Midterm Take-Home

Stat 4/564   **Fall 2014**   **due November 6**

Suppose that the breaking strength of a plastic depends on the amount of a certain additive and on the temperature at which the experiment is conducted. We have two different types of plastic and we use three different amounts of the additive. We cannot control the temperature, so we observe it and record it for use as a predictor. We run two trials within each of the 6 type/additive combinations. The data are given below, where the ordered pairs represent (temperature, breaking strength).

	Level 1	**Level 2**	**Level 3**
Type A	(70,48)	(68,50)	(72,56)
	(70,52)	(72,55)	(72,62)
Type B	(71,40)	(72,40)	(73,41)
	(71,42)	(71,40)	(73,45)

In matrix form, the model can be written:

$$
\begin{pmatrix} 48 \\ 52 \\ 50 \\ 55 \\ 56 \\ 62 \\ 40 \\ 42 \\ 40 \\ 40 \\ 41 \\ 45 \end{pmatrix} =
\begin{pmatrix}
1 & 0 & 1 & 0 & 0 & 70 \\
1 & 0 & 1 & 0 & 0 & 70 \\
1 & 0 & 0 & 1 & 0 & 68 \\
1 & 0 & 0 & 1 & 0 & 72 \\
1 & 0 & 0 & 0 & 1 & 72 \\
1 & 0 & 0 & 0 & 1 & 72 \\
0 & 1 & 1 & 0 & 0 & 71 \\
0 & 1 & 1 & 0 & 0 & 71 \\
0 & 1 & 0 & 1 & 0 & 72 \\
0 & 1 & 0 & 1 & 0 & 71 \\
0 & 1 & 0 & 0 & 1 & 73 \\
0 & 1 & 0 & 0 & 1 & 73
\end{pmatrix}
\begin{pmatrix} \tau_1 \\ \tau_2 \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \beta \end{pmatrix} + \vec{\varepsilon} .
$$

But, notice that the first two columns sum to $\vec{1}$, as do columns 3, 4, and 5. In order for the design matrix to be full rank, it is better to rewrite the model as:

$$
\begin{pmatrix} 48 \\ 52 \\ 50 \\ 55 \\ 56 \\ 62 \\ 40 \\ 42 \\ 40 \\ 40 \\ 41 \\ 45 \end{pmatrix} =
\begin{pmatrix}
1 & 0 & 0 & 0 & 70 \\
1 & 0 & 0 & 0 & 70 \\
1 & 0 & 1 & 0 & 68 \\
1 & 0 & 1 & 0 & 72 \\
1 & 0 & 0 & 1 & 72 \\
1 & 0 & 0 & 1 & 72 \\
1 & 1 & 0 & 0 & 71 \\
1 & 1 & 0 & 0 & 71 \\
1 & 1 & 1 & 0 & 72 \\
1 & 1 & 1 & 0 & 71 \\
1 & 1 & 0 & 1 & 73 \\
1 & 1 & 0 & 1 & 73
\end{pmatrix}
\begin{pmatrix} \mu \\ \tau_2 \\ \lambda_2 \\ \lambda_3 \\ \beta \end{pmatrix} + \vec{\varepsilon} .
$$

(over)

Run the regression and answer the following questions:

1.   What percentage of the variation in breaking strength is explained by the model?

2.   Find the correlation between the parameter estimates for T2 and L2.

3.   Find the estimate of the error variance.

4.   Which parameters are significantly different from 0?

5.   What are the degrees of freedom for the F test in the ANOVA?

6.   How much of the sum of squares is explained by Temperature, given that all of the other predictors are in the model?

7.   What is the estimated variance of the parameter estimate for Temperature?