

## Independent vs. dependent samples

Stat 943  
4-21-15

Matched pairs

①

The primary sampling unit is a pair

$$\begin{array}{c|c|c} H_0: \mu_d = 0 & H_0: \mu_d \leq 0 & H_0: \mu_d \geq 0 \\ H_1: \mu_d \neq 0 & H_1: \mu_d > 0 & H_1: \mu_d < 0 \end{array}$$

Analysis consists of a 1-sample t-test on the differences

Reaction time example

	Left	Right	$d = L - R$
1	.582	.408	.174
2	.481	.404	.077
3	.841	.542	.299
4	.267	.402	-.135
5	.685	.456	.229
6	.450	.533	-.083

Construct a test  
to show that  
times are faster  
with the right hand.  
 $\alpha = .05$

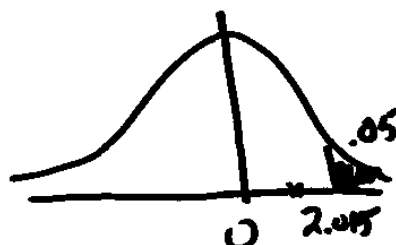
$$\begin{array}{l} H_0: \mu_d \leq 0 \\ H_1: \mu_d > 0 \end{array}$$

$$\text{invT}(.95, 5)$$

$$\text{Test stat} = 1.32$$

$$p\text{-val} = .122$$

Fail to reject  $H_0$

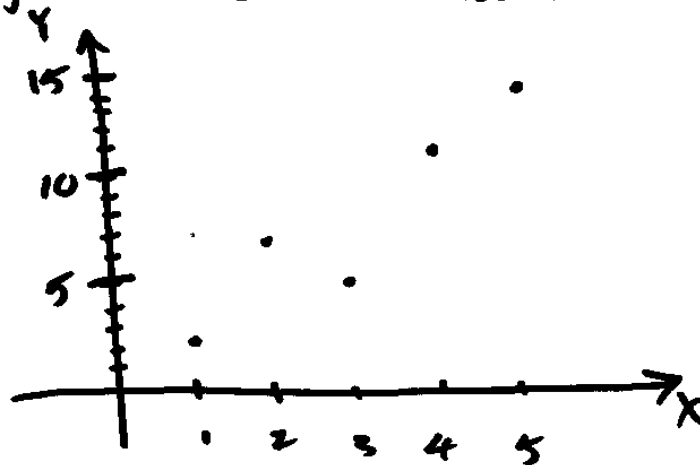


# Regression

- ③
- ① Is there a trend?
  - ② Is it increasing? decreasing?
  - ③ Is it linear?

Simple linear regression

X	Y
1	3
2	7
3	5
4	11
5	14



X = #yrs since shop has been open

Y = #employees

Model: 
$$y_i = \underbrace{\beta_0}_{\text{intercept}} + \underbrace{\beta_1 x_i}_{\text{slope}} + \underbrace{\varepsilon_i}_{\text{unexplained error}}$$

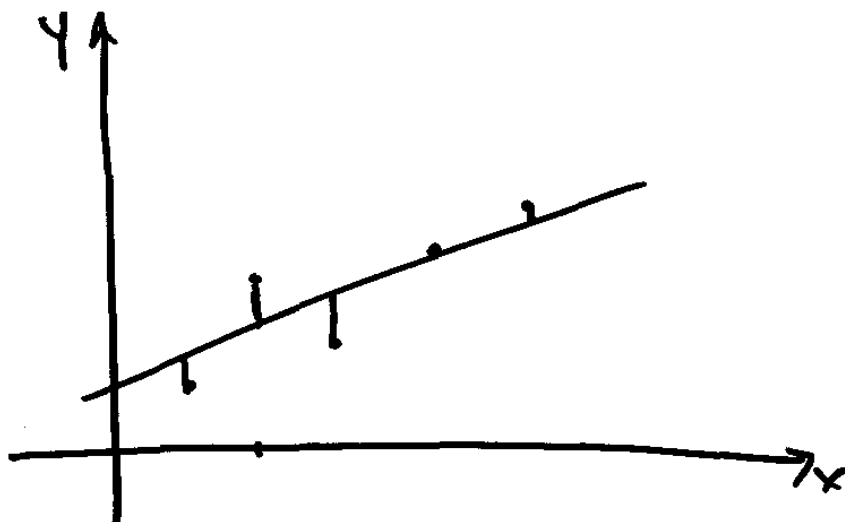
$$y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \varepsilon_i$$

④

is also a linear model  
because it is a linear  
combination of the parameters

$$y_i = \beta_0 e^{\beta_1 x_i} + \varepsilon_i$$

is not a linear model



Least squares method: minimizes the sum of squares of the vertical errors.

⑥

From the calculator (LinRegTTest)

$$a = .2 \quad b = 2.6$$

$$\hat{y} = .2 + 2.6x$$

Note: this line will always pass through the center of mass of the data  $(\bar{x}, \bar{y})$

Coeff. of determination

↓  
 $r^2 = .8445$

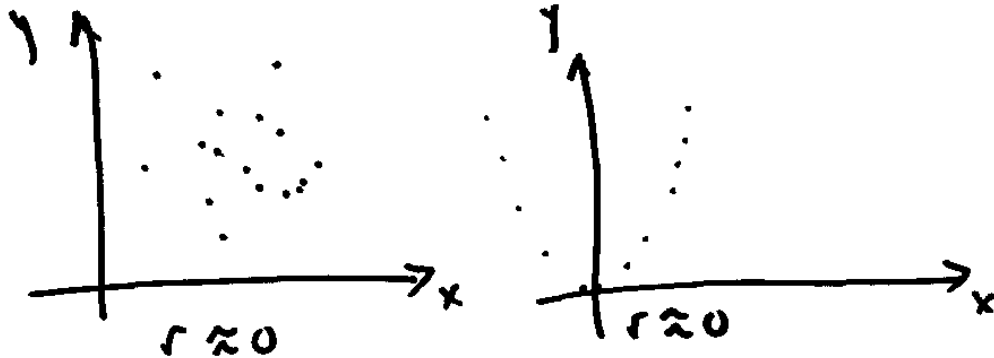
$r = .919$  correlation

$S = 2.033$  is an estimate of the standard deviation of the vertical errors

$$-1 \leq r \leq 1$$

1 and -1 are perfect

(7)



$r$  measures the linear relationship between  $y$  and  $x$   
(straight line)

$$0 \leq r^2 \leq 1$$

(8)

$r^2$  is the proportion of the total variation  
in the  $y$ -values that is explained by  $x$ .

