Stat 565
1-18-18
1

Bonferroni Method

Use Fisher's method, but replace $\alpha/2$ with $\frac{\alpha}{2m}$, where m is the number of comparisons. Usually, $m = \binom{a}{2}$.

Tukey's Method

Compute $T = q_\alpha(a, f)\sqrt{\frac{MSE}{n}}$

a: # means being compared

f: $df_E = N - a$

If $|\bar{y}_{i\cdot} - \bar{y}_{j\cdot}| > T$, declare them signif. diff.

Read q from Table VII, Studentized range

2

Dunnett's Method

Special situation, where one group was a control group, and each other group is compared to the control.

Compute $D = d_\alpha(a-1, f)\sqrt{2\frac{MSE}{n}}$

$a-1$: # groups

f: $df_E = N - a$

Read d from Table VIII

If $|\bar{y}_{i\cdot} - \bar{y}_{1\cdot}| > D$, declare them signif. diff.

Example. Assume you have conducted a 1-way ANOVA with $a = 5$ and H_0 was rejected.

$\bar{y}_{1.} = 9.8$

$\bar{y}_{2.} = 15.4$

$\bar{y}_{3.} = 17.6$

$\bar{y}_{4.} = 21.6$

$\bar{y}_{5.} = 10.8$

Suppose that we use Fisher's method + LSD = 2

1	5	2	3	4
9.8	10.8	15.4	17.6	21.6

4

Suppose we use Tukey's method and find $T = 5$

1	5	2	3	4
9.8	10.8	15.4	17.6	21.6

Power

True	Fail to Rej H_0	Rej H_0
H_0	✓	Type I
H_1	Type II	✓

$1-\alpha$	α
β	$1-\beta$

power

5

α = Prob(Rej H_0 | H_0 true) = level of significance

β = Prob(Fail to Rej H_0 | H_1 true)

$1-\beta$ = power

The power is not a single number.
It is a function of $\alpha, \tau_1, \tau_2, \ldots, \tau_a$

Under H_0, the F statistic has an F distribution with $(a-1)$ and $(N-a)$ df.

Under H_1, the F stat has a noncentral F distribution

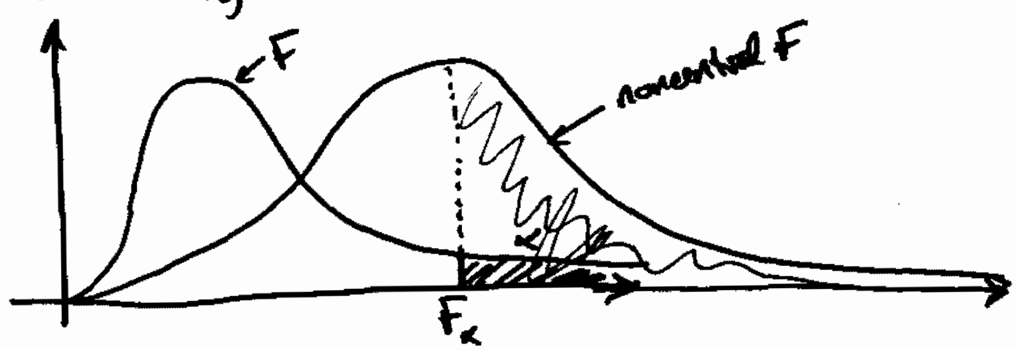
6

with $(a-1)$ and $(N-a)$ df and

noncentrality parameter $\varphi = \sqrt{\dfrac{n\sum_{i=1}^{a}\tau_i^2}{a\sigma^2}}$

So the power is Prob(Rej H_0 | H_1 true),

which is the area under the noncentral F curve, lying to the right of the critical value.



7

Notes:

power $\uparrow$ as $n \uparrow$

power $\downarrow$ as $\alpha \downarrow$

Example: We are comparing 4 population means

$\alpha = .05$

We know that σ^2 is roughly 2.5 for each population.

Our goal is to detect a difference if any pair of the population means differ by more than 2 units.

8

Say $\tau_i - \tau_j = D$ for some i, j

Write $\tau_i = \underbrace{\frac{\tau_i + \tau_j}{2}}_{m} + \frac{D}{2}$ and $\tau_j = \underbrace{\frac{\tau_i + \tau_j}{2}}_{m} - \frac{D}{2}$

Then
$$
\begin{aligned}
\sum_{i=1}^{a} \tau_i^2 &\geq \tau_i^2 + \tau_j^2 \\
&= \left(m + \frac{D}{2}\right)^2 + \left(m - \frac{D}{2}\right)^2 \\
&= m^2 + mD + \frac{D^2}{4} + m^2 - mD + \frac{D^2}{4} \\
&= 2m^2 + \frac{D^2}{2} \geq \frac{D^2}{2}
\end{aligned}
$$

So $\Phi = \sqrt{\frac{n \sum \tau_i^2}{a \sigma^2}} \geq \sqrt{\frac{n D^2}{2 a \sigma^2}}$

In our example, $\phi \geq \sqrt{\frac{n\,2^2}{2 \cdot 4 \cdot 2.5}} = .45\sqrt{n}$ (9)

Use Table V (pp 693–696)

$\nu_1 = \text{num df} = a-1 = 3$

$\nu_2 = \text{den df} = N-a = na-a = a(n-1)$
$= 4(n-1)$

Let's pick a value for n & find the power.

Say $n=16$. $\phi = .45\sqrt{16} = 1.8$, $\nu_2 = 4(15) = 60$

Using p.694, $\alpha = .05$, $\nu_2 = 60$, $\phi = 1.8$, we find $\beta \approx .2$

So the power would be .8.

(10)

HW #2 due Thu 1/25

Complete the remaining parts of 3.7 and 3.12

3.7. The tensile strength of Portland cement is being studied. Four different mixing techniques can be used economically. A completely randomized experiment was conducted and the following data were collected:

Mixing Technique	Tensile Strength (lb/in^2)			
1	3129	3000	2865	2890
2	3200	3300	2975	3150
3	2800	2900	2985	3050
4	2600	2700	2600	2765

(a) Test the hypothesis that mixing techniques affect the strength of the cement. Use $\alpha = 0.05$.

(b) Construct a graphical display as described in Section 3.5.3 to compare the mean tensile strengths for the four mixing techniques. What are your conclusions?

(c) Use the Fisher LSD method with $\alpha = 0.05$ to make comparisons between pairs of means.

(d) Construct a normal probability plot of the residuals. What conclusion would you draw about the validity of the normality assumption?

(e) Plot the residuals versus the predicted tensile strength. Comment on the plot.

(f) Prepare a scatter plot of the results to aid the interpretation of the results of this experiment.

3.12. A pharmaceutical manufacturer wants to investigate the bioactivity of a new drug. A completely randomized single-factor experiment was conducted with three dosage levels, and the following results were obtained.

Dosage		**Observations**		
20 g	24	28	37	30
30 g	37	44	31	35
40 g	42	47	52	38

(**a**) Is there evidence to indicate that dosage level affects bioactivity? Use $\alpha = 0.05$.

(**b**) If it is appropriate to do so, make comparisons between the pairs of means. What conclusions can you draw?

(**c**) Analyze the residuals from this experiment and comment on model adequacy.