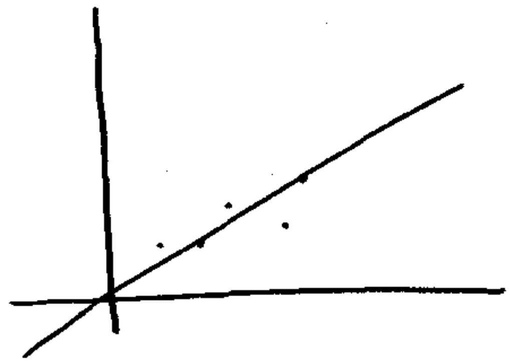
(2) Rethink the entire design + pick a factor that you can live without.

Remove that factor and all of its interactions.

Gain: 2^{k-1} df for error

(3) Estimate <u>all</u> of the effects

Plot the estimates on a normal probability plot



Consider omitting the factors whose effects are closest to the line, since they are most likely to be insignificant.

Then use the "hierarchical principle":

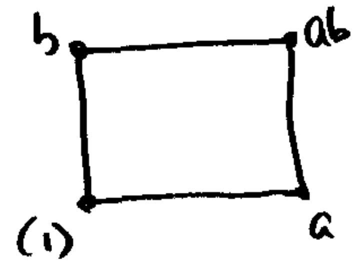
Remove all interactions containing the omitted effect

(4) Add several observations (n_E) at the <u>Center</u> of the design

Then $df_E = n_E - 1$

2^K designs plus blocking

2^2 design in 2 blocks



	I	A	B	AB
(1)	+	−	−	+
a	+	+	−	−
b	+	−	+	−
ab	+	+	+	+

Block 1	Block 2
(1)	a
ab	b

Note that the AB interaction has become <u>confounded</u> with the blocks

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2^3 design

	I	A	B	C	AB	AC	BC	ABC
(1)	+	−	−	−	+	+	+	−
a	+	+	−	−	−	−	+	+
b	+	−	+	−	−	+	−	+
ab	+	+	+	−	+	−	−	−
c	+	−	−	+	+	−	−	+
ac	+	+	−	+	−	+	−	−
bc	+	−	+	+	−	−	+	−
abc	+	+	+	+	+	+	+	+

<u>2 blocks</u> Confound ABC with blocks

1	2
(1)	a
ab	b
ac	c
bc	abc

Note that this block is a subgroup of the original group

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What if we have to do the 2^3 design in 4 blocks?

Pick 2 effects to be confounded with blocks

Select ABC & BC

+ −	+ +	− +	− −
Block 1	Block 2	Block 3	Block 4
(1)	a	b	ab
bc	abc	c	ac

Note that A was unintentionally confounded also.

Note: $(ABC)(BC) = A$

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A smarter choice would be

AB & BC

So AC is also confounded