More confidence intervals

① 452 4-13

Comparing parameters from 2 populations

Confidence interval for $\mu_1 - \mu_2$, based on independent samples from the two populations.

Know that $\bar{X}_1$ and $\bar{X}_2$ will both behave as normal random variables, provided that n_1 and n_2 are sufficiently large.

②

Point estimate of $\mu_1 - \mu_2$ is $\bar{X}_1 - \bar{X}_2$.

$$V(\bar{X}_1 - \bar{X}_2) = V(\bar{X}_1) + V(\bar{X}_2)$$
$$= \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}$$

So $$\frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0,1)$$

$$P\left(-z_{\alpha/2} < \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} < z_{\alpha/2}\right) = 1 - \alpha$$

(3)

$$\bar{x}_1-\bar{x}_2-z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}<\mu_1-\mu_2<\bar{x}_1-\bar{x}_2+z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$$

$\bar{x}_1-\bar{x}_2 \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$ is a

$(1-\alpha)\times 100\%$ C.I for $\mu_1-\mu_2$

What if σ_1^2 and σ_2^2 are unknown?

$$\bar{x}_1-\bar{x}_2 \pm t_{\alpha/2}\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$$

$$df=\frac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\left(\frac{s_1^2}{n_1}\right)^2\frac{1}{n_1-1}+\left(\frac{s_2^2}{n_2}\right)^2\frac{1}{n_2-1}}$$, round down

(4)

Confidence interval for p_1-p_2, the difference between 2 population proportions

Point estimate is $\hat{p}_1-\hat{p}_2$

$$V(\hat{p}_1-\hat{p}_2)=V(\hat{p}_1)+V(\hat{p}_2)$$
$$=\frac{\hat{p}_1\hat{q}_1}{n_1}+\frac{\hat{p}_2\hat{q}_2}{n_2}$$

So $$\frac{\hat{p}_1-\hat{p}_2-(p_1-p_2)}{\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1}+\frac{\hat{p}_2\hat{q}_2}{n_2}}}\sim N(0,1)$$

5

Conf. int. for $p_1 - p_2$:

$$\hat{p}_1 - \hat{p}_2 \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

Group 1	Group 2
Control - no ad	Treatment - ad
$n_1 = 20$	$n_2 = 10$
$x_1 = 8$	$x_2 = 6$
$\hat{p}_1 = \frac{8}{20} = .4$	$\hat{p}_2 = \frac{6}{10} = .6$

C.I. for $p_2 - p_1$: $\hat{p}_2 - \hat{p}_1 \pm z_{\alpha/2}\sqrt{\frac{\hat{p}_2\hat{q}_2}{n_2} + \frac{\hat{p}_1\hat{q}_1}{n_1}}$

95%

6

$$.6 - .4 \pm 1.96\sqrt{\frac{(.6)(.4)}{10} + \frac{(.4)(.6)}{20}}$$

$$.2 \pm .37$$

Paired differences, OR

Matched pairs

Dependent samples

Collect a sample of n matched pairs and make 2 observations on each pair.

	Obs1	Obs2	diffs (Obs1 − Obs2)
1	7	8	−1
2	6	4	2
3	8	9	−1
4	7	7	0
5	4	3	1
6	5	6	−1

Treat this column as a single sample

Confidence interval for μ_d, the population mean difference:

$$\bar{d} \pm t_{\alpha/2} \frac{s_d}{\sqrt{n}} \quad , \quad df = n-1$$

HW#3 due Tuesday 4/20 p.297 9.43, 9.44
p.305 9.58, 9.66