

Confidence intervals of the form  
estimate  $\pm$  margin of error :

Stat 543  
4-9-15  
①

for  $\mu$ :  $\bar{x} \pm t \frac{s}{\sqrt{n}}$  ( $df = n-1$ )

for  $\mu_1 - \mu_2$ :  $\bar{x}_1 - \bar{x}_2 \pm t \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$   $df = *$   
 $* = \left[ \frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right]^2 / \left[ \frac{1}{n_1-1} \left( \frac{s_1^2}{n_1} \right)^2 + \frac{1}{n_2-1} \left( \frac{s_2^2}{n_2} \right)^2 \right]$

for  $p$ :  $\hat{p} \pm z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

for  $p_1 - p_2$ :  $\hat{p}_1 - \hat{p}_2 \pm z \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$

Example 1 In a sample of 200 customers,  
108 say that they would purchase  
a certain new product.

Find a 95% Conf. int. for the percentage  
of the population of customers who would  
buy the product.

$$\hat{p} \pm z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$.54 \pm 1.96 \sqrt{\frac{(.54)(.46)}{200}}$$

$$.54 \pm .069 \quad \text{OR} \quad (.471, .609)$$

$$n = 200$$

$$\hat{p} = \frac{108}{200} = .54$$

$$z = \text{invnorm}(.975)$$

$$= 1.96$$

②

Suppose we have a desired margin of error of .03. What  $n$  do we need?

(3)

Set  $E$  = margin of error

$$E = z \sqrt{\hat{p}(1-\hat{p})} + \text{solve for } n$$

$$n = \frac{z^2 \hat{p}(1-\hat{p})}{E^2} = \frac{1.96^2 (.54)(.46)}{.03^2} = 1061$$

\* If no pilot study is available, you could replace  $\hat{p}(1-\hat{p})$  by .25 in the formula for  $n$ , and be guaranteed to meet or exceed the necessary  $n$ .

Example 2:	no fertilizer	fertilizer
	Control group	Treatment group
	$n_1 = 128$ plants	$n_2 = 120$
	$\bar{x}_1 = 8$ kg	$\bar{x}_2 = 10$ kg
	$s_1 = 1$ kg	$s_2 = 1.5$ kg

(4)

Find a 95% CI for the difference, Trt - Ctrl  
 $\mu_2 - \mu_1$

$$\bar{x}_2 - \bar{x}_1 \pm t \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$(1.6787, 2.3213) \text{ or } 2 \pm .3213$$

Example 3

(placebo)  
Control group

$$n_1 = 25$$

10 improvements

(drug)  
treatment group

$$n_2 = 20$$

12 improvements

⑤

Find a 95% C.I. for the difference

$$\hat{p}_1 - \hat{p}_2 \pm z \sqrt{\hat{p}_1 \frac{(1-\hat{p}_1)}{n_1} + \hat{p}_2 \frac{(1-\hat{p}_2)}{n_2}}$$

$$\hat{p}_1 = .4$$

$$\hat{p}_2 = .6$$

$$(-.4881, .0881) \text{ or } -.2 \pm .2881$$

Hw#2 follows this page.

- 31. Blood Alcohol Concentration** A researcher for the National Highway Traffic Safety Administration (NHTSA) wishes to estimate the average blood alcohol concentration (BAC) for drivers involved in fatal accidents who are found to have positive BAC values. He randomly selects records from 1,200 such drivers and determines the sample mean BAC to be 0.16 g/dL.

*Source:* Based on data from NHTSA, August 2007.

Assuming that the population standard deviation for the BAC of such drivers is 0.08 g/dL, construct and interpret a 90% confidence interval for the mean BAC.

- 34. Travel Time to Work** An urban economist wishes to estimate the mean amount of time people spend traveling to work. He obtains a random sample of 50 individuals who are in the labor force and finds that the mean travel time is 24.2 minutes.

*Source:* Based on data from the American Community Survey

- (a) Assuming that the population standard deviation of travel time is 18.5 minutes, construct and interpret a 95% confidence interval for the mean travel time to work.

**Note:** The standard deviation is large because many people work at home (travel time = 0 minutes) and many have commutes in excess of 1 hour.

- (b) Could the interval in part (a) be used to estimate the mean travel time to work in a rural town? Why?

**45. How Much Do You Read?** A Gallup poll conducted May 20–22, 2005, asked Americans how many books, either hardback or paperback, they read during the previous year. How many subjects are needed to estimate the number of books Americans read the previous year within one book with 95% confidence? Initial survey results indicate that  $\sigma = 16.6$  books.

**16. Tensile Strength** Tensile strength is the amount of stress a material can withstand before it breaks. Researchers wanted to determine the tensile strength of a resin cement used in bonding crowns to teeth. The researchers bonded crowns to 72 extracted teeth. Using a tensile resistance test, they found the mean tensile strength of the resin cement to be 242.2 newtons (N), with a standard deviation of 70.6 N. Based on this information, construct a 90% confidence interval for the mean tensile strength of the resin cement.

**18. Ford Mustang Gas Mileage** For a simple random sample of forty 2007 Ford Mustangs (6-cylinder, 4-liter, 5-speed manual), the mean gas mileage was 20 miles per gallon with a standard deviation of 1.5 miles per gallon.

*Source:* Based on information from [www.fueleconomy.com](http://www.fueleconomy.com)

Construct and interpret a 95% confidence interval for the mean gas mileage of similar 2007 Ford Mustangs.

**19. Lipitor** The drug Lipitor™ is meant to lower cholesterol levels. In a clinical trial of 863 patients who received 10-mg doses of Lipitor daily, 47 reported a headache as a side effect.

Construct a 90% confidence interval for the population proportion of Lipitor users who will report a headache as a side effect.

**25. High-Speed Internet Access** A researcher wishes to estimate the proportion of adults who have high-speed Internet access. What size sample should be obtained if she wishes the estimate to be within 0.03 with 99% confidence if

- (a) she uses a 2007 estimate of 0.69 obtained from Nielsen NetRatings?
- (b) she does not use any prior estimates?

**13. Concrete Strength** An engineer wanted to know whether the strength of two different concrete mix designs differed significantly. He randomly selected 9 cylinders, measuring 6 inches in diameter and 12 inches in height, into which mixture 67-0-301 was poured. After 28 days, he measured the strength (in pounds per square inch) of the cylinder. He also randomly selected 10 cylinders of mixture 67-0-400 and performed the same test. The results are as follows:

Mixture 67-0-301			Mixture 67-0-400			
3,960	4,090	3,100	4,070	4,890	5,020	4,330
3,830	3,200	3,780	4,640	5,220	4,190	3,730
4,080	4,040	2,940	4,120	4,620		

Construct a 90% confidence interval for  $\mu_{400} - \mu_{301}$  and interpret the results.