p.251

243

1

11-28

5. $\sigma = 5500$ Population is normally distributed

$n = 16$

$\bar{x} = 89{,}673.12$

95% C.I. for μ

$$\bar{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$\alpha = 1 -$ conf. level

$= 1 - .95$

$= .05$

$\alpha/2 = .025$

$$89{,}673.12 \pm (1.96)\frac{5500}{\sqrt{16}}$$

$\boxed{89{,}673.12 \pm 2695}$ OR $(86{,}978.12\ ,\ 92{,}368.12)$

2

9. $\sigma = 1.2$

$n = 60$

$\bar{x} = 9.3$

90% C.I. for μ

$\alpha = 1 - .9$

$= .1$

$\alpha/2 = .05$

$$9.3 \pm (1.645)\frac{1.2}{\sqrt{60}}$$

$\boxed{9.3 \pm .255}$ or $(9.045, 9.555)$

In reality, σ is rarely known.

When σ is unknown, estimate it with s.

3

When this is done, the formula becomes

$$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$$

use the t-table with
$df = n-1$

<u>Example</u>: In a sample of 40 people, we find an average height of 70" and a standard deviation of 2.5". Find a 99% confidence interval for the population mean.

4

$n = 40$

$\bar{x} = 70$

$s = 2.5$

$\alpha = 1 - .99$
$= .01$

$\alpha/2 = .005$

$\bar{x} \pm t_{\alpha/2} \frac{s}{\sqrt{n}}$

$df = 40 - 1 = 39$
(go to the closest value in the table)

$70 \pm (2.704) \frac{2.5}{\sqrt{40}}$

$\underline{70 \pm 1.07}$ OR $\underline{(68.93, 71.07)}$

(5)

<u>Example</u> In a sample of 15 people we find an average income of \$42,000 and a standard deviation of \$10,000. Find a 95% C.I. for μ.

$n = 15$
$\bar{x} = 42000$
$s = 10000$

As stated, the problem can't be solved. If we assume that the population is normally distributed, we may proceed.

$\alpha = 1 - .95 = .05$
$\alpha/2 = .025$

$df = 14$

$42000 \pm (2.145)\frac{10000}{\sqrt{15}}$

$\boxed{42000 \pm 5538.37}$

---

(6)

Suppose that we have a desired margin of error, B.

Set $B = \dfrac{t_{\alpha/2}\, s}{\sqrt{n}}$ and solve for n.

$$B\sqrt{n} = t_{\alpha/2} \cdot s$$

$$\sqrt{n} = \frac{t_{\alpha/2} \cdot s}{B}$$

$$n = \left(\frac{t_{\alpha/2} \cdot s}{B}\right)^2$$

/ instead $n = \left[\dfrac{z_{\alpha/2} \cdot s}{B}\right]^2$

(7)

In the most recent example, suppose that we are told that we need a margin of error no more than $\pm \$1000$.

Find the necessary sample size.

$$n = \left[\frac{Z_{\alpha/2}\, s}{B}\right]^2 = \left[\frac{1.96 \cdot 10{,}000}{1000}\right]^2$$

$$= 384.16 \rightarrow 385$$

(round up if there is <u>any</u> <u>fraction</u>)

---

(8)

<u>Confidence interval for a population proportion p</u>

$$\hat{p} \pm Z_{\alpha/2}\sqrt{\frac{\hat{p}\,\hat{q}}{n}} \quad \text{where } \hat{q} = 1-\hat{p}$$

<u>Example</u>: In a random sample of 500 voters, we find that 255 voted "yes".

Find a 95% C.I. for the proportion of the population favoring the ballot measure.

9

$n = 500$

$\hat{p} = \frac{255}{500} = .51$

$\hat{q} = .49$

$\alpha = 1 - .95$
$= .05$

$\alpha/2 = .025$

$$\hat{p} \pm Z_{\alpha/2} \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$.51 \pm 1.96 \sqrt{\frac{(.51)(.49)}{500}}$$

$$.51 \pm .04 \quad \text{OR} \quad (.47, .55)$$

point estimate

margin of error

10

$$\boxed{n = \frac{Z_{\alpha/2}^2 \hat{p}\hat{q}}{B^2}}$$

Say we need a margin of error of .01 .

$$n = \frac{1.96^2 (.51)(.49)}{.01^2} = 9601$$

In the case of proportions, you can skip the pilot study and use $\hat{p} = \hat{q} = .5$ in the "n" formula.

HW: p.257 #19, 21 p.261 #41, 43

p.268 #62 p.268 #59, 63

$\underbrace{\text{p.257 \#19, 21 \quad p.268 \#62}}_{\mu}$ $\underbrace{\text{p.261 \#41, 43 \quad p.268 \#59, 63}}_{p}$

Thursday Quiz #5 covering Chp 5

Probabilities about $\bar{x}$ and $\hat{p}$

Lab #4 due Thursday
and all WebCTs are due Friday 5pm.