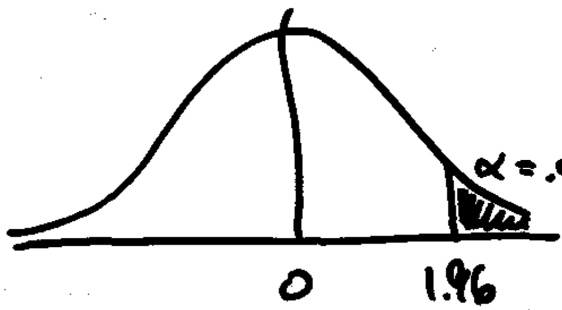
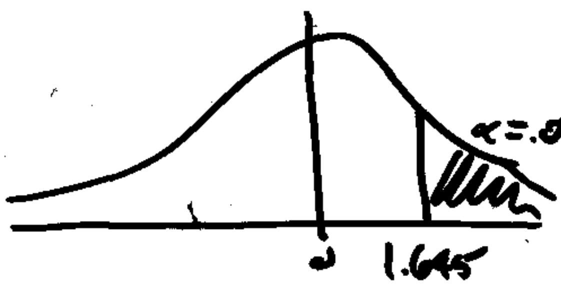
The p-value

Definition: The p-value is the smallest value of α for which you would reject H_0



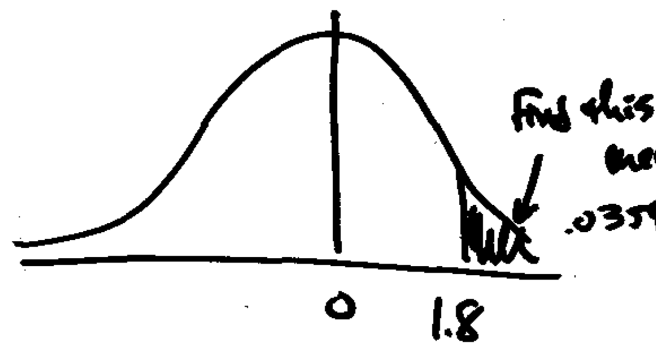


① 452 4-22

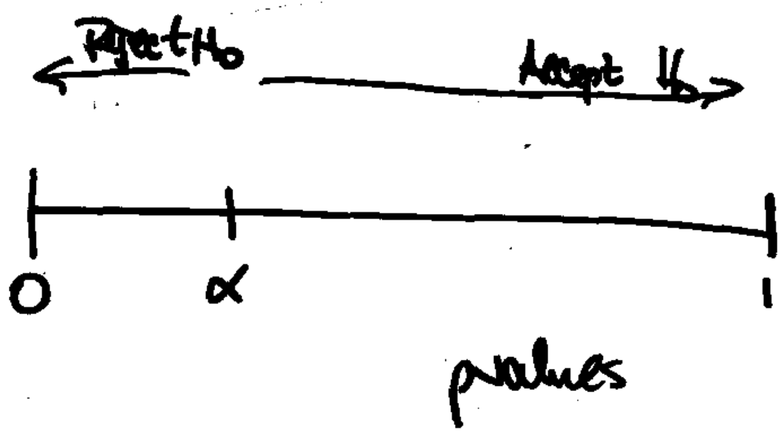
Suppose the test stat is $z = 1.8$. ②

At $\alpha = .05$ you would Reject H_0, but at $\alpha = .025$, you would Accept H_0

so the p-value is somewhere between .025 and .05.



In general, to find the p-value, you find the tail area(s) beyond your test statistic.



Universal rule:

pvalue $< \alpha \iff$ Reject H_0

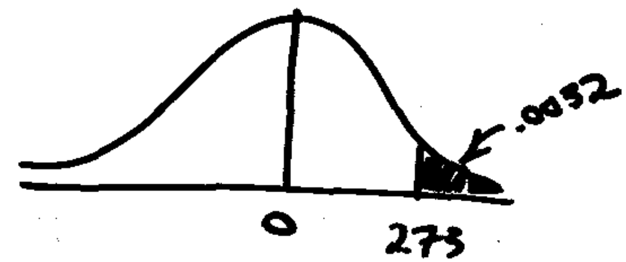
the pvalue is the probability of seeing a test stat as extreme as what you saw, if H_0 is true.

Ex: $H_0: \mu \leq 20$

$H_1: \mu > 20$

Say σ is known & we compute $Z = 2.73$.

Find the p-value.



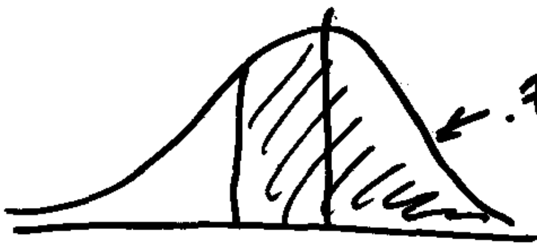
At $\alpha = .05$, Reject H_0

$\alpha = .01$ " "

$\alpha = .001$ Accept H_0

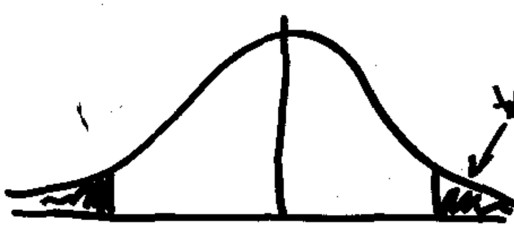
⑤

$H_0: \mu \le 750$ σ is known, +
$H_1: \mu > 750$ we compute $z = -.58$



At $\alpha = .05$, Accept H_0

$H_0: \mu = 12$ σ unknown
$H_1: \mu \ne 12$ $n = 20$, Compute $t = 2.89$



⑥

$.005 < \text{pvalue} < .01$

Type II error: accepting H_0 when H_1 is true.

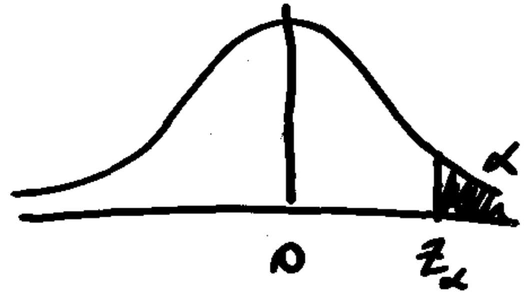
$\beta = \text{Prob}(\text{Type II})$

The <u>power</u> of the test is $1-\beta$.

Suppose we have an upper 1-sided test. Compute the power.

$\beta = P(\text{accept } H_0 \mid H_1 \text{ is true})$

Under H_0:



Accept H_0 if $\quad Z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} < Z_\alpha$,

i.e. if $\quad \bar{x} < \mu_0 + Z_\alpha \dfrac{\sigma}{\sqrt{n}}$

We need

$$P\left(\bar{x} < \mu_0 + Z_\alpha \frac{\sigma}{\sqrt{n}} \;\middle|\; H_1 \text{ is true } (\mu_1) \right)$$

$$P\left(\frac{\bar{x} - \mu_1}{\sigma/\sqrt{n}} < \frac{\mu_0 + Z_\alpha \frac{\sigma}{\sqrt{n}} - \mu_1}{\sigma/\sqrt{n}} \right)$$

$$P\left(Z < Z_\alpha - \frac{(\mu_1 - \mu_0)}{\sigma/\sqrt{n}} \right)$$

let $\delta = \mu_1 - \mu_0$

Then $\quad \beta = P\left(Z < Z_\alpha - \dfrac{\delta}{\sigma/\sqrt{n}} \right)$

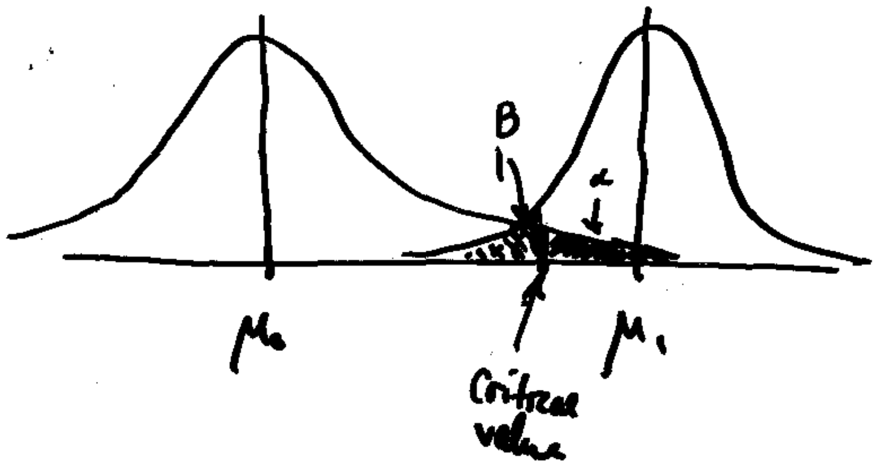
$$= \Phi\left(Z_\alpha - \frac{\delta}{\sigma/\sqrt{n}} \right)$$

↑
cumulative
standard
normal

As $\alpha \uparrow$, $\beta \downarrow$

As $n \uparrow$, $\beta \downarrow$

As $\delta \uparrow$, $\beta \downarrow$



What if β and δ are given?
Find the necessary sample size.

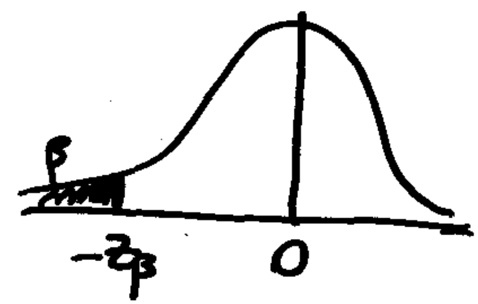
$$\beta = \Phi\left(z_\alpha - \frac{\delta}{\sigma/\sqrt{n}}\right),$$

& solve for n.

$$\Phi^{-1}(\beta) = z_\alpha - \frac{\delta}{\sigma/\sqrt{n}}$$

$$\sqrt{n}\,\frac{\delta}{\sigma} = z_\alpha - \Phi^{-1}(\beta)$$

$$n = \frac{\sigma^2}{\delta^2}\left(z_\alpha - \Phi^{-1}(\beta)\right)^2$$



That is, $\Phi^{-1}(\beta) = -z_\beta$

So $$n = \frac{\sigma^2}{\delta^2}\left(z_\alpha + z_\beta\right)^2$$

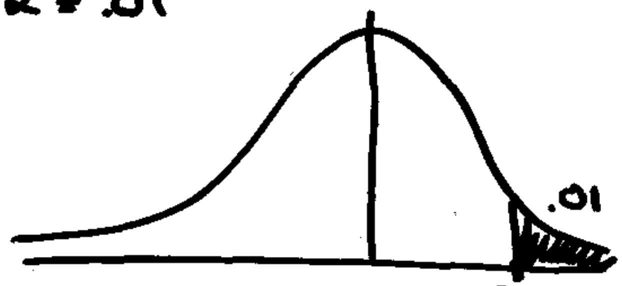
(11)

Test $H_0: \mu \le 3.00$

$H_1: \mu > 3.00$

Say we know $\sigma = 5¢ = .05$

Use $\alpha = .01$



Collect the data.

$n = 25$

$\bar{x} = 3.03$

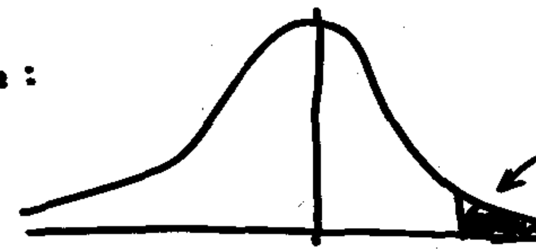
Test stat $= Z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{3.03 - 3.00}{.05/\sqrt{25}}$

(12)

$= 3$

Reject H_0. The true mean is greater than 3.00.

p-value:



pvalue $= .0013$

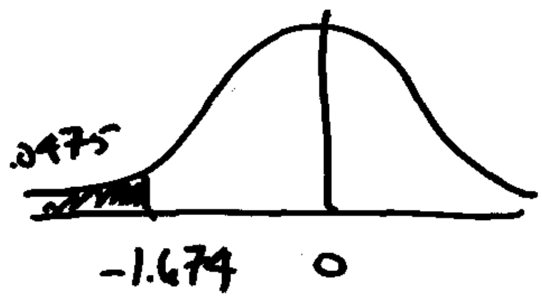
Find the power of the test if the true mean is 3.04

$\beta = \Phi\left(Z_\alpha - \dfrac{\delta}{\sigma/\sqrt{n}}\right)$

(13)

$$= \Phi\left(2.326 - \frac{.04}{.05/\sqrt{25}}\right)$$

$$\beta = \Phi(-1.674)$$



$$\beta = .0475$$

$$\text{power} = 1 - \beta = .9525$$

(14)

To achieve a power of .99 to detect a 4% difference, how large should the sample be?

$$n = \frac{\sigma^2}{\delta^2}(z_\alpha + z_\beta)^2$$

$$= \frac{(.05)^2}{(.04)^2}(2.326 + 2.326)^2$$

$$= 34$$