MTH 621: Homework # 2, due 10/29, in class

To receive full credit, present complete answers that show all work.

Problem 1 (5 points). Find the solution to the initial value problem

$$xu_x + u_y = y, \quad u(x,0) = x^2$$

Problem 2 (10 points). Solve the initial value problem

$$uu_x + u_y = 1, \quad u(0,y) = \frac{y}{2}$$

Problem 3 (10 points). Solve the initial value problem

$$u_x^2 + yu_y - u = 0, \quad u(x,1) = \frac{x^2}{4} + 1$$

Problem 4 (10 points). Solve the initial value problem

$$\sin(u_x) - u_y = y, \quad u(x,0) = \pi x$$

Problem 5 (15 points). Consider the Cauchy problem

$$u = u_x^2 + u_y^2, \quad u(x,0) = \alpha x^2$$

where α is a constant parameter. Find the values of α such that there is a solution to the problem and find the solution(s) in terms of α.