①

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5-25

More nonparametric tests

Rank sum test

(Wilcoxon, Mann, Whitney)

Alternative to the 2-sample t-test

$H_0: \tilde{\mu}_1 = \tilde{\mu}_2$	$H_0: \tilde{\mu}_1 \le \tilde{\mu}_2$	$H_0: \tilde{\mu}_1 \ge \tilde{\mu}_2$
$H_1: \tilde{\mu}_1 \ne \tilde{\mu}_2$	$H_1: \tilde{\mu}_1 > \tilde{\mu}_2$	$H_1: \tilde{\mu}_1 < \tilde{\mu}_2$

① Group 1 must be the sample with the fewest observations

② Rank all observations together, handling ties as before

③ W_1 = sum of ranks from group 1,

②

W_2 = sum of ranks from group 2

④ $U_1 = W_1 - \frac{n_1(n_1+1)}{2}$,

$U_2 = W_2 - \frac{n_2(n_2+1)}{2}$,

$U = \min(U_1, U_2)$

$$\text{Test stat} = \begin{cases} U_1 & \text{lower 1-sided} \\ U_2 & \text{upper 1-sided} \\ U & \text{2-sided} \end{cases}$$

Use table A.18

Reject H_0 if test stat $\le$ crit val.

Group 1		Group 2	
14.2	(1)	21.3	(11)
18.3	(5)	18.7	(7)
		23.0	(12)
17.2	(4)	17.1	(3)
18.4	(6)	16.8	(2)
20.0	(9)	20.9	(10)
		19.7	(8)

$H_0: \tilde{\mu}_1 \geq \tilde{\mu}_2$

$H_1: \tilde{\mu}_1 < \tilde{\mu}_2$ $\qquad \alpha = .05$

$W_1 = 25 \qquad W_2 = 53$

$U_1 = 25 - \frac{5(6)}{2} = 10$ = Test stat

Table A.18 Critical value = 6

Accept H_0. We failed to show that population 1 has a smaller median.

Kruskal-Wallis Test

Alternative to 1-way ANOVA

$H_0: \tilde{\mu}_1 = \tilde{\mu}_2 = \dots = \tilde{\mu}_k$

H_1: Not all of the population medians are equal.

1. Rank all observations, handling ties as before

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② n_i = # items in group i

R_i = sum of ranks in group i

③ Compute

$$H = \frac{12}{n(n+1)} \sum_{i=1}^{K} \frac{R_i^2}{n_i} - 3(n+1)$$

④ Compare to a critical value from χ^2 table with $K-1$ df

Reject H_0 if $H >$ crit. val.

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p.533 #13.12

Group1 29g	Group2 54g	Group3 84g
190 (1)	271 (4)	329 (7)
266 (2)	295 (5)	390 (8)
270 (3)	318 (6)	396 (9)
	402 (11)	399 (10)
	438 (12)	

$H_0: \tilde{\mu}_1 = \tilde{\mu}_2 = \tilde{\mu}_3$

H_1: Not all medians are equal

$n_1 = 3$ $\quad n_2 = 5$ $\quad n_3 = 4$

$R_1 = 6$ $\quad R_2 = 38$ $\quad R_3 = 34$

$$H = \frac{12}{12(13)}\left[\frac{6^2}{3} + \frac{38^2}{5} + \frac{34^2}{4}\right] - 3(13)$$

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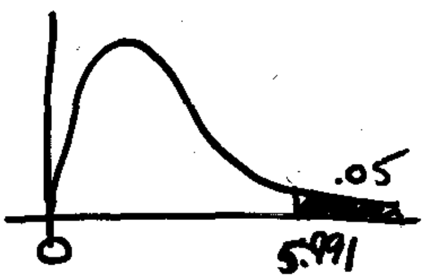
$H = 6.37$

From χ^2 table, .05 upper tail, 2df

5.991 is the crit. value



Reject Ho.

A post hoc analysis is required, since Ho was rejected.

You could perform rank-sum tests for every pair.

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Compare groups 2 + 3

271 (1)	329 (4)
295 (2)	380 (5)
318 (3)	396 (6)
402 (8)	399 (7)
438 (9)	

$w_2 = 23$

$U_2 = 23 - \frac{5(6)}{2}$
$= 8$

$w_1 = 22$

$U_1 = 22 - \frac{4(5)}{2}$
$= 12$

$U = 8$

Crit. value = 1 Acc Ho.

To complete the process, you should also compare grp 1 + 2, grp 1 + 3

Thurs: 5-27 Control charts.

Tues: 6-1 numerical solutions

Thurs: 6-3 2nd midterm, in class