MTH 410/510: Inverse Problems and Data Assimilation I, II (3-, 3-credits)

Course description: This course provides an introduction to mathematical and computational aspects of inverse problems and dynamic data assimilation. Emphasis is placed on the numerical treatment of ill-posed problems, regularization techniques, optimal parameter estimation, forecast error and sensitivity analysis for partially observed dynamical systems. Practical applications of the theory and algorithms are illustrated using examples from image processing and data assimilation.
Prerequisites: Mth 261 and Mth 254. Recommended: knowledge of a programming language; basic notions in probability and statistics. Courses must be taken in sequence.

1. **First term**
   - Introduction to linear inverse problems; diffusion processes and the image restoration model
   - Rank-deficient and ill-posed problems; the need for regularization
   - Computational aspects and regularization methods
     - matrix approximations, numerical rank, singular value decomposition (SVD), generalized SVD, filtering of the SVD components, TSVD & TGSVD; Tikhonov regularization, perturbation bounds, selection of the regularization parameters, the L-curve; discrete smoothing norms, total variation regularization; applications to 2D image deblurring
2. **Second term**
   - Introduction to dynamic data assimilation, statement of the inverse problem
   - Probabilistic formulation: maximum likelihood and Bayesian estimation
   - Linear and nonlinear filtering theory
     - Kalman filter and smoother, extended Kalman filter, particle filters; reduced-rank ensemble methods, sensitivity, filter divergence; evolution of the analysis error covariance, numerical implementation
   - Variational data assimilation methods
     - 3D-Var, 4D-Var methods, equivalence with the Kalman filter and smoother; incorporation of the model error, weak-constrained 4D-Var, hybrid methods; numerical solution to the nonlinear least-square optimization, the SQP approach; practical implementation and illustrative examples from geosciences.

Student Learning Objectives: By the end of the course the students will: become familiar with theoretical and practical aspects of inverse problems and data assimilation; gain hands-on experience with the methods for analysis and numerical solution of discrete ill-posed problems; understand various regularization methods and how to choose the regularization parameters; understand the mathematical principles behind modern data assimilation methods; understand how they would implement data assimilation methods for a real system.

References:

Hansen P.C., *Rank-Deficient and Discrete Ill-Posed Problems.* SIAM 1998.
Hansen P.C., *Discrete Inverse Problems: Insight and Algorithms.* SIAM 2010.
Lewis J.M. et al. *Dynamic Data Assimilation: A least Squares Approach.* Cambridge University Press 2006.