2^K designs with blocking, continued

Stat 566
4-8-24

①

Create a 2^4 design in 4 blocks

	A	B	C	D	ABC	BCD
(1)	−	−	−	−	−	−
a	+	−	−	−	+	−
b	−	+	−	−	+	+
ab	+	+	−	−	−	+
c	−	−	+	−	+	+
ac	+	−	+	−	−	+
bc	−	+	+	−	−	−
abc	+	+	+	−	+	−
d	−	−	−	+	−	+
ad	+	−	−	+	+	+
bd	−	+	−	+	+	−
abd	+	+	−	+	−	−
cd	−	−	+	+	+	−
acd	+	−	+	+	−	−
bcd	−	+	+	+	−	+
abcd	+	+	+	+	+	+

2^2 ← choose 2 effects to be confounded with blocks, but there will be $4-1=3$ altogether

Choose ABC, BCD

− −	+ −	− +	+ +
(1)	a	ab	b
bc	abc	ac	c
abd	bd	d	ad
acd	cd	bcd	abcd

Note AD is also confounded

②

Source	df
A	1
B	1
C	1
D	1
AB	1
AC	1
BC	1
BD	1
CD	1
ABD	1
ACD	1
ABCD	1
BLK	3
TOT	15

Defining constraints

$$L = \alpha_1 X_1 + \alpha_2 X_2 + \dots + \alpha_K X_K \mod 2$$

We select the α_i's.

The X_i's are indicators.

ABC: $L_1 = X_1 + X_2 + X_3$

BCD: $L_2 = X_2 + X_3 + X_4$

(3)

	L_1	L_2
(1)	0	0
a	1	0
b	1	1
ab	0	1
c	1	1
ac	0	1
bc	0	0
abc	1	0
d	0	1
ad	1	1
bd	1	0
abd	0	0
cd	1	0
acd	0	0
bcd	0	1
abcd	1	1

Note: this method gives the same blocks as the +/- method

Fractional 2^k designs

Start with a 2^k design

Divide it into 2^p blocks

But, now only run 1 block

This is a 2^{k-p} design.

(4)

Example: Construct a 2^{3-1} design.

	A	B	C	ABC
(1)	−	−	−	−
a	+	−	−	+
b	−	+	−	+
ab	+	+	−	−
c	−	−	+	+
ac	+	−	+	−
bc	−	+	+	−
abc	+	+	+	+

(2^3 design in 2^1 blocks)

Choose ABC to be confounded with blocks

−	+
(1)	~~a~~
ab	~~b~~
ac	~~c~~
bc	~~abc~~

just run this block

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	I	A	B	C	AB	AC	BC	ABC
(1)	+	−	−	−	+	+	+	−
ab	+	+	+	−	+	−	−	−
ac	+	+	−	+	−	+	−	−
bc	+	−	+	+	−	−	+	−

$$I = -ABC \leftarrow \text{generator word}$$

$$\left.\begin{aligned} A &= -BC \\ B &= -AC \\ C &= -AB \end{aligned}\right\} \text{Alias structure}$$

<u>Defn</u>: The <u>resolution</u> of a 2^{k-p} design is the smallest number of letters in any generator word.

In this example, we have a resolution $\underline{III}$ design 6

We have a $2^{3-1}_{\underline{III}}$ design

Source	df
A	1
B	1
C	1
TOT	3

(7)

Example: 2^{6-2}_{III}

Choose ABC & DEF as generators

Write out the alias structure

I = ABC = DEF = ABCDEF
A = BC = ADEF = BCDEF
B = AC = BDEF = ACDEF
C = AB = CDEF = ABDEF
D = ABCD = EF = ABCEF
E = ABCE = DF = ABCDF
F = ABCF = DE = ABCDE

AD = BCD = AEF = BCEF
AE = BCE = ADF = BCDF
AF = BCF = ADE = BCDE
BD = ACD = BEF = ACEF
BE = ACE = BDF = ACDF
BF = ACF = BDE = ACDE
CD = ABD = CEF = ABEF
CE = ABE = CDF = ABDF
CF = ABF = CDE = ABDE

What if we chose ABCD & CDEF as generators? ABEF

This would give a Res IV design

(8)

Source	df
A	1
B	1
C	1
D	1
E	1
F	1
AD	1
AE	1
AF	1
BD	1
BE	1
BF	1
CD	1
CE	1
CF	1
TOT	15

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Example: Design a 2^{7-4} experiment with the highest possible resolution.

Use Table X on p. 706 (p. 708 for 2^{7-4}_{III})

$I = ABD = ACE = \ldots$

Project onto a 2^3 experiment

A	B	C	D=AB	E=AC	F=BC	G=CD	
−	−	−	+	+	+	−	def
+	−	−	−	−	+	+	afg
−	+	−	−	+	−	+	beg
+	+	−	+	−	−	−	abd
−	−	+	+	−	−	+	cdg
+	−	+	−	+	−	−	ace
−	+	+	−	−	+	−	bcf
+	+	+	+	+	+	+	abcdefg

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Source	df
A	1
B	1
C	1
D	1
E	1
F	1
G	1
TOT	7