FARIBORZ MASEEH DEPARTMENT OF MATHEMATICS + STATISTICS

Portland State UNIVERSITY

Mth 410/510: Inverse Problems & Data Assimilation I

Instructor: Dacian N. Daescu

Lecture 1: Introduction

Course Logistics

- Instructor: Dacian Daescu
  - Office: NH 313, Office hours: TR 10:30-11:30, also by appointment
  - Contact: e-mail daescu@pdu.edu; Phone: 503-725-3581
- Course web page (bookmark or remember my name!☺)
  - http://www.web.pdx.edu/~daescu/mth410_510.html
  - Lecture notes, homework assignments, additional information
- Course Description
  - *Introduction to mathematical and computational aspects of inverse problems (part I) and dynamic data assimilation (part II). Emphasis is placed on the numerical treatment of ill-posed problems, regularization techniques, optimal parameter estimation, sensitivity and observing system analysis.*
- Prerequisites: Mth 261 and Mth 254
  - *Recommended: knowledge of a programming language (Matlab), basic notions in probability and statistics*

Course Logistics

- Grading Policy: The final grade will be based on homework and a final project, as follows
  1. Homework, 60% of the course grade, 3-4 sets of problems will be assigned as homework, at least one week in advance. Hw will include both theoretical and computer assignments (the focus will be on applications however, some programming will be required)
  2. Final Project, 40% of the course grade. A final project will be assigned two weeks in advance and will be due by final examination time. Alternatively, with the approval of the instructor, a student may choose a topic to present as final project.
- Final Examination: Tuesday, December 5, 17:30 - 19:20 in class
- Main criteria for evaluating your work will be: correctness, completeness, and clarity of the presentation
- Working in team for your homework and project is encouraged only if each student in the team is contributing to the problem solving

Inverse Problems: introductory concepts

The answer is Washington.

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What was the question?

Inverse Problems: introductory concepts

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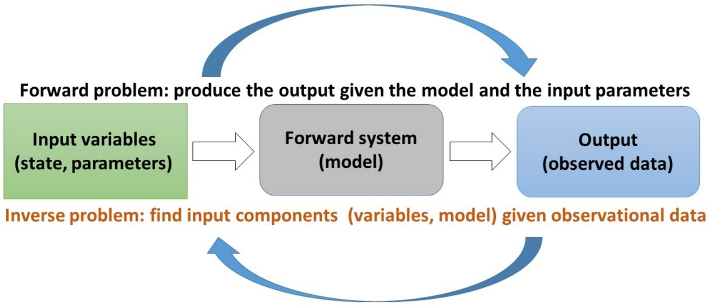

Inverse Problems: introductory concepts

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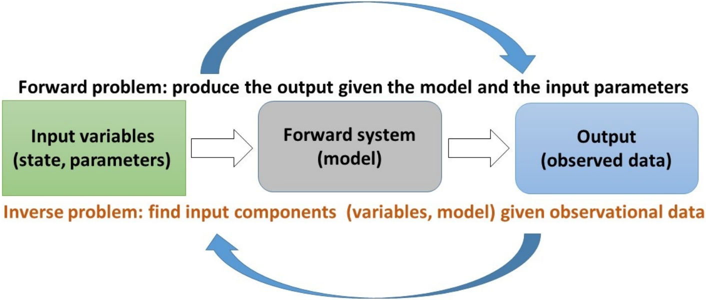

Often the properties of the forward problem are well-understood. In an inverse problem we measure the effect and want to determine the cause.

Inverse Problems: introductory concepts

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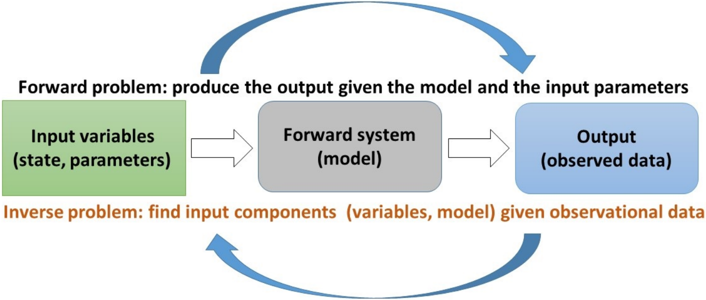

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Inverse problems (Joseph B. Keller, Amer. Mth. Monthly, 1976)

We call two problems inverses to one another if the formulation of each involves all or part of the solution of the other.

Inverse Problems: practical applications

Image restoration, signal processing

Reconstruct an image $\mathbf{M} \in \mathbb{R}^{n \times m}$ from a blurred, noisy version of it $\widetilde{\mathbf{M}} \in \mathbb{R}^{n \times m}$ given the blurring process operator $\mathbf{H}$ (e.g., convolution with a point-spread operator)

$$\widetilde{\mathbf{M}} = \mathbf{M} \star \mathbf{H}, \qquad \widetilde{\mathbf{M}}[i,j] = \sum_{k,l} \mathbf{M}[k,l]\mathbf{H}[i-k, j-l]$$

In practice, we only have access to a noisy version of the blurred image,

$$\widetilde{\mathbf{M}} = \mathbf{M} \star \mathbf{H} + \zeta$$

Simple "deconvolution" may lead to noise amplification

Inverse Problems: practical applications

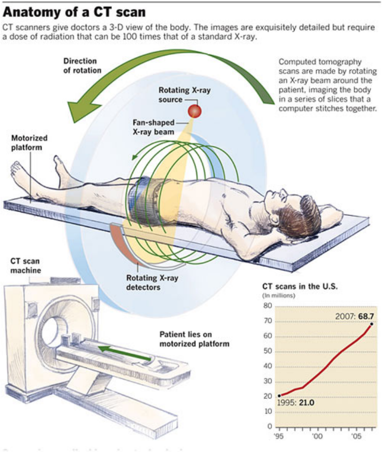
Medical imaging: computerized tomography (CT scan)

Reconstruct a spatially varying density that is exposed to radiation from various directions and that absorbs the radiation according to its material properties

Recover $f(x,y)$ given

$$\int_{\mathcal{C}} f(x,y)\,ds$$

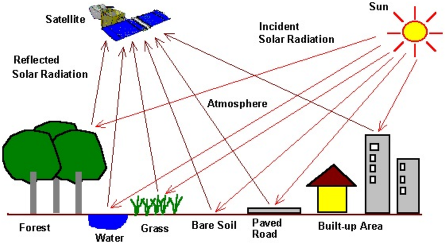
for various line paths $\mathcal{C}$

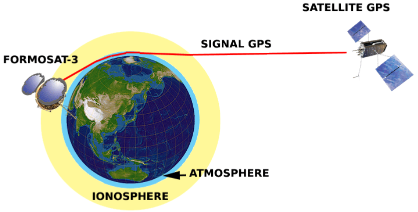


Inverse Problems: practical applications

Inverse scattering, remote sensing

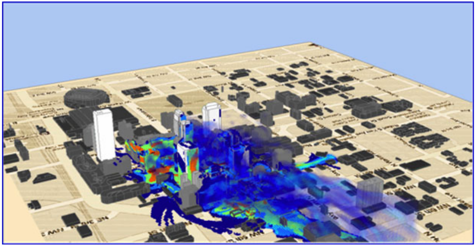
Determine physical properties (e.g., shape, color, density, temperature, humidity, chemical composition) of an object or medium, based on data of how it scatters incoming radiation, signals, or particles.

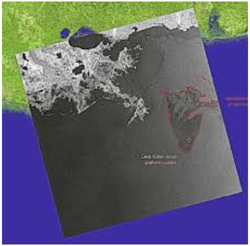




Inverse Problems: practical applications

Source identification: pollutant emissions, air, water, subsurface contaminants



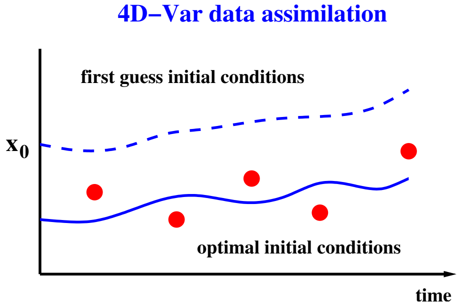


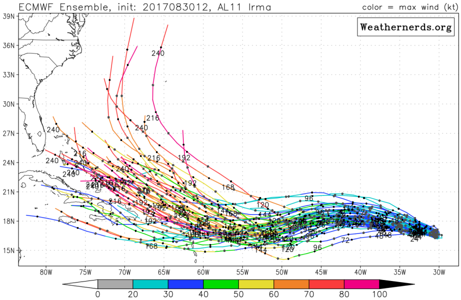
- Inversion of systems of differential or partial differential equations

$$\mathcal{L}(u) = f$$

Inverse Problems: practical applications

Dynamic data assimilation, optimal initial condition retrieval



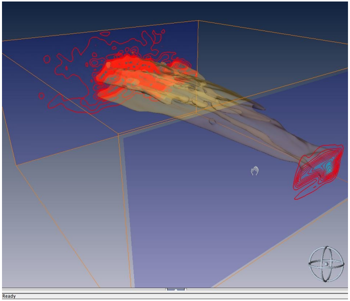


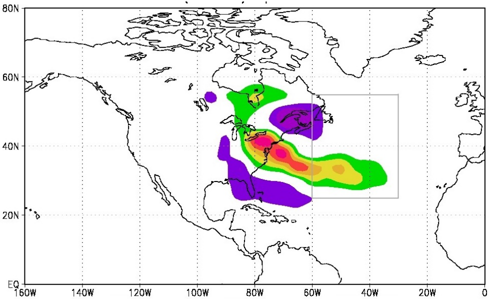
Dynamic Data Assimilation

Model forecasts are highly sensitive to initial condition specification. Optimal initial conditions are obtained by solving a time-evolving large-scale nonlinear inverse problem. Combine information from prior forecast, observational data and error statistics.

Inverse Problems: practical applications

Sensitivity analysis: state and parameter sensitivity





Sensitivity analysis

The key challenge is to determine those input components that contribute most to the uncertainty in the model prediction

- model state and parameter sensitivity
- observing system sensitivity

Inverse Problems (IP): general remarks

Requirements for a well-posed problem (Jacques Hadamard, 1902)

- Existence: the problem has a solution
- Uniqueness: there is only one solution to the problem
- Stability: the solution depends continuously on the data (input)

- IP estimate unknown variables from indirect, noisy measurements
- IP are often more difficult to solve than the direct problems
- In practice, IP require estimation of a large number of parameters
- Solution is often *sensitive to errors in data*

Ill-posed problems

Problems that are not well-posed in the sense of Hadamard are termed ill-posed. Inverse problems are often ill-posed or ill-conditioned: small errors in the input data can result in much larger errors in the estimated variables.

Topics to be covered in the first term

- Introduction to inverse problems, fundamental examples
- Background material: linear algebra, optimization, probability
- Rank-deficient and ill-posed problems, the need for regularization
  - 1D and 2D diffusion processes, the image restoration problem
- Computational aspects and regularization methods
  - singular value decomposition (SVD), generalized SVD, filtering of the SVD components, TSVD & TGSVD; Tikhonov regularization, selection of the regularization parameters, the L-curve; discrete smoothing norms, total variation regularization; nonlinear problems, iterative methods
- Applications to image deblurring, differential and partial differential equations systems, sensitivity analysis

No textbook is required. Lecture notes and additional materials will be posted on the course web page. References include:

- Hansen P.C., Rank-Deficient and Discrete Ill-Posed Problems. SIAM 1998
- Hansen P.C., Discrete Inverse Problems: Insight and Algorithms. SIAM 2010