Stat 543
5-14-15

To complete the example from last time,

(1)

$$\text{Test stat} = \begin{cases} T_+ & \text{lower 1-sided} \\ T_- & \text{upper 1-sided} \\ \min(T_+, T_-) & \text{2-sided} \end{cases}$$

We had $T_+ = 13.5$ 2sided test
$T_- = 22.5$

Test stat = 13.5

In Table XI, read $n = 8$, $\frac{\alpha}{2} = .025$

Crit. val. is 3. Rule: reject Ho if test stat is less than crit. value. Fail to reject Ho

(2)

Mann-Whitney-Wilcoxon rank sum test

(substitute for the 2-sample t-test)

H_0: $\text{median}_1 = \text{median}_2$

H_1: $\text{median}_1 \; > \; < \; \neq \; \text{median}_2$

Example:

Offensive line		Defensive line	
323	313	289	305
320	380	295	300
295	318	250	310
328	309	278	339
305	305		

H_0: $\text{med}_1 \le \text{med}_2$

H_1: $\text{med}_1 > \text{med}_2$

(3)

Rank all obs., combined

	rank	adj
250	1	1
278	2	2
289	3	3
✓295	4	4.5
295	5	4.5
300	6	6
✓305	7	8
✓305	8	8
305	9	8
✓309	10	10
310	11	11
✓313	12	12
✓318	13	13

	rank	adj
✓320	14	14
✓323	15	15
✓328	16	16
339	17	17
✓350	18	18

S = sum of ranks for group 1

= 118.5

T = test stat

$= S - \frac{n_1(n_1+1)}{2}$

$= 118.5 - \frac{10(11)}{2} = 63.5$

(4)

Rule for Table XII :

lower 1-sided, reject H_0 if $T <$ crit. val.

upper " , " " if $T > n_1 n_2 -$ crit. val.

2-sided, " " if either condition occurs, but use $\alpha/2$

$n_1 = 10$, $n_2 = 8$, $\alpha = .05$ ⇒ crit. val = 21

Compare 63.5 to $10(8) - 21 = 59$

Reject H_0. The median weight is higher for the offensive line

5

Kruskal-Wallis test

(substitute for 1-way ANOVA)

$H_0: med_1 = med_2 = \ldots = med_K$

H_1: The population medians are not all equal.

<u>Example</u> 3 sources of raw material

Source 1	Source 2	Source 3
30	30	40
40	40	50
70	80	70
	90	100

6

Rank all observations

		rank	adj
✓	30	1	1.5
✓✓	30	2	1.5
✓	40	3	4
✓✓	40	4	4
	40	5	4
	50	6	6
✓	70	7	7.5
	70	8	7.5
✓✓	80	9	9
✓✓	90	10	10
	100	11	11

$n_1 = 3$
$R_1 = 13$

$n_2 = 4$
$R_2 = 24.5$

$n_3 = 4$
$R_3 = 28.5$

Test stat $= H = \dfrac{12}{n(n+1)} \sum \dfrac{R_i^2}{n_i} - 3(n+1)$

$$H = \frac{12}{11(12)}\left[\frac{13^2}{3} + \frac{24.5^2}{4} + \frac{25.5^2}{4}\right] - 3(12)$$
$$= 1.22$$

Crit val is ≈ 5.58

Rule: reject Ho if $H >$ crit. value

Fail to reject Ho.

If Ho is rejected, you would compare each pair of groups using M-W-W rank sum test.

Multivariate Methods

- used to detect & describe hidden (latent) underlying variables — Factor Analysis
- used to reduce the # of variables — Principal Components
- used to create groups of similar variables — Clustering of vars
- used to create groups of similar observations
- used to classify items into separate populations

Cluster Analysis, Discriminant Analysis

HW#7 follows.

9. Health and Happiness Are health and happiness related? The following data represent the level of happiness and level of health for a random sample of individuals from the General Social Survey.

		Health			
		Excellent	**Good**	**Fair**	**Poor**
Happiness	**Very Happy**	271	261	82	20
	Pretty Happy	247	567	231	53
	Not Too Happy	33	103	92	36

Source: General Social Survey

(a) Does the evidence suggest that health and happiness are related? Use the $\alpha = 0.05$ level of significance.

(b) Write a few sentences that explain the relation, if any, between health and happiness.

Table XI

Critical Values for the Wilcoxon Signed-Rank Test

	α			
n	**0.005**	**0.01**	**0.025**	**0.05**
5	*	*	*	0
6	*	*	0	2
7	*	0	2	3
8	0	1	3	5
9	1	3	5	8
10	3	5	8	10
11	5	7	10	13
12	7	9	13	17
13	9	12	17	21
14	12	15	21	25
15	15	19	25	30
16	19	23	29	35
17	23	27	34	41
18	27	32	40	47
19	32	37	46	53
20	37	43	52	60
21	42	49	58	67
22	48	55	65	75
23	54	62	73	83
24	61	69	81	91
25	68	76	89	100
26	75	84	98	110
27	83	92	107	119
28	91	101	116	130
29	100	110	126	140
30	109	120	137	151

* Indicates that it is not possible to get a value in the critical region.
Source: Jerrold H. Zar, *Biostatistical Analysis,* 4th ed., Prentice Hall, 1999.

Table XII

Critical Values of the Mann–Whitney Test Statistic

n_1	α	n_2 = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
	0.001	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0.005	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
2	0.01	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	1	1	2	2
	0.025	0	0	0	0	0	0	1	1	1	1	2	2	2	2	2	3	3	3	3
	0.05	0	0	0	1	1	1	2	2	2	2	3	3	4	4	4	4	5	5	5
	0.10	0	1	1	2	2	2	3	3	4	4	5	5	5	6	6	7	7	8	8
	0.001	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
	0.005	0	0	0	0	0	0	0	1	1	1	2	2	2	3	3	3	3	4	4
3	0.01	0	0	0	0	0	1	1	2	2	2	3	3	3	4	4	5	5	5	6
	0.025	0	0	0	1	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9
	0.05	0	1	1	2	3	3	4	5	5	6	6	7	8	8	9	10	10	11	12
	0.10	1	2	2	3	4	5	6	6	7	8	9	10	11	11	12	13	14	15	16
	0.001	0	0	0	0	0	0	0	0	1	1	1	2	2	2	3	3	4	4	4
	0.005	0	0	0	0	1	1	2	2	3	3	4	4	5	6	6	7	7	8	9
4	0.01	0	0	0	1	2	2	3	4	4	5	6	6	7	9	8	9	10	10	11
	0.025	0	0	1	2	3	4	5	5	6	7	8	9	10	11	12	12	13	14	15
	0.05	0	1	2	3	4	5	6	7	8	9	10	11	12	13	15	16	17	18	19
	0.10	1	2	4	5	6	7	8	10	11	12	13	14	16	17	18	19	21	22	23
	0.001	0	0	0	0	0	0	1	2	2	3	3	4	4	5	6	6	7	8	8
	0.005	0	0	0	1	2	2	3	4	5	6	7	8	8	9	10	11	12	13	14
5	0.01	0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
	0.025	0	1	2	3	4	6	7	8	9	10	12	13	14	15	16	18	19	20	21
	0.05	1	2	3	5	6	7	9	10	12	13	14	16	17	19	20	21	23	24	26
	0.10	2	3	5	6	8	9	11	13	14	16	18	19	21	23	24	26	28	29	31
	0.001	0	0	0	0	0	0	2	3	4	5	5	6	7	8	9	10	11	12	13
	0.005	0	0	1	2	3	4	5	6	7	8	10	11	12	13	14	16	17	18	19
6	0.01	0	0	2	3	4	5	7	8	9	10	12	13	14	16	17	19	20	21	23
	0.025	0	2	3	4	6	7	9	11	12	14	15	17	18	20	22	23	25	28	28
	0.05	1	3	4	6	8	9	11	13	15	17	18	20	22	24	26	27	29	31	33
	0.10	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	35	37	39
	0.001	0	0	0	0	1	2	3	4	6	7	8	9	10	11	12	14	15	16	17
	0.005	0	0	1	2	4	5	7	8	10	11	13	14	16	17	19	20	22	23	25
7	0.01	0	1	2	4	5	7	8	10	12	13	15	17	18	20	22	24	25	27	29
	0.025	0	2	4	6	7	9	11	13	15	17	19	21	23	25	27	29	31	33	35
	0.05	1	3	5	7	9	12	14	16	18	20	22	25	27	29	31	34	36	38	40
	0.10	2	5	7	9	12	14	17	19	22	24	27	29	32	34	37	39	42	44	47
	0.001	0	0	0	1	2	3	5	6	7	9	10	12	13	15	16	18	19	21	22
	0.005	0	0	2	3	5	7	8	10	12	14	16	18	19	21	23	25	27	29	31
8	0.01	0	1	3	5	7	8	10	12	14	16	18	21	23	25	27	29	31	33	35
	0.025	1	3	5	7	9	11	14	16	18	20	23	25	27	30	32	35	37	39	42
	0.05	2	4	6	9	11	14	16	19	21	24	27	29	32	34	37	40	42	45	48
	0.10	3	6	8	11	14	17	20	23	25	28	31	34	37	40	43	46	49	52	55
	0.001	0	0	0	2	3	4	6	8	9	11	13	15	16	18	20	22	24	26	27
	0.005	0	1	2	4	6	8	10	12	14	17	19	21	23	25	28	30	32	34	37
9	0.01	0	2	4	6	8	10	12	15	17	19	22	24	27	29	32	34	37	39	41
	0.025	1	3	5	8	11	13	16	18	21	24	27	29	32	35	38	40	43	46	49
	0.05	2	5	7	10	13	16	19	22	25	28	31	34	37	40	43	46	49	52	55
	0.10	3	6	10	13	16	19	23	26	29	32	36	39	42	46	49	53	56	59	63

Table XII *(continued)*

Critical Values of the Mann–Whitney Test Statistic

n_1	α	n_2 = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
10	0.001	0	0	1	2	4	6	7	9	11	13	15	18	20	22	24	26	28	30	33
	0.005	0	1	3	5	7	10	12	14	17	19	22	25	27	30	32	35	38	40	43
	0.01	0	2	4	7	9	12	14	17	20	23	25	28	31	34	37	39	42	45	48
	0.025	1	4	6	9	12	15	18	21	24	27	30	34	37	40	43	46	49	53	56
	0.05	2	5	8	12	15	18	21	25	28	32	35	38	42	45	49	52	56	59	63
	0.10	4	7	11	14	18	22	25	29	33	37	40	44	48	52	55	59	63	67	71
11	0.001	0	0	1	3	5	7	9	11	13	16	18	21	23	25	28	30	33	35	38
	0.005	0	1	3	6	8	11	14	17	19	22	25	28	31	34	37	40	43	46	49
	0.01	0	2	5	8	10	13	16	19	23	26	29	32	35	38	42	45	48	51	54
	0.025	1	4	7	10	14	17	20	24	27	31	34	38	41	45	48	52	56	59	63
	0.05	2	6	9	13	17	20	24	28	32	35	39	43	47	51	55	58	62	66	70
	0.10	4	8	12	16	20	24	28	32	37	41	45	49	53	58	62	66	70	74	79
12	0.001	0	0	1	3	5	8	10	13	15	18	21	24	26	29	32	35	38	41	43
	0.005	0	2	4	7	10	13	16	19	22	25	28	32	35	38	42	45	48	52	55
	0.01	0	3	6	9	12	15	18	22	25	29	32	36	39	43	47	50	54	57	61
	0.025	2	5	8	12	15	19	23	27	30	34	36	42	46	50	54	58	62	66	70
	0.05	3	6	10	14	18	22	27	31	35	39	43	48	52	56	61	65	69	73	78
	0.10	5	9	13	18	22	27	31	36	40	45	50	54	59	64	68	73	78	82	87
13	0.001	0	0	2	4	6	9	12	15	18	21	24	27	30	33	36	39	43	46	49
	0.005	0	2	4	8	11	14	18	21	25	28	32	35	39	43	46	50	54	58	61
	0.01	1	3	6	10	13	17	21	24	28	32	36	40	44	48	52	56	60	64	68
	0.025	2	5	9	13	17	21	25	29	34	38	42	46	51	55	60	64	68	73	77
	0.05	3	7	11	16	20	25	29	34	38	43	48	52	57	62	66	71	76	81	85
	0.10	5	10	14	19	24	29	34	39	44	49	54	59	64	69	75	80	85	90	95
14	0.001	0	0	2	4	7	10	13	16	20	23	26	30	33	37	40	44	47	51	55
	0.005	0	2	5	8	12	16	19	23	27	31	35	39	43	47	51	55	59	64	68
	0.01	1	3	7	11	14	18	23	27	31	35	39	44	48	52	57	61	66	70	74
	0.025	2	6	10	14	18	23	27	32	37	41	46	51	56	60	65	70	75	79	84
	0.05	4	8	12	17	22	27	32	37	42	47	52	57	62	67	72	78	83	88	93
	0.10	5	11	16	21	26	32	37	42	48	53	59	64	70	75	81	86	92	98	103
15	0.001	0	0	2	5	8	11	15	18	22	25	29	33	37	41	44	48	52	56	60
	0.005	0	3	6	9	13	17	21	25	30	34	38	43	47	52	56	61	65	70	74
	0.01	1	4	8	12	16	20	25	29	34	38	43	48	52	57	62	67	71	76	81
	0.025	2	6	11	15	20	25	30	35	40	45	50	55	60	65	71	76	81	86	91
	0.05	4	8	13	19	24	29	34	40	45	51	56	62	67	73	78	84	89	95	101
	0.10	6	11	17	23	28	34	40	46	52	58	64	69	75	81	87	93	99	105	111
16	0.001	0	0	3	6	9	12	16	20	24	28	32	36	40	44	49	53	57	61	66
	0.005	0	3	6	10	14	19	23	28	32	37	42	46	51	56	61	66	71	75	80
	0.01	1	4	8	13	17	22	27	32	37	42	47	52	57	62	67	72	77	83	88
	0.025	2	7	12	16	22	27	32	38	43	48	54	60	65	71	76	82	87	93	99
	0.05	4	9	15	20	26	31	37	43	49	55	61	66	72	78	84	90	96	102	108
	0.10	6	12	18	24	30	37	43	49	55	62	68	75	81	87	94	100	107	113	120
17	0.001	0	1	3	6	10	14	18	22	26	30	35	39	44	48	53	58	62	67	71
	0.005	0	3	7	11	16	20	25	30	35	40	45	50	55	61	66	71	76	82	87
	0.01	1	5	9	14	19	24	29	34	39	45	50	56	61	67	72	78	83	89	94
	0.025	3	7	12	18	23	29	35	40	46	52	58	64	70	76	82	88	94	100	106
	0.05	4	10	16	21	27	34	40	46	52	58	65	71	78	84	90	97	103	110	118
	0.10	7	13	19	26	32	39	46	53	59	66	73	80	86	93	100	107	114	121	128
18	0.001	0	1	4	7	11	15	19	24	28	33	38	43	47	52	57	62	67	72	77
	0.005	0	3	7	12	17	22	27	32	38	43	48	54	59	65	71	76	82	88	93
	0.01	1	5	10	15	20	25	31	37	42	48	54	60	66	71	77	83	89	95	101
	0.025	3	8	13	19	25	31	37	43	49	56	62	68	75	81	87	94	100	107	113
	0.05	5	10	17	23	29	36	42	49	56	62	69	76	83	89	96	103	110	117	124
	0.10	7	14	21	28	35	42	49	56	63	70	78	85	92	99	107	114	121	129	136

Table XII (continued)

Critical Values of the Mann–Whitney Test Statistic

n_1	α	n_2 = 2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
19	0.001	0	1	4	8	12	16	21	26	30	35	41	46	51	56	61	67	72	78	83
	0.005	1	4	8	13	18	23	29	34	40	46	52	58	64	70	75	82	88	94	100
	0.01	2	5	10	16	21	27	33	39	45	51	57	64	70	76	83	89	95	102	108
	0.025	3	8	14	20	26	33	39	46	53	59	66	73	79	86	93	100	107	114	120
	0.05	5	11	18	24	31	38	45	52	59	66	73	81	88	95	102	110	117	124	131
	0.10	8	15	22	29	37	44	52	59	67	74	82	90	98	105	113	121	129	136	144
20	0.001	0	1	4	8	13	17	22	27	33	38	43	49	55	60	66	71	77	83	89
	0.005	1	4	9	14	19	25	31	37	43	49	55	61	68	74	80	87	93	100	106
	0.01	2	6	11	17	23	29	35	41	48	54	61	68	74	81	88	94	101	108	115
	0.025	3	9	15	21	28	35	42	49	56	63	70	77	84	91	99	106	113	120	128
	0.05	5	12	19	26	33	40	48	55	63	70	78	85	93	101	108	116	124	131	139
	0.10	8	16	23	31	39	47	55	63	71	79	87	95	103	111	120	128	136	144	152

Source: Adapted from L. R. Verdooren, Extended Tables of Critical Values for Wilcoxon's Test Statistic,' *Biometrika*, 60 (1963), 177–186; used by permission of the Biometrika Trustees. The adaptation is due to W. J. Conover, *Practical Nonparametric Statistics*, New York: Wiley, 1971, 384–388.

Table XIV

Critical Values of the Kruskal–Wallis Test Statistic

Sample Sizes n_1	n_2	n_3	Critical Value	α
2	1	1	2.7000	0.500
2	2	1	3.6000	0.200
2	2	2	4.5714	0.067
			3.7143	0.200
3	1	1	3.2000	0.300
3	2	1	4.2857	0.100
			3.8571	0.133
3	2	2	5.3572	0.029
			4.7143	0.048
			4.5000	0.067
			4.4643	0.105
3	3	1	5.1429	0.043
			4.5714	0.100
			4.0000	0.129
3	3	2	6.2500	0.011
			5.3611	0.032
			5.1389	0.061
			4.5556	0.100
			4.2500	0.121
3	3	3	7.2000	0.004
			6.4889	0.011
			5.6889	0.029
			5.6000	0.050
			5.0667	0.086
			4.6222	0.100
4	1	1	3.5714	0.200
4	2	1	4.8214	0.057
			4.5000	0.076
			4.0179	0.114
4	2	2	6.0000	0.014
			5.3333	0.033
			5.1250	0.052
			4.4583	0.100
			4.1667	0.105
4	3	1	5.8333	0.021
			5.2083	0.050
			5.0000	0.057
			4.0556	0.093
			3.8889	0.129
4	3	2	6.4444	0.008
			6.3000	0.011
			5.4444	0.046
			5.4000	0.051
			4.5111	0.098
			4.4444	0.102
4	3	3	6.7455	0.010
			6.7091	0.013
			5.7909	0.046
			5.7273	0.050
			4.7091	0.092

Sample Sizes n_1	n_2	n_3	Critical Value	α
			4.7000	0.101
4	4	1	6.6667	0.010
			6.1667	0.022
			4.9667	0.048
			4.8667	0.054
			4.1667	0.082
			4.0667	0.102
4	4	2	7.0364	0.006
			6.8727	0.011
			5.4545	0.046
			5.2364	0.052
			4.5545	0.098
			4.4455	0.103
4	4	3	7.1439	0.010
			7.1364	0.011
			5.5985	0.049
			5.5758	0.051
			4.5455	0.099
			4.4773	0.102
4	4	4	7.6538	0.008
			7.5385	0.011
			5.6923	0.049
			5.6538	0.054
			4.6539	0.097
			4.5001	0.104
5	1	1	3.8571	0.143
5	2	1	5.2500	0.036
			5.0000	0.048
			4.4500	0.071
			4.2000	0.095
			4.0500	0.119
5	2	2	6.5333	0.008
			6.1333	0.013
			5.1600	0.034
			5.0400	0.056
			4.3733	0.090
			4.2933	0.122
5	3	1	6.4000	0.012
			4.9600	0.048
			4.8711	0.052
			4.0178	0.095
			3.8400	0.123
5	3	2	6.9091	0.009
			6.8218	0.010
			5.2509	0.049
			5.1055	0.052
			4.6509	0.091
			4.4945	0.101
5	3	3	7.0788	0.009
			6.9818	0.011

Table XIV *(continued)*

Critical Values of the Kruskal–Wallis Test Statistic

Sample Sizes					Sample Sizes				
n_1	n_2	n_3	Critical Value	α	n_1	n_2	n_3	Critical Value	α
5	3	3	5.6485	0.049	5	5	1	6.8364	0.011
			5.5152	0.051				5.1273	0.046
			4.5333	0.097				4.9091	0.053
			4.4121	0.109				4.1091	0.086
5	4	1	6.9545	0.008				4.0364	0.105
			6.8400	0.011	5	5	2	7.3385	0.010
			4.9855	0.044				7.2692	0.010
			4.8600	0.056				5.3385	0.047
			3.9873	0.098				5.2462	0.051
			3.9600	0.102				4.6231	0.097
5	4	2	7.2045	0.009				4.5077	0.100
			7.1182	0.010	5	5	3	7.5780	0.010
			5.2727	0.049				7.5429	0.010
			5.2682	0.050				5.7055	0.046
			4.5409	0.098				5.6264	0.051
			4.5182	0.101				4.5451	0.100
5	4	3	7.4449	0.010				4.5363	0.102
			7.3949	0.011	5	5	4	7.8229	0.010
			5.6564	0.049				7.7914	0.010
			5.6308	0.050				5.6657	0.049
			4.5487	0.099				5.6429	0.050
			4.5231	0.103				4.5229	0.099
5	4	4	7.7604	0.009				4.5200	0.101
			7.7440	0.011	5	5	5	8.0000	0.009
			5.6571	0.049				7.9600	0.010
			5.6176	0.050				5.7800	0.049
			4.6187	0.100				5.6600	0.051
			4.5527	0.102				4.5600	0.100
5	5	1	7.3091	0.009				4.5000	0.102

Source: W. H. Kruskal and W. A. Wallis, Use of Ranks in One-Criterion Analysis of Variance, *J. Amer. Statist. Assoc.*, **47** (1952), 583–621, Addendum, *Ibid.*, **48** (1953), 907–911.