Chapter 13 ANOVA

① 452

5-13

$H_0: \mu_1 = \mu_2 = \ldots = \mu_a$

H_1: Not all means are equal

Notation: Model

$$y_{ij} = \mu + \tau_i + \varepsilon_{ij}$$

group $i = 1, \ldots, a$

item within the group $j = 1, \ldots, n_i$

y_{ij} = j^{th} observation in group i

$$y_{i\cdot} = \sum_{j=1}^{n_i} y_{ij}$$

②

$$\bar{y}_{i\cdot} = \frac{y_{i\cdot}}{n_i}$$

$$y_{\cdot\cdot} = \sum_{i=1}^{a} \sum_{j=1}^{n_i} y_{ij}$$

$$\bar{y}_{\cdot\cdot} = \frac{y_{\cdot\cdot}}{\sum_{i=1}^{a} n_i} = \frac{y_{\cdot\cdot}}{n_\cdot}$$

Special case: all sample sizes equal,

i.e. $n_1 = n_2 = \ldots = n_a = n$

In this case, $n_\cdot = an$

③

$$SST = \sum_{i=1}^{a} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{..})^2$$

↑

Total sum of squares

$$= \sum_{i=1}^{a} \sum_{j=1}^{n_i} \left(\underbrace{y_{ij} - \bar{y}_{i.}} + \underbrace{\bar{y}_{i.} - \bar{y}_{..}} \right)^2$$

$$= \overset{①}{\sum_{i=1}^{a} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{i.})^2} + \overset{②}{\sum_{i=1}^{a} \sum_{j=1}^{n_i} (\bar{y}_{i.} - \bar{y}_{..})^2}$$

$$+ \; 2 \overset{③}{\sum_{i=1}^{a} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{i.})(\bar{y}_{i.} - \bar{y}_{..})}$$

④

③ is 0 :

$$\sum_{i=1}^{a} \left[(\bar{y}_{i.} - \bar{y}_{..}) \underbrace{\sum_{j=1}^{n_i} (y_{ij} - \bar{y}_{i.})}_{y_{i.} - n_i \bar{y}_{i.} = 0} \right]$$

② $$\sum_{i=1}^{a} \sum_{j=1}^{n_i} (\bar{y}_{i.} - \bar{y}_{..})^2$$

$$= \sum_{i=1}^{a} n_i (\bar{y}_{i.} - \bar{y}_{..})^2 = SSA$$

Treatment sum of squares

Between groups " " "

⑤

① $$\sum_{i=1}^{a}\sum_{j=1}^{n_i}(y_{ij}-\bar{y}_{i\cdot})^2 = SSE$$

Error sum of squares

Residual " " "

within groups " " "

So $SST = SSA + SSE$

Source	SS	df	MS	F
TRT	SSA	$a-1$	$SSA/a-1$	$\frac{MSA}{MSE}$
ERR	SSE	$n_\cdot - a$	$SSE/n_\cdot - a$	
TOT	SST	$n_\cdot - 1$		

⑥

Example: $H_0: \mu_1 = \mu_2 = \mu_3$

H_1: Not all means are equal

$n_1 = 5$	$n_2 = 5$	$n_3 = 5$
$\bar{y}_1 = 85$	$\bar{y}_2 = 54$	$\bar{y}_3 = 52$
$s_1^2 = 275$	$s_2^2 = 337.5$	$s_3^2 = 312.5$

Note: $$SSE = \sum_{i=1}^{a}\sum_{j=1}^{n_i}(y_{ij}-\bar{y}_{i\cdot})^2$$

$$= \sum_{i=1}^{a}(n_i - 1)s_i^2$$

$$= 4(275) + 4(337.5) + 4(312.5)$$
$$= 3700$$

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$$SSA = \sum_{i=1}^{a} n_i(\bar{y}_{i\cdot} - \bar{y}_{\cdot\cdot})^2$$

Note: $$\bar{y}_{\cdot\cdot} = \sum_{i=1}^{a}\sum_{j=1}^{n_i} y_{ij} \Big/ n_{\cdot}$$

$$= \frac{\sum_{i=1}^{a} n_i \bar{y}_{i\cdot}}{n_{\cdot}}$$

$$= \frac{5(85) + 5(54) + 5(52)}{15}$$

$$= 63.67$$

$$SSA = 3416.97$$

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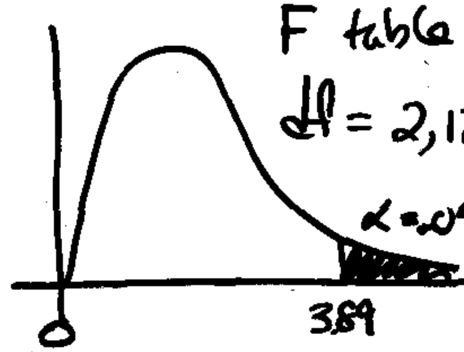
Source	SS	df	MS	F
TRT	3416.97	2	1708.48	5.54
ERR	3700	12	308.33	
TOT	7116.97	14		

Test stat

F table

df = 2,12



Reject H_0. The 3 population means are not all equal.

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Post hoc analysis:

Multiple comparisons

Tukey's Studentized Range

(1) Write the sample means in increasing order.

(3)	(2)	(1)
52	54	85

(2) Compute Tukey's "yardstick"

$$= \underbrace{q(\alpha, a, v)}_{} \cdot \sqrt{\frac{MSE}{n_h}}$$

α: same α as in F test

a: # groups

v: dfE

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n_h is the harmonic mean of the sample sizes

$$n_h = \left[\frac{\frac{1}{n_1} + \frac{1}{n_2} + \cdots + \frac{1}{n_a}}{a}\right]^{-1}$$

q is read from Table A.12

For our example, $\alpha = .05$
$a = 3$
$v = 12$

$q = 3.77$

$$\text{yardstick} = 3.77\sqrt{\frac{308.33}{5}} = 29.6$$

3 2 1 11

52 54 85

You may have overlapping groups:

1 2 3 4

HW #7 due 5/20

p. 522 #13.4
p. 534 #13.14

must use stat software