

MTH 410/510: Inverse Problems & Data Assimilation I, II

Instructor: Dacian N. Daescu

Office: Neuberger Hall 313

E-mail: daescu@pdx.edu

Phone: (503) 725-3581

Office Hours: TR 10:30 - 11:30. Also by appointment.

Class Time and Location: TR 17:15 - 18:30 Neuberger Hall 387

Course web site: Lecture notes, homework assignments, and other information about the course will be available on the web site: http://www.web.pdx.edu/~daescu/mth410_510.html

Students are responsible for checking this site on a regular basis.

Final Examination: Tuesday, December 5, 17:30 - 19:20 in class

Course Description: This course provides an introduction to mathematical and computational aspects of inverse problems (part I) and dynamic data assimilation (part II). Emphasis is placed on the numerical treatment of ill-posed problems, regularization techniques, optimal parameter estimation, sensitivity and observing system analysis. Topics will be covered as follows.

1. First term

- Introduction to linear inverse problems, fundamental examples
- Rank-deficient and ill-posed problems, the need for regularization: 1D and 2D diffusion processes, the image restoration model
- Computational aspects and regularization methods
 - numerical optimization, matrix approximations, numerical rank, singular value decomposition (SVD), generalized SVD, filtering of the SVD components, TSVD & TGSVD; Tikhonov regularization, perturbation bounds, selection of the regularization parameters, the L-curve; discrete smoothing norms, total variation regularization; nonlinear problems, iterative methods
- Applications to image deblurring, differential and partial differential equations systems

2. Second term

- Introduction to dynamic data assimilation, statement of the inverse problem
- Probabilistic formulation: maximum likelihood and Bayesian estimation
- Linear and nonlinear filtering theory
 - Kalman filter and smoother, extended Kalman filter, particle filters; reduced-rank ensemble methods, sensitivity, filter divergence; evolution of the analysis error covariance, numerical implementation
- Variational data assimilation methods
 - 3D-Var, 4D-Var methods, equivalence with the Kalman filter and smoother; incorporation of the model error, weak-constrained 4D-Var, hybrid methods; numerical solution to the nonlinear least-square optimization, the incremental approach; observing system analysis
- Practical implementation and illustrative examples from applied sciences

No textbook is required. Lecture notes and additional materials will be posted on the course web page.

References include:

Hansen P.C., *Rank-Deficient and Discrete Ill-Posed Problems*. SIAM 1998

Hansen P.C., *Discrete Inverse Problems: Insight and Algorithms*. SIAM 2010

Lewis J.M. et al. *Dynamic Data Assimilation: A least Squares Approach*. Cambridge Univ. Press 2006

Student Learning Objectives: By the end of the course the students will: become familiar with theoretical and practical aspects of inverse problems and data assimilation; gain hands-on experience with the methods for analysis and numerical solution of discrete ill-posed problems; understand various regularization methods and how to choose the regularization parameters; understand the mathematical principles behind modern data assimilation methods; understand how they would implement data assimilation methods for a real system.

Prerequisites: Mth 261 and Mth 254. *Recommended: knowledge of a programming language, basic notions in probability and statistics.*

Grading Policy: The final grade will be based on homework and a final project, as follows:

1. **Homework, 60% of the course grade.** Four sets of problems will be assigned as homework, at least one week in advance.
2. **Final Project, 40% of the course grade.** A final project will be assigned two weeks in advance and will be due by final examination time.

In assigning final course grades, plus/minus grading will be used. Main criteria for evaluating your work will be: *correctness, completeness, and clarity of the presentation.* Working in team for your homework and project is encouraged *only if each student in the team is contributing to the problem solving.*

Special requests: If you require special arrangements for seating, testing or other class requirements, please contact me after class or during my office hours.

Access and Inclusion for Students with Disabilities

PSU values diversity and inclusion; we are committed to fostering mutual respect and full participation for all students. My goal is to create a learning environment that is equitable, useable, inclusive, and welcoming. If any aspects of instruction or course design result in barriers to your inclusion or learning, please notify me. The Disability Resource Center (DRC) provides reasonable accommodations for students who encounter barriers in the learning environment.

If you have, or think you may have, a disability that may affect your work in this class and feel you need accommodations, contact the Disability Resource Center to schedule an appointment and initiate a conversation about reasonable accommodations. The DRC is located in 116 Smith Memorial Student Union, 503-725-4150, drc@pdx.edu, <https://www.pdx.edu/drc>.

If you already have accommodations, please contact me to make sure that I have received a faculty notification letter and discuss your accommodations. For information about emergency preparedness, please go to the Fire and Life Safety webpage (<https://www.pdx.edu/environmental-health-safety/fire-and-life-safety>) for information.

Title IX of the Education Amendments of 1972

As an instructor, one of my responsibilities is to help create a safe learning environment for my students and for the campus as a whole. We expect a culture of professionalism and mutual respect in our department and class. You may report any incident of discrimination or discriminatory harassment, including sexual harassment, to either the Office of Equity and Compliance or the Office of the Dean of Student Life. Please be aware that as a faculty member, I have the responsibility to report any instances of sexual harassment, sexual violence and/or other forms of prohibited discrimination. If you would rather share information about sexual harassment or sexual violence to a confidential employee who does not have this reporting responsibility, you can find a list of those individuals at <https://www.pdx.edu/sexual-assault/get-help>. For more information about Title IX please complete the required student module Creating a Safe Campus in your D2L.