

Create a  $2^4$  design in 4 blocks  $= 2^2 \leftarrow$  choose 2 effects to be confounded Stat 525  
2-15-18

	A	B	C	D	ABC	BCD	Confounded
(1)	-	-	-	-	-	-	
a	+	-	-	-	+	-	Choose ABC, BCD
b	-	+	-	-	+	+	
ab	+	+	-	-	-	+	ABC $\times$ BCD = AD
c	-	-	+	-	+	+	
ac	+	-	+	-	-	+	
bc	-	+	+	-	-	-	
abc	+	+	+	-	+	-	
d	-	-	-	+	-	+	
ad	+	-	-	+	+	+	
bd	-	+	-	+	+	-	
abd	+	+	-	+	-	-	
cd	-	-	+	+	+	-	
acd	+	-	+	+	-	-	
bcd	-	+	+	+	-	+	
abcd	+	+	+	+	+	+	

(1)	a
bc	abc
abd	bd
acd	cd

ab	b
ac	c
d	ad
bcd	abcd

Source	df
A	1
B	1
C	1
D	1
AB	1
AC	1
BC	1
BD	1
CD	1
ABD	1
ACD	1
ABCD	1
Blocks	3
Total	15

Defining constraints (2)

$$L = \alpha_1 x_1 + \alpha_2 x_2 + \alpha_3 x_3 + \alpha_4 x_4 \pmod{2}$$

We choose the  $\alpha_i$ 's.

The  $x_i$ 's are indicators: + : 1  
- : 0

For ABC,  $L_1 = x_1 + x_2 + x_3 \pmod{2}$

BCD  $L_2 = x_2 + x_3 + x_4 \pmod{2}$

	$L_1$	$L_2$
(1)	0	0
a	1	0
b	1	1
ab	0	1
c	1	1
ac	0	1
bc	0	0
abc	1	0
d	0	1
ad	1	1
bd	1	0
abd	0	0
cd	1	0
acd	0	0
bcd	0	1
abcd	1	1

Note that this gives  
exactly the same 4 blocks  
as our  $+1-$  method.

(3)

### Fractional $2^k$ designs

(4)

Start with a  $2^k$  design

Divide the runs into  $2^p$  blocks

But now, only run 1 of the blocks

This is called a  $2^{k-p}$  design

Example: Construct a  $2^{3-1}$  design

	A	B	C	ABC
(1)	-	-	-	-
a	+	-	-	+
b	-	+	-	+
ab	+	+	-	-
c	-	-	+	+
ac	+	-	+	-
bc	-	+	+	-
abc	+	+	+	+

Choose ABC  
to confound  
with blocks

(5)

(1)	a
ab	b
ac	c
bc	abc

↑  
principal  
block

Just run  
this one

	I	A	B	C	AB	AC	BC	ABC
(1)	+	-	-	-	+	+	+	-
ab	+	+	+	-	+	-	-	-
ac	+	+	-	+	-	+	-	-
bc	+	-	+	+	-	-	+	-

(6)

Note  $I = -ABC$  } generator word  
 $A = -BC$  }  
 $B = -AC$  } This is the  
 $C = -AB$  } alias structure

Defn: The resolution of a  $2^{k-p}$  design is the  
smallest number of letters in any generator word

(7)

Our example is resolution III,

so we have a  $2^{3-1}_{III}$  design.

In a resolution III design,

each main effect will be aliased

with a 2-way interaction

Source	df
A	1
B	1
C	1
Total	3

Example  $2^{6-2}$  design

(8)

Suppose we choose ABC and DEF as generators  
 ABCDEF is also a generator

So our resolution is III

$$I = ABC = DEF = ABCDEF$$

$$A = BC = ADEF = BCDEF$$

$$B = AC = BDEF = ACDEF$$

etc.

⑨

Try Again:

Choose  $ABCD$  &  $CDEF$  as generators

$$I = ABCD = CDEF = AB EF \quad (\text{Res IV})$$

$$A = BCD = ACDEF = B EF$$

$$B = ACD = BCDEF = A EF$$

⋮

$$AB = CD = ABCDEF = EF$$

HW #5 due 2/22

P. 181 #40, 42

P. 229 #30

**4.40.** An engineer is studying the mileage performance characteristics of five types of gasoline additives. In the road test he wishes to use cars as blocks; however, because of a time constraint, he must use an incomplete block design. He runs the balanced design with the five blocks that follow. Analyze the data from this experiment (use  $\alpha = 0.05$ ) and draw conclusions.

Additive	Car				
	1	2	3	4	5
1		17	14	13	12
2	14	14		13	10
3	12		13	12	9
4	13	11	11	12	
5	11	12	10		8

**4.42.** Seven different hardwood concentrations are being studied to determine their effect on the strength of the paper produced. However, the pilot plant can only produce three runs each day. As days may differ, the analyst uses the balanced incomplete block design that follows. Analyze the data from this experiment (use  $\alpha = 0.05$ ) and draw conclusions.

Hardwood Concentration (%)	Days			
	1	2	3	4
2	114			
4	126	120		
6		137	117	
8	141		129	149
10		145		150
12			120	
14				136

Hardwood Concentration (%)	Days		
	5	6	7
2	120		117
4		119	
6			134
8			
10	143		
12	118	123	
14		130	127

**5.30.** An experiment was performed to investigate the keyboard feel on a computer (crisp or mushy) and the size of the keys (small, medium, or large). The response variable is typing speed. Three replicates of the experiment were performed. The experimental design and the data are as follow.

Key Size	Keyboard Feel	
	Mushy	Crisp
Small	31, 33, 35	36, 40, 41
Medium	36, 35, 33	40, 41, 42
Large	37, 34, 33	38, 36, 39

- (a) Analyze the data from this experiment.
- (b) Investigate model adequacy by constructing appropriate residual plots.
- (c) What conclusions can you draw?