χ^2 test for independence

(Chi-squared)

① 452 5-20

Observed

Opinion \ Income	Low	Medium	High	
For	182	213	203	598
Against	154	138	110	402
	336	351	313	1000

Contigency table or cross-tabulation

H_0: The distribution of incomes is the same in each row (row homogeneity)

OR

H_0: The distribution of opinions is the same in each column (column homogeneity)

OR ②

H_0: Opinion and Income are independent (not related, not associated)

In each case, H_1 will be the negation of H_0.

Expected

Opinion \ Income	Low	Med	High	
For	200.9	209.9	187.2	598
Against	135.1	141.1	125.8	402
	336	351	313	1000

$\frac{598}{1000} \cdot 336$ $\frac{598}{1000} \cdot 351$

(3)

$df = (\#rows - 1)(\#cols - 1)$

$= (2-1)(3-1) = 2$

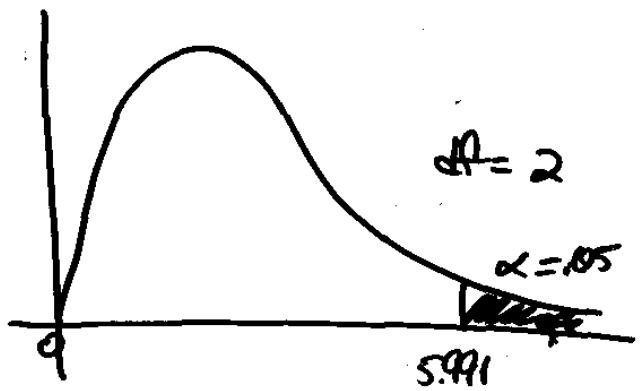
$\text{Cell } \chi^2 = \frac{(O-E)^2}{E}$

	Low	Med	High
For	1.783	.040	1.338
Against	2.652	.068	1.991

$\frac{(182-200.9)^2}{200.9}$

$\text{Test stat} = \chi^2 = \sum \frac{(O-E)^2}{E} = 7.878$

(4)



Reject H_0. The distribution of opinions is not the same in all columns.

(1) Most important cell was lower left

There were more low income people than expected that were against the tax measure.

(2) lower right

There were fewer people than expected who had high incomes and were against the measure.

Nonparametric Tests

Wilcoxon Signed rank test:

Alternative to the 1-sample t-test

$H_0: \tilde{\mu} = \tilde{\mu}_0$	$H_0: \tilde{\mu} \leq \tilde{\mu}_0$	$H_0: \tilde{\mu} \geq \tilde{\mu}_0$
$H_1: \tilde{\mu} \neq \tilde{\mu}_0$	$H_1: \tilde{\mu} > \tilde{\mu}_0$	$H_1: \tilde{\mu} < \tilde{\mu}_0$

$\tilde{\mu}$ represents the population median.

60
62
68
70
70
75
75
75
56
86

$H_0: \tilde{\mu} = 70$
$H_1: \tilde{\mu} \neq 70$

① Subtract $\tilde{\mu}_0$ from each observation

② Delete any zeroes

-10
-8
-2
~~0~~
~~0~~
5
5
5
16
16

③ Rank the items, smallest to largest, in absolute value, average ranks for tied values

	rank	
-2	1	
5	2	3
5	3	
5	4	
-8	5	
-10	6	
16	7	7.5
16	8	

④ Compute W_+ = Sum of ranks for positive values
Compute W_- for neg. values
$W = \min(W_+, W_-)$

$W_+ = 24$
$W_- = 12$
$W = 12$

Test Stat $= \begin{cases} W_+ & \text{lower 1-sided} \\ W_- & \text{upper 1-sided} \\ W & \text{2-sided} \end{cases}$

Reject H_0 if test stat (≤) critical value from A.17

Use $\alpha = .05$, 2-sided, $n = 8$

Critical value from A.17 is 4

Test stat = 12

Accept H_0. We were unable to show that the median differs from 70.

HW #8 p. 455 # 12.13 mult. reg. } Stat
p. 384 # 10.93 χ^2 } plotting

	C1	C2	C3
	Reform	**Income**	**Count**
1	For	Low	182
2	For	Medium	213
3	For	High	203
4	Against	Low	154
5	Against	Medium	138
6	Against	High	110

Tabulated statistics: Reform, Income

Using frequencies in Count

Rows: Reform Columns: Income

	High	Low	Medium	All
Against	110	154	138	402
	125.8	135.1	141.1	402.0
	1.9905	2.6524	0.0682	*
For	203	182	213	598
	187.2	200.9	209.9	598.0
	1.3381	1.7831	0.0458	*
All	313	336	351	1000
	313.0	336.0	351.0	1000.0
	*	*	*	*

Cell Contents: Count
Expected count
Contribution to Chi-square

Pearson Chi-Square = 7.878, DF = 2, P-Value = 0.019
Likelihood Ratio Chi-Square = 7.874, DF = 2, P-Value = 0.020