Some discrete distributions

Stat 451
1-25-18

The Bernoulli Distribution (p)

①

$$X = \begin{cases} 1 & \text{with probability } p \\ 0 & \text{" \quad " } \quad 1-p = q \end{cases}$$

x	$p(x)$
0	q
1	p
	1

$$\mu = E[X] = \sum x\, p(x) = 0 \cdot q + 1 \cdot p = p$$

$$E[X^2] = \sum x^2 p(x) = 0^2 \cdot q + 1^2 \cdot p = p$$

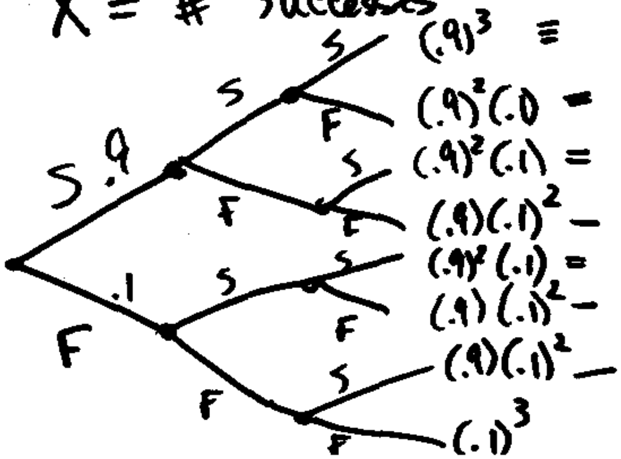
$$\sigma^2 = V[X] = E[X^2] - \mu^2 = p - p^2 = p(1-p)$$
$$= pq$$

The Binomial Distribution (n, p)

②

Consider a system with 3 independent components. Suppose that each component has a .1 chance of failure (.9 chance of success)

Let $X = \#$ successes



x	$p(x)$
0	$(.1)^3$
1	$3(.9)(.1)^2$
2	$3(.9)^2(.1)$
3	$(.9)^3$
	1

③

In general, a binomial experiment has the following properties:

1. Run a sequence of $\underline{n}$ independent trials
2. Each trial has 2 possible outcomes (S,F)
3. The probability of success, p, is the same on each trial
4. $X =$ # successes

Then X will takes on the values $0, 1, 2, \ldots, n$

④

$$p(x) = \binom{n}{x} (p)^x (q)^{n-x}$$

---

2 more properties of μ & σ^2

$$E[X+Y] = E[X] + E[Y]$$

$$V[X+Y] = V[X] + V[Y] + 2\,\mathrm{Cov}(X,Y)$$

If X & Y are independent, $\mathrm{Cov}(X,Y) = 0$

If $X \sim \mathrm{Bino}(n,p)$, then $X = X_1 + \ldots + X_n$ where each $X_i \sim \mathrm{Bern}(p)$, independent

So $E[X] = np$ and $V[X] = npq$ (5)

The Hypergeometric Distribution (N, N_1, n)

1. Population consists of N items
2. N_1 of those items are of type 1 and the remaining items are of type 2.
3. Take a random sample (without replacement) of n items
4. X = # of type 1 items in the sample

(6)

X takes on the values

$$0, 1, 2, \ldots, \min(n, N_1)$$

$N_2 = N - N_1$

$$p(x) = \frac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N}{n}}$$

$$\mu = n\frac{N_1}{N}, \quad \sigma^2 = n\frac{N_1}{N}\left(1 - \frac{N_1}{N}\right)\underbrace{\left(\frac{N-n}{N-1}\right)}$$

finite population correction

(7)

<u>Example</u>: Batch of 100 monitors, 3 of which are defective

Take a sample of 10 monitors.

Let X = # defective items in sample.

We will accept the batch if we find no defects.

$$\text{Hyper}(N=100, N_1=3, n=10) \Rightarrow p(x) = \frac{\binom{N_1}{x}\binom{N_2}{n-x}}{\binom{N}{n}}$$

$$P(X=0) = \frac{\binom{3}{0}\binom{97}{10}}{\binom{100}{10}} = .727$$

(8)

HW #3 due Thur Feb 1

p.89 #9, 12

p.97 #10

p.122 #4

p.145 #5, 7, 13, 16

9. You ask your roommate to water a sickly plant while you are on vacation. Without water the plant will die with probability 0.8 and with water it will die with probability 0.1. With probability 0.85, your roommate will remember to water the plant.

(a) What is the probability that your plant is alive when you return? (You may use a tree diagram.)

(b) If the plant is alive when you return, what is the probability that your roommate remembered to water it?

12. Seventy percent of the light aircraft that disappear while in flight in a certain country are subsequently discovered. Of the aircraft that are discovered, 60% have an emergency locator, whereas 10% of the aircraft not discovered have such a locator. Suppose a light aircraft disappears while in flight.

(a) What is the probability that it has an emergency locator and it will not be discovered?

(b) What is the probability that it has an emergency locator?

(c) If it has an emergency locator, what is the probability that it will not be discovered?

10. The system of components shown in Figure 2-15 below functions as long as components 1 and 2 both function or components 3 and 4 both function. Each of the four components functions with probability 0.9 independently of the others. Find the probability that the system functions.

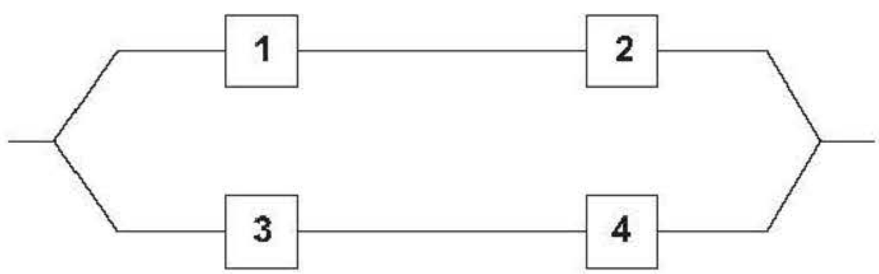


Figure 2-15 System of four components.

4. A metal fabricating plant currently has five major pieces under contract, each with a deadline for completion. Let X be the number of pieces completed by their deadlines. Suppose that X is a random variable with PMF $p(x)$ given by

x	0	1	2	3	4	5
$p(x)$	0.05	0.10	0.15	0.25	0.35	0.10

(a) Compute the expected value and variance of X.

(b) For each piece completed by the deadline, the plant receives a bonus of \$15,000. Find the expected value and variance of the total bonus amount.

5. The probability that a letter will be delivered within three working days is 0.9. You send out 10 letters on Tuesday to invite friends for dinner. Only those who receive the invitation by Friday (i.e., within 3 working days) will come. Let X denote the number of friends who come to dinner.

(a) The random variable X is (choose one)
   (i) binomial (ii) hypergeometric (iii) negative binomial (iv) Poisson.

(b) Give the expected value and variance of X.

(c) Determine the probability that at least 7 friends will come.

(d) A catering service charges a base fee of \$100 plus \$10 for each guest coming to the party. What is the expected value and variance of the catering cost?

7. In the grafting context of Exercise 1, suppose that grafts are done one at a time and the process continues until the first failed graft. Let X denote the number of grafts up to and including the first failed graft.

(a) The random variable X is (choose one)
   (i) binomial (ii) hypergeometric (iii) negative binomial (iv) Poisson.

(b) Give the sample space and PMF of X.

(c) Give the expected value and variance of X.

13. A distributor receives a new shipment of 20 iPods. He draws a random sample of five iPods and thoroughly inspects the click wheel of each of them. Suppose that the shipment contains three iPods with a malfunctioning click wheel. Let X denote the number of iPods with a defective click wheel in the sample of five.

(a) The random variable X is (choose one)
   (i) binomial (ii) hypergeometric (iii) negative binomial (iv) Poisson.

(b) Give the sample space and the formula for the PMF of X.

(c) Compute $P(X = 1)$.

(d) Find the expected value and variance of X.

16. A particular website generates income when people visiting the site click on ads. The number of people visiting the website is modeled as a Poisson process with rate $\alpha = 30$ per second. Of those visiting the site, 60% click on an ad. Let Y denote the number of those who will click on an ad over the next minute.

(a) The distribution of the random variable Y is (choose one)
   (i) binomial (ii) hypergeometric (iii) negative binomial (iv) Poisson.
   (*Hint.* See Example 3.4-16.)

(b) Give the mean and variance of Y.