Stat 565
2-6-18

Factorial models, continued

①

3 factors:

$$y_{ijkl} = \mu + \tau_i + \beta_j + \gamma_k + (\tau\beta)_{ij} + (\tau\gamma)_{ik} + (\beta\gamma)_{jk} + (\tau\beta\gamma)_{ijk} + \varepsilon_{ijkl}$$

$i = 1, \ldots, a$
$j = 1, \ldots, b$ $\quad N = abcn$
$k = 1, \ldots, c$
$l = 1, \ldots, n$

②

Source	df
A	$a-1$
B	$b-1$
C	$c-1$
AB	$(a-1)(b-1)$
AC	$(a-1)(c-1)$
BC	$(b-1)(c-1)$
ABC	$(a-1)(b-1)(c-1)$
ERR	$abc(n-1)$
TOT	$N-1$

③

Estimable functions

Defn: An estimable function of the parameters is a linear combination of the left hand sides of the normal equations

$$\frac{\partial SSE}{\partial \mu}, \quad \frac{\partial SSE}{\partial \tau_i}, \text{ etc.} \quad \stackrel{\text{set}}{=} 0$$

2-factor example

$$SSE = \sum_i \sum_j \sum_k \left(y_{ijk} - \mu - \tau_i - \beta_j - (\tau\beta)_{ij}\right)^2$$

④

$\frac{\partial SSE}{\partial \mu} \stackrel{\text{set}}{=} 0$:

$$(1) \quad \mu + \bar{\tau}_. + \bar{\beta}_. + (\overline{\tau\beta})_{..} = \bar{y}_{...}$$

$$\frac{\partial SSE}{\partial \tau_i} : (2) \quad \mu + \tau_i + \bar{\beta}_. + (\overline{\tau\beta})_{i.} = \bar{y}_{i..} \quad \forall i$$

$$\frac{\partial SSE}{\partial \beta_j} : (3) \quad \mu + \bar{\tau}_. + \beta_j + (\overline{\tau\beta})_{.j} = \bar{y}_{.j.} \quad \forall j$$

$$\frac{\partial SSE}{\partial (\tau\beta)_{ij}} : (4) \quad \mu + \tau_i + \beta_j + (\tau\beta)_{ij} = \bar{y}_{ij.} \quad \forall i,j$$

These are the normal equations

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Consider (2) with $i=1$, (2) with $i=2$

+ subtract

$$\mu + \tau_1 + \bar{\beta}_{\cdot} + \overline{(\tau\beta)}_{1\cdot} - \left(\mu + \tau_2 + \bar{\beta}_{\cdot} + \overline{(\tau\beta)}_{2\cdot}\right)$$

$$= (\tau_1 - \tau_2) + \underbrace{\overline{(\tau\beta)}_{1\cdot} - \overline{(\tau\beta)}_{2\cdot}}$$

If there are no missing values, $= 0$ and

$\tau_1 - \tau_2$ would be estimable.

Its estimate would be $\bar{y}_{1\cdot\cdot} - \bar{y}_{2\cdot\cdot}$

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Blocking in factorial designs

2-factor model, and on each of n days

we run 1 entire replication of the experiment

$$y_{ijk} = \mu + \tau_i + \beta_j + (\tau\beta)_{ij} + \underset{\substack{\uparrow \\ \text{blocks}}}{\delta_k} + \varepsilon_{ijk}$$

Source	df
A	$a-1$
B	$b-1$
AB	$(a-1)(b-1)$
BLK	$n-1$
ERR	$(ab-1)(n-1)$
	$N-1$

$abn-1-(a-1)-(b-1)$
$-(a-1)(b-1)-(n-1)$

(7)

Midterm in-class 2/13

1 take home problem will be posted 2/8, due at beginning of exam

Take-home: RCBD with a missing value

In-class: - Take a new model & write it out in matrix form $Y = X\beta + \varepsilon$

- Use least squares to find parameter estimates
- Figure out the df for a new model

(8)

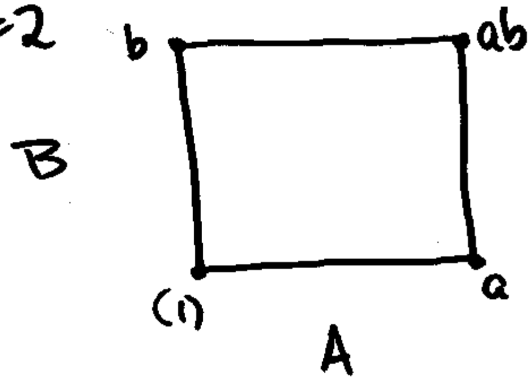
- 1-way, RCBD, BIBD, Latin square, Graeco-Latin square, Factorial

You can bring 1 page of notes (front & back) + calculator

The 2^k factorial design

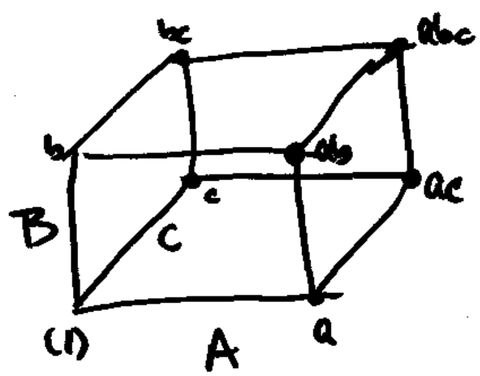
K factors, each with 2 levels

$k=2$



	A	B	replication
(1)	−	−	x x x
a	+	−	x x x
b	−	+	x x x
ab	+	+	x x x

$k=3$ 2^3 design



	A	B	C	AB	AC	BC	ABC
(1)	−	−	−	+	+	+	−
a	+	−	−	−	−	+	+
b	−	+	−	−	+	−	+
ab	+	+	−	+	−	−	−
c	−	−	+	+	−	−	+
ac	+	−	+	−	+	−	−
bc	−	+	+	−	−	+	−
abc	+	+	+	+	+	+	+