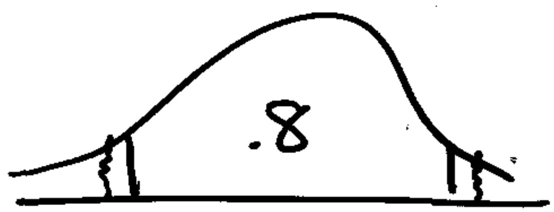
①

452

4-8

Tolerance interval:

an interval that captures a certain percentage $(1-\alpha)\times 100\%$ of the population, with confidence $(1-\gamma)\times 100\%$.



$\bar{x} \pm ks$

k from Table A.7

②

Example: Suppose that a population of project completion times is normally distributed. We observe 100 projects and find $\bar{x} = 5$ days and $s = 1$ day.

Find a tolerance interval that captures 90% of the population with 95% confidence.

$n = 100$ $\quad \alpha = .1$

$\bar{x} = 5$ $\quad \gamma = .05$

$s = 1$ $\quad k = 1.874$ from Table A.7

$\bar{x} \pm ks = 5 \pm 1.874(1)$

$= \boxed{5 \pm 1.874}$

Binomial Distribution

with n known but p unknown.

X = # successes observed in n trials

We know from last week that $\hat{p} = \frac{X}{n}$

Find its properties:

$$E(\hat{p}) = E\left[\frac{X}{n}\right] = \frac{1}{n}E[X]$$
$$= \frac{1}{n}(np) = p$$

So $\hat{p}$ is an unbiased estimator of p

$$V(\hat{p}) = V\left(\frac{X}{n}\right) = \frac{1}{n^2}V(X) = \frac{1}{n^2}(npq)$$
$$= \frac{pq}{n}$$

Think of $\hat{p} = \frac{X}{n}$ as $\frac{\sum_{i=1}^{n} X_i}{n}$,

where $X_i = \begin{cases} 1 & \text{success} \\ 0 & \text{failure} \end{cases}$ on trial i

The Central Limit Theorem then says that $\hat{p}$ will be approximately normal.

$$\frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}} \sim N(0,1)$$

Follow the same steps as for $\bar{x}$ and μ:

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} < p < \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

5

Ex: In 100 trials, we find 70 successes. Estimate p with 95% confidence.

$$\hat{p} \pm z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$.7 \pm 1.96\sqrt{\frac{(.7)(.3)}{100}}$$

$$.7 \pm .089$$

Suppose that we need a margin of error no more than e. How large must n be? Set $z_{\alpha/2}\sqrt{\frac{\hat{p}\hat{q}}{n}} = e$ to get $n = z_{\alpha/2}^2\,\hat{p}\hat{q}/e^2$

6

In our example, find n so that $e = \pm .05$

$$n = \frac{1.96^2(.7)(.3)}{.05^2} = 323$$

---

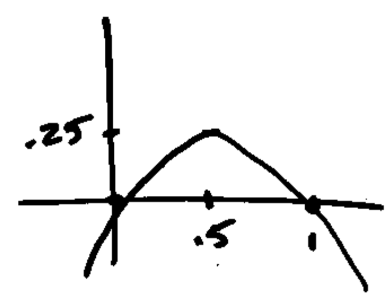
from pilot

$$n = \left(\frac{z_{\alpha/2}\,\sigma}{e}\right)^2 \qquad n = \frac{z_{\alpha/2}^2\,\hat{p}\,(1-\hat{p})}{e^2}$$

from pilot

OR use $\hat{p} = .5$ as a planning value & overestimate n.

$$y = x(1-x) = x - x^2$$

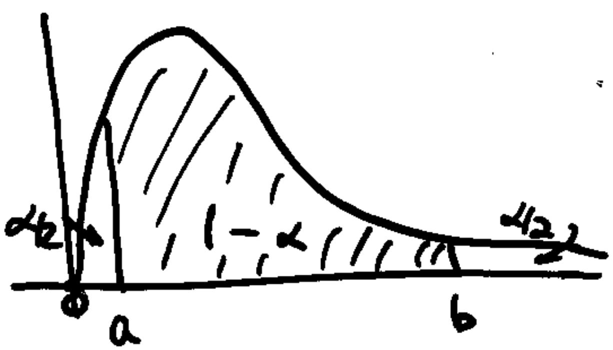


7

New goal: Estimate σ^2 or σ

Know that s^2 is an unbiased estimator of σ^2.

Know that $\frac{(n-1)s^2}{\sigma^2} \sim \chi^2_{n-1}$



$$P\left(a < \frac{(n-1)s^2}{\sigma^2} < b\right) = 1-\alpha$$

8

$$a < \frac{(n-1)s^2}{\sigma^2} < b$$

$$\frac{1}{a} > \frac{\sigma^2}{(n-1)s^2} > \frac{1}{b}$$

$$\frac{(n-1)s^2}{a} > \sigma^2 > \frac{(n-1)s^2}{b}$$

$$\boxed{\frac{(n-1)s^2}{\chi^2_{\alpha/2}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}}$$ $(1-\alpha)100\%$ C.I. for σ^2

$$\boxed{\sqrt{\frac{(n-1)s^2}{\chi^2_{\alpha/2}}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_{1-\alpha/2}}}}$$ $(1-\alpha)100\%$ C.I. for σ