243

1

10-24

p.108

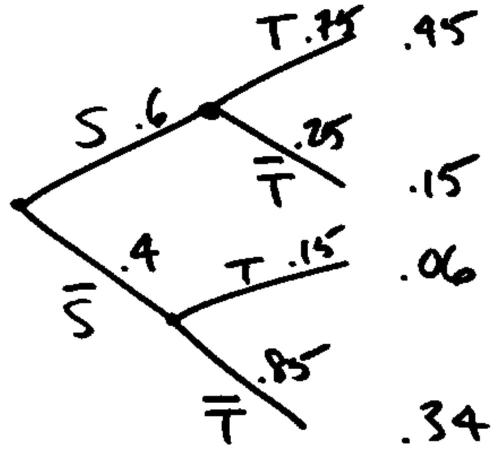
69. S: success

T: positive test

$P(S) = .6$

$P(T \mid S) = .75$

$P(T \mid \bar{S}) = .15$



	T	$\bar{T}$	
S	.45	.15	.6
$\bar{S}$	.06	.34	.4
	.51	.49	1

$$P(S \mid T) = \frac{.45}{.51} = .882$$

2

p.125, p.136 ← cumulative distribution

1.

x	P(x)	F(x)
0	.3	.3
1	.2	.5
2	.2	.7
3	.1	.8
4	.1	.9
5	.1	1
	1	

← $P(X=3) = .1$, $P(X \le 3) = .8$

$$P(X > 2) = 1 - P(X \le 2) = 1 - .7 = .3$$

11. $\mu = E(X) = \sum x p(x)$

$$= 0(.3) + 1(.2) + 2(.2) + 3(.1) + 4(.1) + 5(.1) = 1.8$$

3.

x	$p(x)$	$F(x)$
0	.10	.10
10	.20	.30
20	.35	.65
30	.20	.85
40	.10	.95
50	.05	1
	1	

$\leftarrow P(X=40) = .10,\ P(X \le 40) = .95$

$$P(X > 20) = .35$$

13. $\mu = E(X) = 0(.1) + 10(.2) + 20(.35) + 30(.2) + 40(.1) + 50(.05)$

$= 21.5$

4

From last time, $\sigma^2 = V(X) = \sum (x-\mu)^2 p(x)$

<u>Example</u>:

x	$p(x)$	$x-\mu$	$(x-\mu)^2$	$(x-\mu)^2 p(x)$
0	1/8	-1.5	2.25	.28125
1	3/8	-.5	.25	.09375
2	3/8	.5	.25	.09375
3	1/8	1.5	2.25	.28125
				$\sigma^2 = .75$

We previously found $\mu = 1.5$

The standard deviation $\sigma = \sqrt{\sigma^2} = \sqrt{.75} = .866$

⑤

Shortcut formula for $\sigma^2 = V(x) = E(X^2) - \mu^2$

$$= \sum x^2 p(x) - \mu^2$$

Same example

x	$p(x)$	x^2	$x^2 p(x)$
0	1/8	0	0
1	3/8	1	.375
2	3/8	4	1.5
3	1/8	9	1.125
			$3 = E(X^2)$

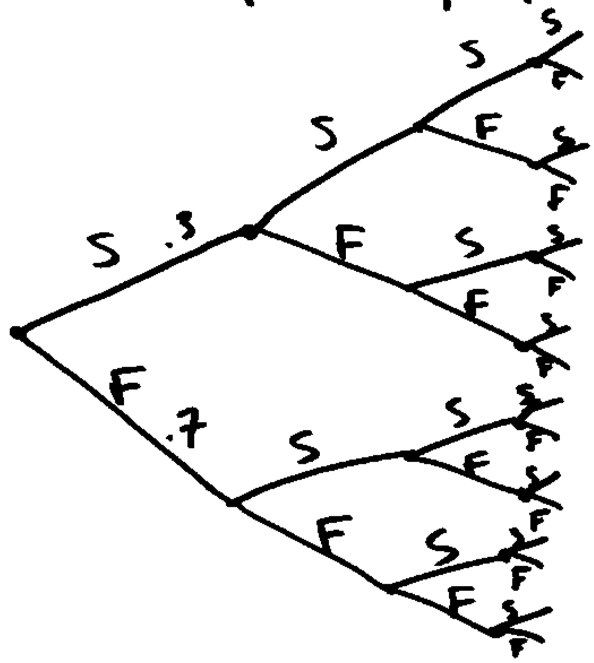
$$\sigma^2 = E(X^2) - \mu^2$$
$$= 3 - (1.5)^2$$
$$= 3 - 2.25$$
$$= .75$$

⑥

<u>Binomial Experiment</u>

- Run a sequence of n trials
- Trials are independent
- Each trial results in 1 of 2 possible outcomes (success / failure)
- The probability of success (p) is the same on each trial
- X counts the number of successes

Consider a Binomial experiment with $n=4$ trials and success probability $p=.3$



x	$p(x)$
0	$(.7)^4$
1	$4(.3)(.7)^3$
2	$6(.3)^2(.7)^2$
3	$4(.3)^3(.7)$
4	$(.3)^4$

Notice $p(x)=\binom{4}{x}(.3)^x(.7)^{4-x}$

In general, for a Binomial Experiment with n trials and success rate p,

$$\boxed{p(x)=\binom{n}{x}p^x(1-p)^{n-x}} \quad x=0,1,2,\dots,n$$

$$\boxed{\begin{aligned}\mu &= E(x) = np \\ \sigma^2 &= V(x) = np(1-p)\end{aligned}}$$

Example: Shoot 20 baskets

X = number of successes

Say $p = .7$

We need to assume that the trials are independent

1. Identify the probability distribution: $\text{Bino}(n=20, p=.7)$
2. Write the formula for the probabilities

$$p(x) = \binom{20}{x} .7^x \, .3^{20-x}$$

3. Find the desired probabilities

Find the prob. that he makes 15 out of the 20.

$$p(15) = \binom{20}{15} .7^{15} \, .3^5$$
$$= .1789$$

Find the prob. that he makes all 20.

$$p(20) = \binom{20}{20} .7^{20} (.3)^0 = .0008$$

How many successes do you expect?

$$\mu = E(X) = np = 20(.7) = 14$$

Find the prob. that he makes at least 18 of his shots.

$$P(X \geq 18) = p(18) + p(19) + p(20)$$

HW p.143 #39a, 40

Midterm is Tues 10/31

- Bring 1 page of notes
- Calculator
- Scan Tron #882
- pencil