

Continued from last time:

3^4 in 9 blocks

We had selected $ABC \neq AB^2D^2$ to

confound with blocks

We listed the 8 effects that would all be confounded with blocks.

$$L_1 = X_1 + X_2 + X_3 \pmod{3}$$

$$L_2 = X_1 + 2X_2 + 2X_4 \pmod{3}$$

Stat 565

2.27.18

①

ANOVA	Source	df
	A	2
	B	2
	C	2
	D	2
	AB	4
	AC	4
	AD	4
	BC	4
	BD	4
	CD	4
	ABC	6
	ABD	6
	ACD	6
	BCD	6
	ABCD	16
	Blocks	8
	Err	0
	Total	80

Confound:

ABC $A^2B^2C^2$

AB^2D^2 A^2BD

A^2CD^2 AC^2D

B^2CD BC^2D^2

②

(3)

runs	L_1	L_2			
(1)	0	0	00	01	02
a	1	1	(1)	a^2b	
a^2	2	2	+8	+8	
b	1	2			
ab	2	0			
a^2b	0	1			
b^2	2	1	10	11	12
ab^2	0	2	a^2b^2	a	b
a^2b^2	1	0	+8	+8	+8
...			
$a^2b^2c^2d^2$	0	1	20	21	22

3^{k-p} design: k factors, 3 levels each,
divided into 3^p blocks, but
only 1 block is run.

(4)

($\frac{1}{3^p}$ fraction of a 3^k design)

Example Construct a 3^{3-1} design
($\frac{1}{3}$ fraction of a 3^3 design)

Choose 1 effect to be a generator.

Choose AB^2C (and $A^2B^2C^2$)

$$I = AB^2C = A^2BC^2 \quad (\text{Resolution III}) \textcircled{5}$$

$$A = A^2 = \cancel{A^2B^2C} = BC^2 = B^2C = ABC^2$$

$$B = B^2 = AC = A^2B^2C^2 = ABC = A^2C^2$$

$$C = C^2 = A^2BC^2 = A^2B = AB^2 = A^2BC$$

$$AB = A^2B^2 = A^2C = B^2C^2 = BC = AC^2$$

Source	df	
A	2	
B	2	
C	2	
AB	2	(Should be 4, but A^2B & AB^2 are absorbed with C)
ERP	0	
TOT	8	

Which 4 runs do we conduct?

⑥

Project our 3^{3-1} design onto a 3^2 design

A	B	$C = A^2B$	$L = 2x_1 + x_2$
0	0	0	(1)
1	0	2	AC^2
2	0	1	A^2C
0	1	1	bc
1	1	0	ab
2	1	2	A^2bc^2
0	2	2	b^2c^2
1	2	1	Ab^2c
2	2	0	A^2b^2

Example: Construct a 3^{4-2}_{III} design

(7)

Choose 2 generators, get 8 total

$$\begin{aligned} I &= AB^2C = BCD \\ &= A^2BC^2 = B^2C^2D^2 \\ &= AC^2D = A^2CD^2 \\ &= ABD^2 = A^2B^2D \end{aligned}$$

Res III

Write out the alias structure, keeping track of only main effects & 2-way interactions

$$\begin{aligned} A &= A^2 = BC^2 = CD^2 = B^2D \\ &= B^2C = C^2D = BD^2 \end{aligned}$$

$$\begin{aligned} B &= B^2 = AC = C^2D^2 = AD \\ &= CD = A^2C^2 = AB^2 \end{aligned}$$

$$\begin{aligned} C &= C^2 = A^2B = B^2D^2 = AD \\ &= AB^2 = BD = A^2D^2 \end{aligned}$$

$$\begin{aligned} D &= D^2 = B^2C^2 = A^2C = AB \\ &= BC = AC^2 = A^2B^2 \end{aligned}$$

(8)

Source	df
A	2
B	2
C	2
D	2
ERR	0
TOT	8

Find the 9 runs to conduct

⑨

Project the 3^{4-2} design onto a full 5^2 design

A	B	$C = A^2B$	$D = B^2C^2$	$L_1 = 2x_1 + x_2$ (i) $L_2 = 2x_2 + 2x_3$
0	0	0	0	
1	0	2	1	ac^2d
2	0	1	2	a^2cd^2
0	1	1	1	bcd
1	1	0	2	abd^2
2	1	2	0	a^2bc^2
0	2	2	2	$b^2c^2d^2$
1	2	1	0	ab^2c
2	2	0	1	a^2b^2d