

Midterm Exam

Stat 4/564

Fall 2014

November 4

1. Assume the model $y_i = -2 + \beta x_i + \varepsilon_i$, $i = 1, \dots, n$.
 - a) Find the least-squares estimator, $\hat{\beta}$, of β .
 - b) Find the expected value of $\hat{\beta}$.
 - c) Find the variance of $\hat{\beta}$.
 - d) Find the standard error of $\hat{\beta}$.
2. Consult the attached SAS output, and answer the following questions.
 - a) If the least significant predictor were to be removed, which one should it be?
 - b) Which predictor shows the highest degree of linear dependence on the others?
 - c) Which pair of predictors shows the most significant correlation?
 - d) Which predictor has the most significant correlation with y ?
 - e) Estimate the standard deviation of the residuals.
 - f) Estimate the variance of $\hat{\beta}_1$.
3. Suppose that in the model in problem 2, you wanted to test the following set of restrictions:

$$\begin{aligned}\beta_1 &= \beta_3 \\ \beta_2 &= 2\beta_3 \\ \beta_4 &= 0\end{aligned}$$
 - a) Write the matrix A that would allow these restrictions to be written in the form $A\beta = \vec{0}$.
 - b) What is the rank of A ?
 - c) What would the numerator and denominator degrees of freedom be for the corresponding F test?
4. Suppose that the breaking strength of a plastic depends on the temperature at which the experiment is conducted. We have two different types of plastic, which are coded as “0” and “1”. In the following 12 observations, you are given the type of plastic, then the temperature, and then the breaking strength.

(0,70,48), (0,68,50), (0,72,56), (0,70,52), (0,72,55), (0,72,62)
 (1,71,40), (1,72,40), (1,73,41), (1,71,42), (1,71,40), (1,73,45)

Write out the usual regression model in matrix form, showing all of the contents of Y , X , β , and ε .

The SAS System
The CORR Procedure

5 Variables: x1 x2 x3 x4 y

Simple Statistics						
Variable	N	Mean	Std Dev	Sum	Minimum	Maximum
x1	10	4.70000	1.88856	47.00000	2.00000	8.00000
x2	10	2.79800	0.73720	27.98000	1.66000	3.84000
x3	10	25.30000	19.24145	253.00000	4.00000	64.00000
x4	10	2.30000	3.94546	23.00000	-5.00000	8.00000
y	9	25.07000	9.62573	225.63000	13.75000	41.67000

Pearson Correlation Coefficients Prob > r under H0: Rho=0 Number of Observations					
	x1	x2	x3	x4	y
x1	1.00000 0.0042 10	0.81355 0.0042 10	0.98426 <.0001 10	-0.26990 0.4508 10	0.85580 0.0032 9
x2	0.81355 0.0042 10	1.00000 0.0042 10	0.71725 0.0195 10	-0.37643 0.2837 10	0.73535 0.0240 9
x3	0.98426 <.0001 10	0.71725 0.0195 10	1.00000 0.0195 10	-0.20476 0.5704 10	0.89448 0.0011 9
x4	-0.26990 0.4508 10	-0.37643 0.2837 10	-0.20476 0.5704 10	1.00000 0.4794 10	0.27171 0.4794 9
y	0.85580 0.0032 9	0.73535 0.0240 9	0.89448 0.0011 9	0.27171 0.4794 9	1.00000 0.4794 9

The SAS System
The REG Procedure
Model: MODEL1

Model Crossproducts X'X X'Y Y'Y						
Variable	Intercept	x1	x2	x3	x4	y
Intercept	9	39	24.98	189	22	225.63
x1	39	189	117.7	999	82	1081.93
x2	24.98	117.7	74.1792	607.46	51.5	670.3201
x3	189	999	607.46	5637	378	5732.83
x4	22	82	51.5	378	192	638.51
y	225.63	1081.93	670.3201	5732.83	638.51	6397.7811

The SAS System
The REG Procedure
Model: MODEL1
Dependent Variable: y

X'X Inverse, Parameter Estimates, and SSE						
Variable	Intercept	x1	x2	x3	x4	y
Intercept	11.423026437	-4.349390554	-0.790172668	0.4861368488	-0.196470795	3.4640542668
x1	-4.349390554	4.6242798229	-3.057008152	-0.345897851	0.0243799695	5.2133463655
x2	-0.790172668	-3.057008152	3.9779879484	0.1353854856	0.0625852827	-3.126907577
x3	0.4861368488	-0.345897851	0.1353854856	0.0309377395	-0.005198984	0.2426054093
x4	-0.196470795	0.0243799695	0.0625852827	-0.005198984	0.0107566349	1.0632151023
y	3.4640542668	5.2133463655	-3.126907577	0.2426054093	1.0632151023	2.0506584161

Analysis of Variance					
Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	4	739.18634	184.79659	360.46	<.0001
Error	4	2.05066	0.51266		
Corrected Total	8	741.23700			

Root MSE	0.71601	R-Square	0.9972
Dependent Mean	25.07000	Adj R-Sq	0.9945
Coeff Var	2.85603		

Parameter Estimates						
Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > t	Variance Inflation
Intercept	1	3.46405	2.41995	1.43	0.2256	0
x1	1	5.21335	1.53971	3.39	0.0276	92.48560
x2	1	-3.12691	1.42807	-2.19	0.0937	19.27662
x3	1	0.24261	0.12594	1.93	0.1263	51.60415
x4	1	1.06322	0.07426	14.32	0.0001	1.48681
