Transln Cost Fuction

E. Berndt and D. Wood, "Technology, Prices, and the Derived Demand for Energy," *Review of Economics and Statistics*, 57, 1975, pp 376-384.

Production and Cost Functions

- Production function:
  $Q = f(\mathbf{x})$
- Cost minimizing factor demands:
  $x_i = x_i(Q,\mathbf{p})$
- Cost function:
  $C = \sum_{i=1,\ldots M} p_i x_i(Q,\mathbf{p}) = C(Q,\mathbf{p})$

Theory of Cost Function

- Shephard's Lemma:
  $x_i = x_i(Q,\mathbf{p}) = \partial C(Q,\mathbf{p})/\partial p_i$
  $p_i x_i/C = (p_i/C)\, \partial C(Q,\mathbf{p})/\partial p_i$
- Factor Shares: $s_i = \partial\, \mathit{ln} C(Q,\mathbf{p})/\partial\, \mathit{ln} p_i$
- Elasticity of Factor Substitution:

$$\theta_{ij} = \frac{C \cdot (\partial^2 C / \partial p_i \partial p_j)}{(\partial C / \partial p_i)(\partial C / \partial p_j)}$$

- (Own and Cross) Price Elasticity:

$$\eta_{ij} = s_j \theta_{ij}, \quad \eta_{ij} \neq \eta_{ji}$$

Theory of Cost Function

- Constant returns to scale: C = Qc(p)
- Average cost function: c(**p**) = C/Q
- Marginal cost function: ∂C/∂Q = c(**p**)
- Linear homogeneity in prices: λc(**p**)=c(λ**p**)
- 2nd order Taylor approximation of *ln*c(**p**) at *ln***p** = 0:

$$\ln c \approx \beta_0 + \sum_{i=1}^{M} \left(\frac{\partial \ln c}{\partial \ln p_i} \right) \ln p_i + \frac{1}{2} \sum_{i=1}^{M} \sum_{j=1}^{M} \left(\frac{\partial^2 \ln c}{\partial \ln p_i \partial \ln p_j} \right) \ln p_i \ln p_j$$

Berndt-Wood Model

- U.S. Manufacturing, 1947-1971
- Output and Four Factors: Q, K, L, E, M
- Prices: P_K, P_L, P_E, P_M
- The constant return to scale translog cost function:
  $\ln(C) = b_0 + \ln(Q) + b_K\ln(P_K) + b_L\ln(P_L) + b_E\ln(P_E) + b_M\ln(P_M) +$
  $\frac{1}{2} b_{KK}\ln(P_K)^2 + \frac{1}{2} b_{LL}\ln(P_L)^2 + \frac{1}{2} b_{EE}\ln(P_E)^2 + \frac{1}{2} b_{MM}\ln(P_M)^2 +$
  $b_{KL}\ln(P_K)\ln(P_L) + b_{KE}\ln(P_K)\ln(P_E) + b_{KM}\ln(P_K)\ln(P_M) + b_{LE}\ln(P_L)\ln(P_E)$
  $+ b_{LM}\ln(P_L)\ln(P_M) + b_{EM}\ln(P_E)\ln(P_M)$
- Symmetric conditions: $\beta_{ij} = \beta_{ji}$, i,j = K,L,E,M

Berndt-Wood Model

- Factor shares:
  $S_K = P_K K/C$, $S_L = P_L L/C$, $S_E = P_E E/C$, $S_M = P_M M/C$

  $S_K+S_L+S_E+S_M = 1$ (because $P_K K+P_L L+P_E E+P_M M = C$)
- Factor share equations:
  $S_K = b_K + b_{KK} \ln(P_K) + b_{KL} \ln(P_L) + b_{KE} \ln(P_E) + b_{KM} \ln(P_M)$
  $S_L = b_L + b_{KL} \ln(P_K) + b_{LL} \ln(P_L) + b_{LE} \ln(P_E) + b_{LM} \ln(P_M)$
  $S_E = b_E + b_{KE} \ln(P_K) + b_{LE} \ln(P_L) + b_{EE} \ln(P_E) + b_{EM} \ln(P_M)$
  $S_M = b_M + b_{KM} \ln(P_K) + b_{LM} \ln(P_L) + b_{EM} \ln(P_E) + b_{MM} \ln(P_M)$
- Elasticities:
  $\theta_{ij} = b_{ij}/(S_i S_j) + 1$ if $i \neq j$; $\theta_{ij} = b_{ij}/(S_i S_i) + 1 - 1/S_i$,
  $\eta_{ij} = S_j \theta_{ij}$, i,j=K,L,E,M

Berndt-Wood Model

- Linear restrictions:

  $b_K + b_L + b_E + b_M = 1$
  $b_{KK} + b_{KL} + b_{KE} + b_{KM} = 0$
  $b_{KL} + b_{LL} + b_{LE} + b_{LM} = 0$
  $b_{KE} + b_{LE} + b_{EE} + b_{EM} = 0$
  $b_{KM} + b_{LM} + b_{EM} + b_{MM} = 0$