Normal approximation to the binomial

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11-14

Ex: p199 #37

Bino($n = 45, p = .8$)

$P(X \geq 30)$ . . .

2nd example: Bino($n = 20, p = .6$)

Find $P(10 \leq x \leq 12)$

$= p(10) + p(11) + p(12)$

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$P(10 \leq x \leq 12) \longrightarrow P(9.5 \leq x \leq 12.5)$

(2) $\mu = np = 20(.6) = 12$

$\sigma = \sqrt{np(1-p)} = \sqrt{20(.6)(.4)} = \sqrt{4.8}$

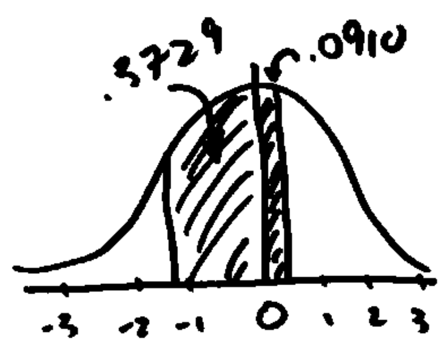
$= 2.191$

(3) $P(9.5 \leq x \leq 12.5)$

$= P\left(\frac{9.5-12}{2.191} \leq Z \leq \frac{12.5-12}{2.191}\right)$

$= P(-1.14 \leq Z \leq .23)$

$= .3729 + .0910 = .4639$ Approx. answer



$$= \binom{20}{10}(.6)^{10}(.4)^{10} + \binom{20}{11}(.6)^{11}(.4)^{9}$$

$$+ \binom{20}{12}(.6)^{12}(.4)^{8}$$

$$= .1171 + .1597 + .1797 = .4565$$

<u>Exact</u> <u>answer</u>

Normal approximation:

① Continuity correction

Decrease the lower bound by .5
Increase the upper " " "

Back to the 1st example:

① $P(X \geq 30) \longrightarrow P(X \geq 29.5)$

② $\mu = np = 45(.8) = 36$

$\sigma = \sqrt{45(.8)(.2)} = \sqrt{7.2} = 2.683$

③ $P(X \geq 29.5) = P\left(Z \geq \frac{29.5 - 36}{2.683}\right)$

$= P(Z \geq -2.42)$

$= .4922 + .5$

$= .9922$

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HW p. 199 #52, 53.

Quiz 4 : Tues 11-21
Uniform, Normal, Normal approx to bino.

Chapter 5 Compare the following 2 experiments:

Experiment #1
Draw 1 slip of paper from a box containing slips numbered 1, 2, 3, 4.

x	$p(x)$
1	.25
2	.25
3	.25
4	.25
	1

$\mu = \sum x p(x)$
$= 2.5$
$\sigma^2 = \sum x^2 p(x) - \mu^2$
$= 1.25$

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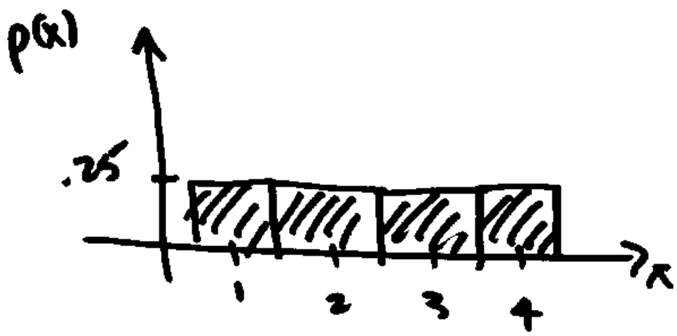

Experiment #2 Draw 2 slips from the same box, with replacement & let $\bar{x}$ be the average of the 2 numbers.

$\bar{x}$	1	2	3	4
1	1	1.5	2	2.5
2	1.5	2	2.5	3
3	2	2.5	3	3.5
4	2.5	3	3.5	4

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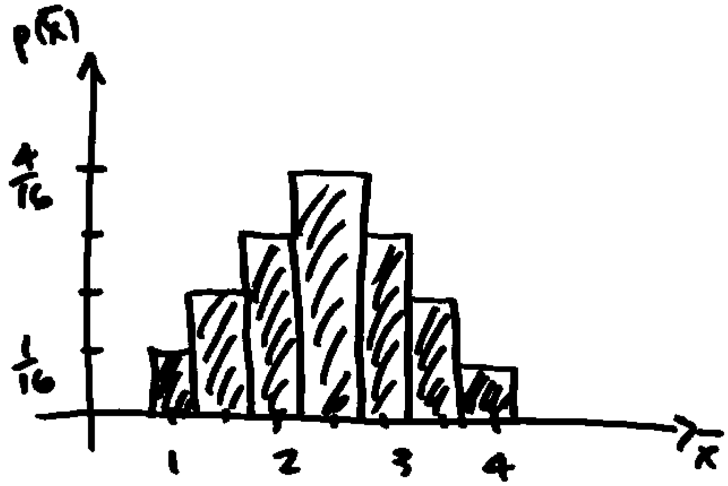
$\bar{x}$	$p(\bar{x})$
1	1/16
1.5	2/16
2	3/16
2.5	4/16
3	3/16
3.5	2/16
4	1/16
	1



$$\mu_{\bar{x}} = \sum \bar{x}\, p(\bar{x}) = 2.5$$

$$\sigma^2_{\bar{x}} = \sum \bar{x}^2 p(\bar{x}) - \mu^2_{\bar{x}} = .625$$

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Observations:

1. The shape of the distribution of $\bar{x}$ starts looking <u>normal</u> as n increases

2. The mean of the distribution of $\bar{x}$ is the same as the mean of the original distribution $\boxed{\mu_{\bar{x}} = \mu}$

3. The variance of the distribution of $\bar{x}$ is equal to the variance of the original distribution, divided by n. $\boxed{\sigma^2_{\bar{x}} = \sigma^2/n}$

These 3 observations form the

<u>Central</u> <u>Limit</u> <u>Theorem</u>