p.199 243

①

52. Bino$(n=20, p=.7)$ [Exact answer $=.2375$] 11–16

$$P(X>15) = P(X \geq 16) \longrightarrow P(X \geq 15.5)$$

$$\mu = np = 20(.7) = 14$$

$$\sigma = \sqrt{np(1-p)} = \sqrt{20(.7)(.3)} = \sqrt{4.2} = 2.049$$

$$P(X \geq 15.5) = P\left(Z \geq \frac{15.5-14}{2.049}\right) = P(Z \geq .73)$$

$$= .5 - .2673$$
$$= .2327$$

②

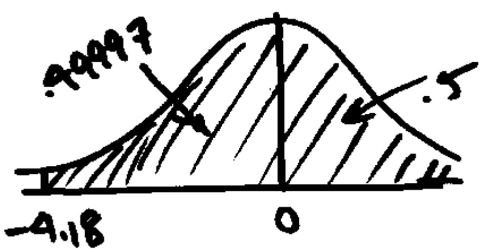
53. Bino$(n=100, p=.4)$

$$P(X \geq 20) \longrightarrow P(X \geq 19.5)$$

$$\mu = 100(.4) = 40$$

$$\sigma = \sqrt{100(.4)(.6)} = \sqrt{24} = 4.899$$

$$P(X \geq 19.5) = P\left(Z \geq \frac{19.5-40}{4.899}\right) = P(Z \geq -4.18)$$



$$= .99997$$

We will use the Central Limit Theorem to compute probabilities concerning $\bar{x}$.

Example: In a certain population, the average age is 22 and the std. dev. is 3.
We take a sample of 40 people.
Find the probability that their average age is greater than 23.

$P(\bar{x} > 23)$. The central limit theorem says that

$\bar{x}$ is approx. normal (since $n \geq 30$)

with $\mu_{\bar{x}} = \mu = 22$ and

$$\sigma^2_{\bar{x}} = \frac{\sigma^2}{n} = \frac{9}{40}$$

so $\sigma_{\bar{x}} = \sqrt{\frac{\sigma^2}{n}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{40}}$

$$P(\bar{x} > 23) = P\left(z > \frac{23-22}{(3/\sqrt{40})}\right)$$

$= P(z > 2.11)$

$= .5 - .4826 = .0174$

5

p226

20. $\mu = 3.4 \quad \sigma = 1.5$

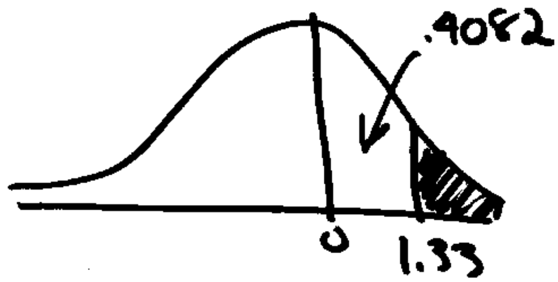
$n = 100$

$$P(\bar{X} \geq 3.6) = P\left(Z \geq \frac{3.6 - 3.4}{\left(1.5/\sqrt{100}\right)}\right)$$

$$= P(Z \geq 1.33)$$

$$= .5 - .4082$$

$$= .0918$$



6

<u>Note</u>: In the previous examples, if we were asked $P(X > 23)$ or $P(X > 3.6)$, the correct answer would be "not enough info", since we don't know the shape of the original distributions.

7

22. $\sigma = 4500$
$n = 225$

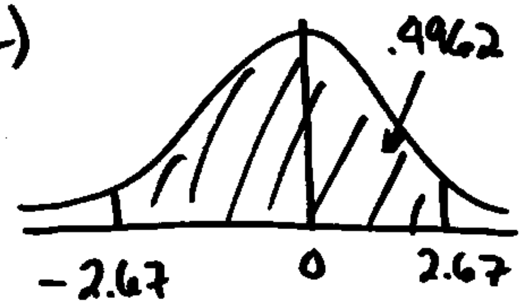
$$P(\mu - 800 \leq \bar{X} \leq \mu + 800)$$

$$= P\left(\frac{\mu - 800 - \mu}{4500/\sqrt{225}} \leq Z \leq \frac{\mu + 800 - \mu}{4500/\sqrt{225}}\right)$$

$$= P(-2.67 \leq Z \leq 2.67)$$

$$= 2(.4962)$$

$$= .9924$$



8

The Central Limit Theorem can also be used to find probabilities concerning a sample proportion.

The sample proportion $\hat{p}$ is approx. normal, with mean p and standard deviation $\sqrt{\frac{p(1-p)}{n}}$, provided that n is sufficiently large.

9

24. $p = .58$

$n = 250$

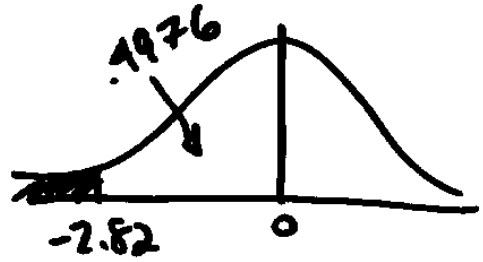
$P\left(\hat{p} \le \frac{123}{250}\right)$

$= P(\hat{p} \le .492) = P\left(Z \le \dfrac{.492 - .58}{\sqrt{\frac{.58(.42)}{250}}}\right)$

$= P(Z \le -2.82)$

$= .5 - .4976$

$= .0024$



10

The normal distr. & its applications.

(1) If the original population is normal, then $P(X > a) = P\left(Z > \frac{a-\mu}{\sigma}\right)$

(2) No matter what the original distribution is,

$P(\overline{X} > a) = P\left(Z > \dfrac{a-\mu}{\sigma/\sqrt{n}}\right)$,

if $n \ge 30$.

3. $P(\hat{p} > a) = P\left(z > \dfrac{a-p}{\sqrt{\frac{p(1-p)}{n}}}\right)$,

provided n is large.

HW p.225 # 21, 23, 25a, 27

Quiz 4 Tuesday Uniform, Normal, Normal approx to Bino

LAB 3 due Tuesday