Stat 543
4-7-15
1

Confidence intervals for population means

[Estimate ± margin of error]

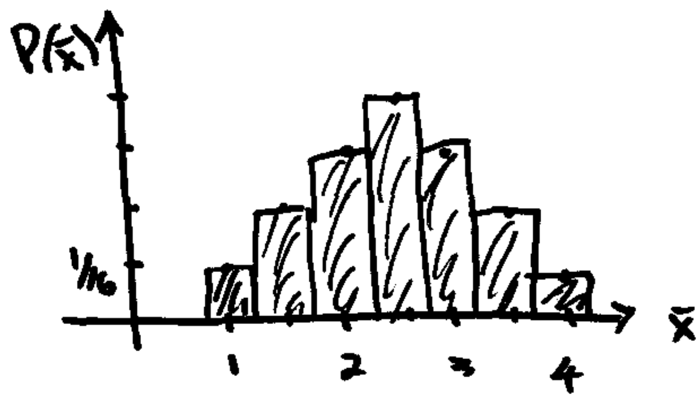
Example: A box has 4 slips of paper, numbered 1-4

Draw 2 slips, with replacement, + record $\bar{x}$

1st \ 2nd	1	2	3	4
1	1	1.5	2	2.5
2	1.5	2	2.5	3
3	2	2.5	3	3.5
4	2.5	3	3.5	4

$\bar{x}$	Prob($\bar{x}$)
1	1/16
1.5	2/16
2	3/16
2.5	4/16
3	3/16
3.5	2/16
4	1/16
	1

2



Central Limit Theorem: Regardless of the original population, the sample mean, $\bar{x}$, will behave* (approximately) as a normal random variable, provided that the sample size is sufficiently large.

*subject to some conditions

3

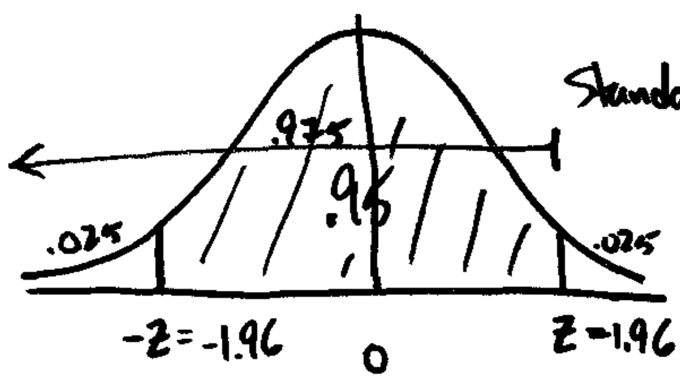
Collect a random sample from the population.

The population mean, μ, and standard deviation, σ are unknown. But we can calculate $\bar{x}$ and s from the sample.

The z-score for $\bar{x}$ is $\dfrac{\bar{x}-\mu}{(\sigma/\sqrt{n})}$.

The Central Limit Theorem guarantees that these z-scores will (approximately) follow a standard normal distribution. (mean $= 0$, st. dev. $= 1$)

4



Standard normal distribution

(TI-84: invnorm(.975))

$\dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}}$ will fall between ± 1.96, 95% of the time

(5)

$$-1.96 < \frac{\bar{x}-\mu}{\sigma/\sqrt{n}} < 1.96$$ with 95% confidence

$$-1.96\frac{\sigma}{\sqrt{n}} < \bar{x} - \mu < 1.96\sigma/\sqrt{n}$$

$$-\bar{x} - 1.96\frac{\sigma}{\sqrt{n}} < -\mu < -\bar{x} + 1.96\sigma/\sqrt{n}$$

$$\bar{x} + 1.96\frac{\sigma}{\sqrt{n}} > \mu > \bar{x} - 1.96\frac{\sigma}{\sqrt{n}}$$

That is, μ lies between $\bar{x} \pm 1.96\frac{\sigma}{\sqrt{n}}$,
with 95% confidence

(6)

In general: $\bar{x} \pm z\frac{\sigma}{\sqrt{n}}$

→ 95%: invnorm(.975) = 1.96
99%: invnorm(.995) = 2.576
90%: invnorm(.95) = 1.645

Since σ is almost always unknown, s is used in its place.

$$\bar{x} \pm t\frac{s}{\sqrt{n}}$$

degrees of freedom = df = n-1

invT(.975, df)

(7)

Example: We take a sample of 25 people, + find an average weight of 170 lbs + a standard deviation of 20 lbs.

Find a 95% conf. interval for μ.

$$\bar{x} \pm \overbrace{t \underbrace{\frac{s}{\sqrt{n}}}_{\text{standard error}}}^{\text{margin of error}} \qquad 170 \pm (2.064)\frac{20}{\sqrt{25}}$$

$$\uparrow$$

invT(.975, 24)

$$\boxed{170 \pm 8.256}$$

or $(161.744, 178.256)$

(8)

How large must n be to get a desired margin of error?

Set $E = t\frac{s}{\sqrt{n}}$ + solve for n

$$n = \left(\frac{ts}{E}\right)^2$$

We'll use z instead of t

$$\boxed{n = \left(\frac{zs}{E}\right)^2}$$

In our example, we want $E = 2$.

$$n = \left[\frac{1.96(20)}{2}\right]^2 = 384.16 \to \underline{\underline{385}}$$