

2^k designs with blocking, Continued

Stat 566
4-8-24

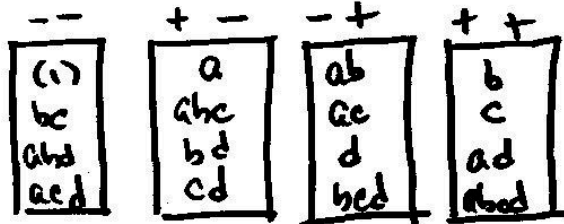
①

Create a 2⁴ design in 4 blocks

	A	B	C	D	ABC	BCD
(1)	-	-	-	-	-	-
a	+	-	-	-	+	-
b	-	+	-	-	+	+
ab	+	+	-	-	-	+
c	-	-	+	-	+	+
ac	+	-	+	-	-	+
bc	-	+	+	-	-	-
abc	+	+	+	-	+	-
d	-	-	-	+	-	+
ad	+	-	-	+	+	+
bd	-	+	-	+	+	-
abd	+	+	-	+	-	-
cd	-	-	+	+	+	-
acd	+	-	+	+	-	+
bcd	-	+	+	+	-	+
abcd	+	+	+	+	+	+

Choose 2 effects to be confounded with blocks, but there will be $4-1=3$ altogether

Choose ABC, BCD



Note AD is also confounded

Source	df
A	1
B	1
C	1
D	1
AB	1
AC	1
BC	1
BD	1
CD	1
ABD	1
ACD	1
ABCD	1
BLK	3
TOT	15

Defining Constraints

②

$$L = \alpha_1 X_1 + \alpha_2 X_2 + \dots + \alpha_k X_k \pmod{2}$$

We select the α_i 's.

The X_i 's are indicators.

$$ABC: L_1 = X_1 + X_2 + X_3$$

$$BCD: L_2 = X_2 + X_3 + X_4$$

	L ₁	L ₂
(1)	0	0
a	1	0
b	1	1
ab	0	1
c	1	1
ac	0	0
bc	0	0
abc	1	0
d	0	1
ad	1	1
bd	1	0
abd	0	0
cd	1	0
acd	0	0
bcd	0	1
abcd	1	1

Note: this method gives the same blocks as the +/- method

(3)

Fractional 2^k designs

Start with a 2^k design

Divide it into 2^p blocks

But, now only run 1 block

This is a 2^{k-p} design.

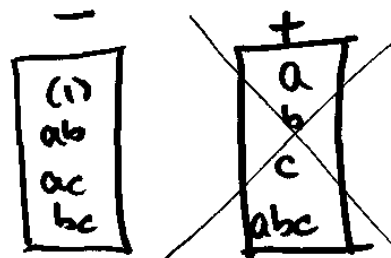
Example: Construct a 2^{3-1} design.

(4)

	A	B	C	ABC
(1)	-	-	-	-
a	+	-	-	+
b	-	+	-	+
ab	+	+	-	-
c	-	-	+	+
ac	+	-	+	-
bc	-	+	+	-
abc	+	+	+	+

(2^3 design in 2^1 blocks)

Choose ABC to be confounded with blocks



↑
just run this block

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	I	A	B	C	AB	AC	BC	ABC
(i)	+	-	-	-	+	+	+	-
ab	+	+	+	-	+	-	-	-
ac	+	+	-	+	-	+	-	-
bc	+	-	+	+	-	-	+	-

$I = -ABC$ ← generator word
 $A = -BC$
 $B = -AC$
 $C = -AB$

} Alias structure

Defn: The resolution of a 2^{k-p} design is the smallest number of letters in any generator word.

In this example, we have a resolution III design (6)

We have a 2^{3-1}_{III} design

Source	df
A	1
B	1
C	1
TOT	3

Example: 2^{6-2}_{III}

(7)

Choose ABC & DEF as generators

Write out the alias structure

$I = ABC = DEF = ABCDEF$
 $A = BC = ADEF = BCDEF$
 $B = AC = BDEF = ACDEF$
 $C = AB = CDEF = ABCDF$
 $D = ABCD = EF = ABCDF$
 $E = ABCE = DF = ABCDF$
 $F = ABCF = DE = ABCDE$

$AD = BCD = AEF = BCEF$

$AE = BCE = ADF = BCD$

$AF = BCF = ADE = BCDE$

$BD = ACD = BEF = ACEF$

$BE = ACE = BDF = ACDF$

$BF = ACF = BDE = ACDE$

$CD = ABD = CEF = ABDF$

$CE = ABE = CDF = ABDF$

$CF = ABF = CDE = ABDE$

What if we chose ABCD & CDEF as generators? ABCE

This would give a res IV design

(8)

Source	df
A	1
B	1
C	1
D	1
E	1
F	1
AB	1
AC	1
AD	1
AE	1
AF	1
BC	1
BD	1
BE	1
BF	1
CD	1
CE	1
CF	1
TOT	15

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Example: Design a 2^{7-4} experiment with the highest possible resolution.

Use Table X on p. 706 (p. 708 for 2^{7-4}_{III})

$I = ABD = ACE = \dots$

Project onto a 2^3 experiment

A	B	C	D=AB	E=AC	F=BC	G=CD	
-	-	-	+	+	+	-	def
+	-	-	-	-	+	+	afg
-	+	-	-	+	-	+	beg
+	+	-	+	-	-	-	abd
-	-	+	+	-	-	+	cdg
+	-	+	-	+	-	-	ace
-	+	+	-	-	+	-	bcf
+	+	+	+	+	+	+	abcdefg

Source	df
A	1
B	1
C	1
D	1
E	1
F	1
G	1
TOT	7

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