Stat 561
10-4-18

Assume $P(M) = P(F) = .5$

Assume independent births

(1)

You see a person with one of their children, a son.

You know that they have 2 children.

Find the prob. that their other child is a son.

Let A: 1st child is M

B: 2nd child is M

Find $P(A \cap B \mid A \cup B) = \dfrac{P((A \cap B) \cap (A \cup B))}{P(A \cup B)}$

(2)

$= \dfrac{P(A \cap B)}{P(A \cup B)}$

	P
MM	1/4
MF	1/4
FM	1/4
FF	1/4

$= \dfrac{1/4}{3/4} = \dfrac{1}{3}$

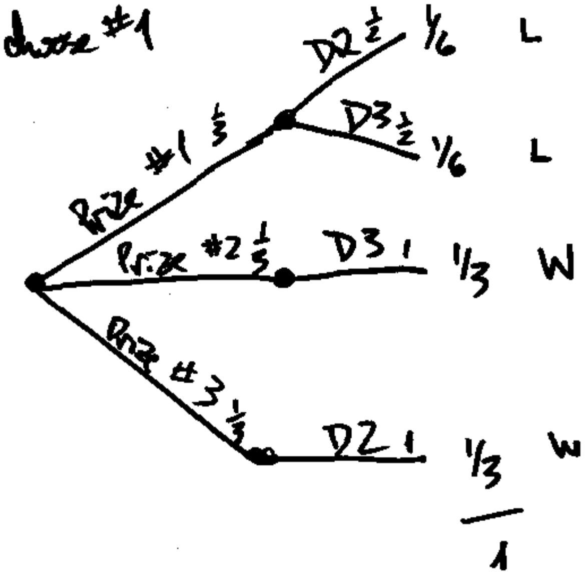
<u>Monty Hall problem</u>

You choose door #1

Monty Hall opens one of the 2 remaining doors to reveal a joke prize. He then gives you the chance to switch your choice. Should you?

Assume that you switch

You chose #1



Switch $\Rightarrow$ $P(W) = 2/3$

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Random variables

Defn: A random variable is a function from the sample space into the set of real numbers.

$$X : S \to \mathbb{R}$$

Defn: The range of the random variable X is the set of real numbers that X maps onto.

⑤

<u>Example</u>: Flip 2 coins

$S = \{HH, HT, TH, TT\}$

Let X be the # of heads

$X(HH) = 2$
$X(HT) = 1$
$X(TH) = 1$
$X(TT) = 0$

range of X is $\{0,1,2\}$

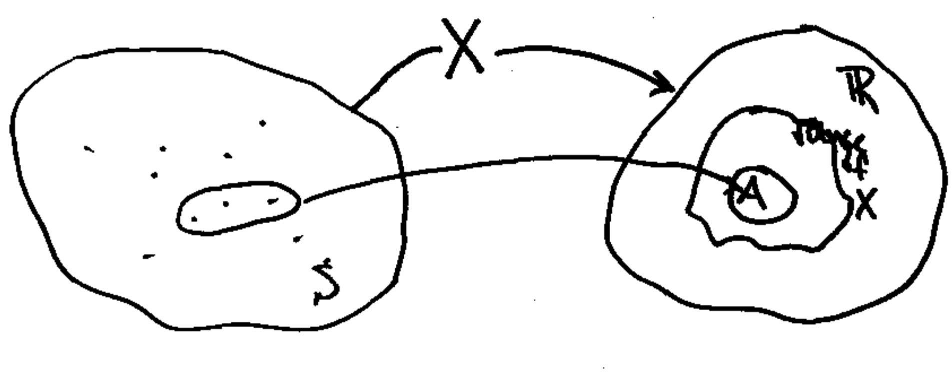
<u>Defn</u>: If the range of X is countable (finite or countably infinite), then X is a <u>discrete</u> random variable.

⑥

If the range of X is an interval, then X is a <u>continuous</u> random variable.

<u>Defn</u>. Let A be a subset of the range of X.

Define $P_X(A) = P[\{s \in S : X(s) \in A\}]$



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<u>Example</u>: Roll 2 dice

$$S = \left\{ \begin{matrix} 11 & 12 & \cdots & 16 \\ 21 & 22 & \cdots & 26 \\ \vdots & & & \\ 61 & 62 & \cdots & 66 \end{matrix} \right\}$$

Let X be the maximum of the 2 dice.

Find the probability that $X = 5$.

$$P_X(\{5\}) = P\left[\{15, 25, 35, 45, 51, 52, 53, 54, 55\}\right]$$
$$= \frac{9}{36} = \frac{1}{4}$$

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Consider the <u>discrete</u> case

<u>Defn</u>: Let $r_i \in$ range of X

Let $p_X(r_i) = P_X(\{r_i\})$.

p_X is called a <u>probability mass function</u> (pmf)

<u>Defn</u>: Let $F_X(x) = P_X\left[(-\infty, x]\right]$

This is called the <u>cumulative distribution function</u> (cdf) of the random variable X.

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Example: Flip 3 coins.

Let $X = \#$ heads.

Find the pmf & cdf for X.

$$S = \{HHH, HTH, HHT, HTT, THH, THT, TTH, TTT\}$$

$$\begin{array}{cccccccc} HHH & HTH & HHT & HTT & THH & THT & TTH & TTT \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 3 & 2 & 2 & 1 & 2 & 1 & 1 & 0 \end{array}$$

pmf:

$p_X(0) = \frac{1}{8}$

$p_X(1) = 3/8$

$p_X(2) = 3/8$ $\qquad p_X(1.5) = 0$

$p_X(3) = \frac{1}{8}$

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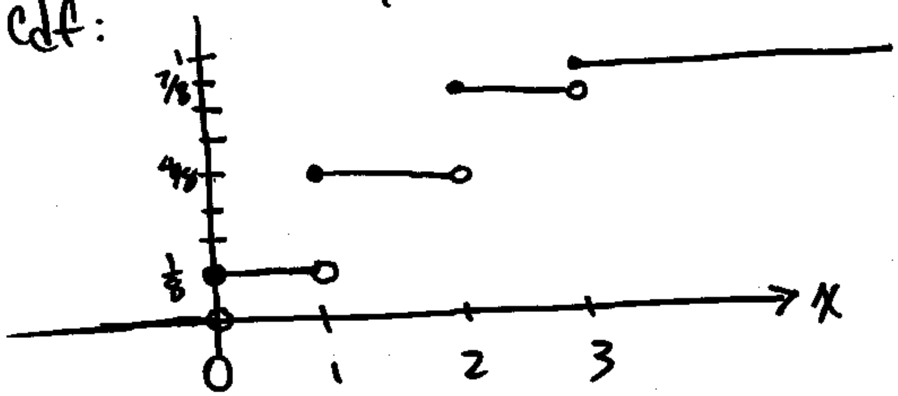
cdf: $F(x) = P(X \leq x)$



HW #2 due 10/11

p.42 #39, 47, 54, 55

(scanned pages follow)

1.39 A pair of events A and B cannot be simultaneously *mutually exclusive* and *independent*. Prove that if $P(A) > 0$ and $P(B) > 0$, then:

(a) If A and B are mutually exclusive, they cannot be independent.

(b) If A and B are independent, they cannot be mutually exclusive.

1.47 Prove that the following functions are cdfs.

(a) $\frac{1}{2} + \frac{1}{\pi}\tan^{-1}(x)$, $x \in (-\infty, \infty)$ (b) $(1 + e^{-x})^{-1}$, $x \in (-\infty, \infty)$

(c) $e^{-e^{-x}}$, $x \in (-\infty, \infty)$ (d) $1 - e^{-x}$, $x \in (0, \infty)$

(e) the function defined in (1.5.6)

1.54 For each of the following, determine the value of c that makes $f(x)$ a pdf.

(a) $f(x) = c\sin x$, $0 < x < \pi/2$ (b) $f(x) = ce^{-|x|}$, $-\infty < x < \infty$

1.55 An electronic device has lifetime denoted by T. The device has value $V = 5$ if it fails before time $t = 3$; otherwise, it has value $V = 2T$. Find the cdf of V, if T has pdf

$$f_T(t) = \frac{1}{1.5}e^{-t/(1.5)}, \quad t > 0.$$