The Central Limit Theorem had given this result:

\[ P \left( -1.96 < \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} < 1.96 \right) = 0.95 \]

\[ -1.96 < \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} < 1.96 \quad \text{is an event that occurs with probability 0.95} \]

\[ -1.96 \frac{\sigma}{\sqrt{n}} < \bar{x} - \mu < 1.96 \frac{\sigma}{\sqrt{n}} \]

\[ \bar{x} - 1.96 \frac{\sigma}{\sqrt{n}} < -\mu < \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}} \]

\[ \bar{x} + 1.96 \frac{\sigma}{\sqrt{n}} > \mu > \bar{x} - 1.96 \frac{\sigma}{\sqrt{n}} \]

That is, \( \mu \) will lie between \( \bar{x} \pm 1.96 \frac{\sigma}{\sqrt{n}} \) with 95\% confidence.

What if you want 99\% confidence?

- \( \bar{x} \pm 2.576 \frac{\sigma}{\sqrt{n}} \)
- \( \frac{.995}{.99} \)

\[ -2 \quad 0 \quad 2 = 2.576 \]
But \( \sigma \) is usually unknown.

Use \( s \), the sample standard deviation, as its estimate:

\[
X \pm (t) \frac{s}{\sqrt{n}} \quad \text{df} = n-1
\]

Example: In a sample of 25 people, we find an average weight of 170 lbs and a std. dev. of 20 lbs. \( \bar{x} \).
Find a 95\% C.I. for the population mean.

\[
\text{n = 25} \Rightarrow \text{df} = 24
\]

95\% C.I. \( \Rightarrow 97.5\% \) column

\[
\Rightarrow t = 2.064
\]

\[
170 \pm (2.064) \frac{20}{\sqrt{25}}
\]

\[
(170 \pm 8.256) \text{ or } (161.744, 178.256)
\]

We are 95\% confident that our procedure has created an interval that captures the true value of \( \mu \).
How large should the sample be, to have a margin of error of $E$?

Set $t \cdot \frac{s}{\sqrt{n}} = E$ \hspace{1cm} \text{solve for } n

$$n = \left(\frac{ts}{E}\right)^2$$

But this requires that you already know $n$!!

So use $$n = \left(\frac{ts}{E}\right)^2$$

In our example, suppose we need a margin of error of $\pm 2$ lbs. Find $n$.

$$n = \left(\frac{ts}{E}\right)^2 = \left[\frac{1.96(2)}{2}\right]^2 = 384.16$$

$\rightarrow 385$

Note: For sample size determinations, always round up to a whole number.
### List of C.I.s of the form $\text{estim} \pm \text{margin of err}.$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>C. I</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>$\bar{x} \pm t \frac{s}{\sqrt{n}}$</td>
</tr>
<tr>
<td>$\mu_1 - \mu_2$</td>
<td>$\bar{x}_1 - \bar{x}_2 \pm t \sqrt{\frac{s^2_1}{n_1} + \frac{s^2_2}{n_2}}$</td>
</tr>
<tr>
<td></td>
<td>$df$ has complicated formula</td>
</tr>
<tr>
<td></td>
<td>Use &quot;non-pooled&quot; version</td>
</tr>
<tr>
<td>Population proportion $\hat{p}$</td>
<td>$\hat{p} \pm z \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$</td>
</tr>
<tr>
<td>Sample proportion $\hat{p}_1 - \hat{p}_2$</td>
<td>$\hat{p}_1 - \hat{p}_2 \pm z \sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1} + \frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}$</td>
</tr>
</tbody>
</table>

### Example:
In a sample of 200 customers, 108 say that they will purchase a new version of the product. Find a 95% C.I. for the true population proportion.

\[
\hat{p} = \frac{108}{200} = 0.54
\]

\[
\text{or } 0.54 \pm 0.069 \quad \text{or } (0.471, 0.609)
\]

\[n = 200 \quad z = 1.96\]