Homework solution #1(PH312)

1-

For this non-relativistic speed we have $E = \frac{1}{2}mv^2 = \frac{1}{2}\left(mc^2\right)\beta^2 = 0.002555$ eV. Using $E = n^2h^2/8mL^2$ we find

$$n^2 = \frac{8mL^2E}{h^2} = \frac{8mc^2L^2E}{h^2c^2} = \frac{8\left(511 \times 10^3 \text{ eV}\right)\left(48.5 \text{ nm}\right)^2\left(0.002555 \text{ eV}\right)}{\left(1240 \text{ eV} \cdot \text{nm}\right)^2} = 16.0$$

and therefore $n = 4$.

2-

$$\Delta E_n = E_{n+1} - E_n = \left(n + 1 + \frac{1}{2}\right)\hbar\omega - \left(n + \frac{1}{2}\right)\hbar\omega = \hbar\omega \qquad \text{for all } n$$

This is true for all n, and there is no restriction on the number of levels.

3-

Lacking an explicit equation for finite square well energies, we will approximate using the infinite square well formula. In order to contain three energy levels the depth of the well should be at least

$$E = \frac{n^2h^2}{8mL^2} = \frac{9h^2}{8mL^2}$$

Evaluating numerically with the given mass

$$E = \frac{9h^2}{8mL^2} = \frac{9h^2c^2}{8mc^2L^2} = \frac{9\left(1240 \text{ eV} \cdot \text{nm}\right)^2}{8\left(2 \times 10^9 \text{ eV}\right)\left(3 \times 10^{-6} \text{ nm}\right)^2} = 96.1 \text{ MeV}$$