Midterm Modern Physics, Phys 312

your name PSU ID#

1. Somebody quite famous wrote: *According to classical mechanics, it is not possible for a particle to be in a place where its total energy is less than its potential energy. In quantum mechanics this impossibility is changed into an improbability.*

1a. What effect is she/he talking about? Does her/his statement make sense? If you see fit, try to argue with the most general solution to the Schrödinger equation within a square finite height potential barrier of finite thickness:

$$\psi = Ce^{-k_2 x} + De^{k_2 x}$$, where $$k_2 = \frac{\sqrt{2m(U-E)}}{\hbar}$$. **5 points**

tunnel effect, yes is does, potential energy is U, total energy is E and also U + KE, the latter being kinetic energy,

so U > E implies negative kinetic energy which does not exist classically, in quantum mechanics this is not a problem at all, the wave function above is for such a scenario U > E, the result of the square root is, therefore, positive, we have to set D = 0 for this scenario and end up with an exponentially decaying wave function,

the square of the wave function is the probability density, since the wave function is approaching zero without reaching it as long as x is finite, the square of the wave function will not reach zero either, this being the probability density of finding the "particle" in the barrier, the "particle" has a probability to be there and could "in principle" be found at these places. However, if will not move into any kind of detector, so it is only a presence "in principle"

1b. Does the *improbability* she/he mentions mean that there is still a finite probability that a quantum mechanical object could be in a place where its total energy is less than its potential energy?

Yes ✓ in principle

No (no is acceptable if well argued due to the measurement problem, it's no in practice as the wavicals cannot be detected there, so it is questionable if it is really there) **2 point**

1c. If your answer to 1b was yes, from what physical fundamental equation and by which kind of maths is this probability calculated?

Schrödinger equation, differential (and integral) calculus, rules of complex numbers and functions

If your answer to 1b was no, why do you think so? **3 points**

Measurement problem of quantum mechanics, it is not really clear if there is an external physical world

2. Within a hypothetical microelectronic device electrons with a total energy of 10 meV are incident on a square barrier of height 100 meV and widths 0.2 nm. Calculate both the transmission and the reflection coefficients.

U = 0.1 eV, E = 0.01 eV, L = 0.2 10^{-9} m

$$T = e^{-2k_2 L} = e^{-2\frac{\sqrt{2m(U-KE)}}{\hbar}L}$$

$$e^{-2\pi 0.4\cdot 10^{-9}\frac{\sqrt{2\cdot 9.1\cdot 10^{-31}(0.1-0.01)\cdot 1.602\cdot 10^{-19}}}{6.625\cdot 10^{-34}}} = e^{-0.6145287} = \frac{1}{e^{0.6145287}} \approx 0.5408958$$

as R + T = 1 3 points

R = 0.4591042

it is also OK to use the approximation formula from the Serway book, p. 231

in that case, you should obtain T ≈ 0.787 and R ≈ 0.213

Out of 10^6 electrons, how many will be transmitted trough this barrier, how many will get stuck for ever in the barrier, and how many will be reflected by the barrier? (Round to full electrons and make pretty accurate calculations above.)

approximately 540896 will be transmitted

approximately 459104 will be reflected **5 points**

none will get stuck

if you had other values due to the other approximation form the book or just miscalculated, you still get the points (if your calculation is not too far off, that is)

If the electrons arrive at a rate of 10 per second at the barrier from a time t = t_0 onwards, how much time Δt will have elapsed before the first electron can be detected outside the barrier?

the transmission probability being 0.5408958 or 54, …% , 5.4… will tunnel per second and it will take about 0.185 seconds for the first full electron to emerge **3 points** in other words: 1 electron emerges at a time T $^{10\ \text{electrons}}/_{\text{second}}$ Δt, so $\Delta t = \dfrac{1 electron \cdot \sec}{T \cdot 10 electron}$

Why must this time interval Δt be greater than zero? only a full electron can be detected, neither 10 % of it nor 99%, …, so there must be a finite time greater zero, there is a finite probability density at $\Delta t = 0$ as we are talking about a steady state, used the time independent Schrödinger equation, … but a particle can only materialize as a whole particle not a fraction of a particle

2 points

3. Show that the energy level spacing of a harmonic oscillator is in accord with the correspondence principle by finding the ratio $\dfrac{\Delta E_n}{E_n}$ between adjacent energy levels and seeing what happens to this ratio as n ? ∞ Hint: $E_n = (n + {}^1/_2)$ h f **4 points**

5-26: Taking ΔE_n to be $E_{n+1} - E_n$ in Equation (5.70),

$$\frac{\Delta E_n}{E_n} = \frac{\left(n+1+\frac{1}{2}\right)h\nu - \left(n+\frac{1}{2}\right)h\nu}{\left(n+\frac{1}{2}\right)h\nu} = \frac{1}{n+\frac{1}{2}},$$

which goes to zero as $n \to \infty$. This is in accordance with the correspondence principle, in that for larger energies, the energy-level spacing is not noticeable.

4. Calculate the zero-point energy in Joule and MeV of a harmonic oscillator with *angular* frequency 10^{10} Hz $E_n = (n + {}^1/_2)$ h f, n = 0 is the lowest quantum number

4 points

? = 2p f = 10^{10} s^{-1}

f = 1.5915 10^9 s^{-1}

$E_n = {}^1/_2$ 6.625 10^{-34} Ws^2 1.5915 10^9 s^{-1} = 5.272 10^{-25} J = 3.29110^{-12} MeV

1 J = 6.242 10^{12} MeV,

5. **Find the energies of the second, third, forth, and fifth levels for a proton in an infinitely deep three-dimensional cubic box of 10 nm length . Which are degenerate?**
5 points

$$E = \pi^2\hbar^2 \Big/ 2mL^2 \cdot (n_1^2 + n_2^2 + n_3^2)$$

mp = 1.672 10^{-27} kg

quotient without quantum numbers 3.2813 10^{-25} Ws (OK if in eV or other energy unit)

first level 111, non degenerate
second level 112, degenerate into 211 and 211,

third level 122, degenerate into 212 and 221 2.953 10^{-24} Ws
fourth level 113, degenerate into 311, and 131 3.609 10^{-24} Ws
fifth level 222, non degenerate 3.938 10^{-24} Ws

sixth 223, degenerate into 232 and 322

36 points total