Rules for finding expected mean squares (restricted model version)

1. Write \( E_{ijkl} = \frac{E_{(ijk)}a}{l} \)

2. If a term has a subscript in parentheses, then there will be no interactions between the factors represented by indices in ( ) and those outside.

   Example: \( \beta_{ij(c)} \) ⇒ no AB interaction

3. Subscripts outside the ( ) are live
   " inside the ( ) are dead
   " not present in the term are absent

4. For a random effect, the variance component is of the form \( \Sigma \)
   For a fixed effect, the variance component is of the form \( \frac{\sigma^2}{a-1} \)
**EMS Table (Example)**

<table>
<thead>
<tr>
<th>Factor</th>
<th>i</th>
<th>j</th>
<th>k</th>
<th>EMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_i )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \beta_j )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \tau_{ij} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \varepsilon_{ijk} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Enter a "1" in the table for each fixed subscript in that term.

6. Enter a "0" for a live subscript in an F column and a "1"... "R".

7. For absent subscripts, enter the number of levels of that factor.

Complete parts 5, 6, 7 for last Thursday's example (A fixed, B random)

<table>
<thead>
<tr>
<th>Factor</th>
<th>F</th>
<th>R</th>
<th>R</th>
<th>EMS</th>
<th>( F )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \gamma_i )</td>
<td>O</td>
<td>b</td>
<td>n</td>
<td>( bn \frac{\Sigma \gamma_i^2}{a^2} + n \sigma_\beta^2 + \sigma^2 )</td>
<td>( MSA/MSAB )</td>
</tr>
<tr>
<td>( \beta_j )</td>
<td>a</td>
<td>1</td>
<td>n</td>
<td>( an \sigma_\beta^2 + \sigma^2 )</td>
<td>( MSB/MS )</td>
</tr>
<tr>
<td>( \tau_{ij} )</td>
<td>O</td>
<td>1</td>
<td>n</td>
<td>( n \sigma_\gamma^2 + \sigma^2 )</td>
<td>( MSAB/MS )</td>
</tr>
<tr>
<td>( \varepsilon_{ijk} )</td>
<td>I</td>
<td>I</td>
<td>I</td>
<td>( \sigma^2 )</td>
<td></td>
</tr>
</tbody>
</table>
(1) In each row, cover all of the columns corresponding to the five subscripts for that term. Then the EMS will contain terms corresponding to each row with at least the same subscripts, and the coefficient of the variance is the product of the visible numbers.

(2) For each H_0, find a term whose EMS matches the EMS of the numerator, under H_0. Use that term as the denominator.

**Example: 3 factors: A fixed, B & C random**

<table>
<thead>
<tr>
<th>Factor</th>
<th>F R R R</th>
<th>EMS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \tau_i )</td>
<td>0 1 3 8</td>
<td>( b \text{en} \frac{\varepsilon^2}{a-1} + c \sigma^2 + b \sigma^2 + n \sigma^2 + \sigma^2 )</td>
</tr>
<tr>
<td>( \tau_j )</td>
<td>0 1 3 8</td>
<td>( a \text{en} \sigma^2 + c \sigma^2 + \sigma^2 )</td>
</tr>
<tr>
<td>( \tau_k )</td>
<td>0 1 3 8</td>
<td>( a \text{en} \sigma^2 + c \sigma^2 + \sigma^2 )</td>
</tr>
<tr>
<td>( \tau_{ij} )</td>
<td>0 1 3 8</td>
<td>( b \text{en} \sigma^2 + n \sigma^2 + \sigma^2 )</td>
</tr>
<tr>
<td>( \tau_{jk} )</td>
<td>0 1 3 8</td>
<td>( b \text{en} \sigma^2 + n \sigma^2 + \sigma^2 )</td>
</tr>
<tr>
<td>( \tau_{ik} )</td>
<td>0 1 3 8</td>
<td>( a \text{en} \sigma^2 + \sigma^2 )</td>
</tr>
<tr>
<td>( \tau_{ijk} )</td>
<td>0 1 3 8</td>
<td>( n \sigma^2 + \sigma^2 )</td>
</tr>
<tr>
<td>( \varepsilon_{ijkl} )</td>
<td>0 1 3 8</td>
<td>( \sigma^2 )</td>
</tr>
</tbody>
</table>
A: no exact F test ← next time, we'll see an approximate F test
B: MSB/MSBC
C: MSce/MSBC
AB: MSAC/MSABC
AC: MSae/MSAeC
BC: MSbc/MSce
ABC: MSac/MSacB

Rule for df:
\[ df = \text{product of } (b\text{-levels} - 1) \]
\[ \text{for each main subscript times} \]
\[ (c\text{-levels}) \text{ for each short subscript} \]
In our last example,
\[ df_{AB} = (a-1)(b-1) \]
\[ df_e = abc(a-1) \]

Rule for find SS terms:
Start by symbolically multiplying the df
\[ AB: df = (a-N)(b-1) = ab - a - b + 1 \]
For each term in the expansion, put a dot
For each missing index, square the term, divide by the # levels of each missing index, t sum over all present indices.
\[ SS_{AB} = \sum \sum \frac{y_{ij}^2}{cn} - \sum \frac{y_i^2}{bcn} - \sum \frac{y_{ij}^2}{acen} + \frac{y_{...}^2}{aben} \]
<table>
<thead>
<tr>
<th>Factor</th>
<th>FF</th>
<th>RR</th>
<th>EMS</th>
<th>Denom</th>
</tr>
</thead>
<tbody>
<tr>
<td>τ</td>
<td></td>
<td></td>
<td>(b_{\text{a}} \tau_{1}^2 + b_{\text{c}} \tau_{2}^2 + \sigma^2)</td>
<td>(\text{AC})</td>
</tr>
<tr>
<td>β</td>
<td></td>
<td></td>
<td>(c_{a} \beta_{1}^2 + c_{b} \beta_{2}^2 + \sigma^2)</td>
<td>(\text{BC})</td>
</tr>
<tr>
<td>γ</td>
<td></td>
<td></td>
<td>(a_{n} \gamma_{x}^2 + \sigma^2)</td>
<td>(\text{ERR})</td>
</tr>
<tr>
<td>(2\beta)</td>
<td></td>
<td></td>
<td>(c_{a} \tau_{1} \beta_{1} + \sigma_{p}^2)</td>
<td>(\text{ABC})</td>
</tr>
<tr>
<td>(2\gamma)</td>
<td></td>
<td></td>
<td>(b_{n} \gamma_{x}^2 + \sigma^2)</td>
<td>(\text{ERR})</td>
</tr>
<tr>
<td>(\text{ijk})</td>
<td></td>
<td></td>
<td>(a_{n} \beta_{x} + \sigma^2)</td>
<td>(\text{ERR})</td>
</tr>
<tr>
<td>(2\text{ijk})</td>
<td></td>
<td></td>
<td>(n \sigma_{p}^2 + \sigma^2)</td>
<td>(\text{ERR})</td>
</tr>
<tr>
<td>(\varepsilon_{\text{ijk}})</td>
<td>1</td>
<td>1</td>
<td>(\sigma^2)</td>
<td>1</td>
</tr>
</tbody>
</table>