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TECHNOLOGY, PRICES, AND THE DERIVED DEMAND FOR ENERGY

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I. Introduction

INDUSTRIAL demand for energy is essentially a derived demand: the firm's demand for energy as an input is derived from the demand for the firm's output. Inputs other than energy typically also enter the firm's production process. Since firms tend to choose that bundle of inputs which minimizes the total cost of producing a given level of output, the derived demand for inputs, including energy, depends on the level of output, the substitution possibilities among inputs allowed by the production technology, and the relative prices of all inputs.

Empirical studies typically have not simultaneously considered the derived demand for energy and nonenergy inputs. Studies of industrial energy demand have been restricted in one of two ways: (1) by focussing attention solely on the level of output and ignoring input prices (Darmstadter et al. 1971), Dupree and West (1972), Morrison and Readling (1968), National Energy Board (1971), National Petroleum Council (1971) (Schurr et al. 1960), or (2) by confining the analysis to output and the demand response of some particular energy type to a change in its own price and perhaps that of close energy substitutes, but neglecting the prices of all nonenergy inputs (Anderson 1971), Baxter and Rees (1968), Mount, Chapman, and Tyrrell (1973). In a similar fashion, empirical studies of demand for nonenergy inputs have excluded from their concern the role of energy prices. For example, the voluminous literature on determinants of investment behavior typically either (i) restricts attention to changes in output and ignores all input prices (the "accelerator" models), or (ii) expands the analysis to output and the prices of capital and labor services, but ignores the possible role of all intermediate inputs including energy (the "neoclassical" models). To our knowledge, no empirical study has explicitly investigated cross-substitution possibilities between energy and nonenergy inputs.[1]

The absence of studies on substitution possibilities between energy and nonenergy inputs becomes particularly apparent if one is interested in deriving implications of increasingly scarce and higher priced energy inputs. Consider the effect of higher priced energy on corporate investment behavior. If energy and capital are substitutable, ceteris paribus, then higher priced energy will increase the demand for new capital goods. If energy and capital are complementary, however, then ceteris paribus, higher priced energy will dampen the demand for energy and the demand for new plant and equipment. More generally, if it were found that possibilities for substitution between energy and nonenergy inputs are extremely limited, then we might expect that the adjustment by industry to higher priced energy will be somewhat difficult, that unit cost may rise considerably, that the composition of output may shift away from energy intensive products, and

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[1] Parks (1971) has investigated substitution possibilities between capital, labor, and three intermediate goods, but does not specify energy as a separate input.

that significant changes in the underlying technological structure may be required.

In this paper we report results of an attempt to characterize more completely the structure of technology in United States manufacturing, 1947–1971, by providing evidence on the possibilities for substitution between energy and nonenergy inputs. Our principal finding is that technological possibilities for substitution between energy and nonenergy inputs are present, but to a somewhat limited extent. Specifically, we find that (i) energy demand is price responsive — the own price elasticity is about −.5; (ii) energy and labor are slightly substitutable — the Allen partial elasticity of substitution between energy and labor (σ_{EL}) is about 0.65; and (iii) energy and capital are complementary — σ_{EK} is about −3.2. We also consider the validity of the value added specification typically used in studies of production and investment behavior, and find that our data do not support this specification.

II. Theoretical Model and Stochastic Specification

We assume that there exists in United States manufacturing a twice differentiable aggregate production function relating the flow of gross output Y to the services of four inputs: capital (K), labor (L), energy (E), and all other intermediate materials (M). Further, we assume that production is characterized by constant returns to scale and that any technical change affecting K, L, E, and M is Hicks-neutral. Corresponding to such a production function there exists a cost function which reflects the production technology. In its general form we write this cost function as $G = G\ (Y, P_K, P_L, P_E, P_M)$, where G is total cost, and P_K, P_L, P_E, and P_M are the input prices of K, L, E, and M, respectively.

For purposes of estimation we must employ a specific functional form for G. We choose to specify a highly general functional form, one that places no a priori restrictions on the Allen partial elasticities of substitution and one that can be interpreted as a second order approximation to an arbitrary twice-differentiable cost function. A variety of functional forms satisfy these requirements — the generalized Leontief, generalized Cobb-Douglas, and translog cost functions all are sufficiently flexible.[2] We arbitrarily choose to employ the translog cost function. For our four-input KLEM model, we write this cost function with symmetry and constant returns to scale imposed as

$$\begin{aligned}\ln G = {} & \ln \alpha_0 + \ln Y + \alpha_K \ln P_K + \alpha_L \ln P_L \\ & + \alpha_E \ln P_E + \alpha_M \ln P_M + \frac{1}{2}\gamma_{KK} (\ln P_K)^2 \\ & + \gamma_{KL} \ln P_K \ln P_L + \gamma_{KE} \ln P_K \ln P_E \\ & + \gamma_{KM} \ln P_K \ln P_M + \frac{1}{2}\gamma_{LL} (\ln P_L)^2 \\ & + \gamma_{LE} \ln P_L \ln P_E + \gamma_{LM} \ln P_L \ln P_M \\ & + \frac{1}{2}\gamma_{EE} (\ln P_E)^2 + \gamma_{EM} \ln P_E \ln P_M \\ & + \frac{1}{2}\gamma_{MM} (\ln P_M)^2. \qquad (1)\end{aligned}$$

Linear homogeneity in prices imposes the following restrictions on (1)

$$\begin{aligned}& \alpha_K + \alpha_L + \alpha_E + \alpha_M = 1 \\ & \gamma_{KK} + \gamma_{KL} + \gamma_{KE} + \gamma_{KM} = 0 \\ & \gamma_{KL} + \gamma_{LL} + \gamma_{LE} + \gamma_{LM} = 0 \\ & \gamma_{KE} + \gamma_{LE} + \gamma_{EE} + \gamma_{EM} = 0 \\ & \gamma_{KM} + \gamma_{LM} + \gamma_{EM} + \gamma_{MM} = 0. \qquad (2)\end{aligned}$$

Assuming perfect competition in the factor markets, we treat input prices as fixed. Given the level of output, cost minimizing input demand functions are derived as follows. First we logarithmically differentiate (1),

$$\frac{\partial \ln G}{\partial \ln P_i} = \frac{\partial G}{\partial P_i}\frac{P_i}{G} = \alpha_i + \sum_j \gamma_{ij} \ln P_j,\ i,\quad j = K, L, E, M$$

and then, using Shephard's Lemma,[3]

$$x_i = \frac{\partial G}{\partial P_i},\quad i = K, L, E, M$$

we obtain the KLEM input demand equations

$$\begin{aligned}M_K = \frac{P_K K}{G} & = \alpha_K + \gamma_{KK} \ln P_K + \gamma_{KL} \ln P_L \\ & + \gamma_{KE} \ln P_E + \gamma_{KM} \ln P_M \\ M_L = \frac{P_L L}{G} & = \alpha_L + \gamma_{KL} \ln P_K + \gamma_{LL} \ln P_L \\ & + \gamma_{LE} \ln P_E + \gamma_{LM} \ln P_M \qquad (3)\end{aligned}$$

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[2] The generalized Leontief cost function is discussed by Diewert (1971), the generalized Cobb-Douglas by Diewert (1973a) and the translog by Christensen, Jorgenson, and Lau (1971, 1973).

[3] For further discussion, see Diewert (1974).

$$M_E = \frac{P_E E}{G} = \alpha_E + \gamma_{KE} \ln P_K + \gamma_{LE} \ln P_L + \gamma_{EE} \ln P_E + \gamma_{EM} \ln P_M$$

$$M_M = \frac{P_M M}{G} = \alpha_M + \gamma_{KM} \ln P_K + \gamma_{LM} \ln P_L + \gamma_{EM} \ln P_E + \gamma_{MM} \ln P_M$$

where the total cost $G = P_K K + P_L L + P_E E + P_M M$. The M_i are of course the cost shares of the inputs in the total cost of producing Y.

Uzawa (1962) has derived the Allen partial elasticities of substitution (AES) between inputs i and j as

$$\sigma_{ij} = \frac{G\,G_{ij}}{G_i G_j}$$

where

$$G_i = \frac{\partial G}{\partial P_i}, \quad G_{ij} = \frac{\partial^2 G}{\partial P_i\, \partial P_j}$$

and by definition, $\sigma_{ij} = \sigma_{ji}$. With the translog cost function the AES are

$$\sigma_{ii} = \frac{\gamma_{ii} + M_i^2 - M_i}{M_i^2}, \quad i = K, L, E, M \qquad (4)$$

$$\sigma_{ij} = \frac{\gamma_{ij} + M_i M_j}{M_i M_j}, \quad i, j = K, L, E, M, \; i \neq j. \qquad (5)$$

These AES are not constrained to be constant but may vary with the values of the cost shares.

The price elasticity of demand for factors of production, E_{ij}, is conventionally defined as

$$E_{ij} = \frac{\partial \ln x_i}{\partial \ln P_j}$$

where output quantity and all other input prices are fixed. Allen (1938) has shown that the AES are analytically related to the price elasticities of demand for factors of production

$$E_{ij} = M_j \sigma_{ij}. \qquad (6)$$

Hence even though $\sigma_{ij} = \sigma_{ji}$, in general $E_{ij} \neq E_{ji}$.

We characterize the structure of technology in United States Manufacturing 1947–1971 by estimating the input demand equations (3) subject to the restrictions imposed by linear homogeneity in prices (2). Such an empirical implementation requires that our translog model (3) be imbedded within a stochastic framework. We assume that deviations of the cost shares from the logarithmic derivatives of the translog cost function are the result of random errors in cost minimizing behavior; we append to each of the equations in (3) an additive disturbance term. Since the cost shares of the four equations in (3) always sum to unity, the sum of the disturbances across the four equations is zero at each observation. This implies that the disturbance covariance matrix is singular and nondiagonal. We arbitrarily drop the disturbance from the M_K equation and specify that the disturbance column vector $\epsilon(t)$,

$$\epsilon(t) = [\epsilon_L(t)\ \epsilon_E(t)\ \epsilon_M(t)]$$

is independently and identically normally distributed with mean vector zero and nonsingular covariance matrix Ω, $t = 1, \ldots, T$.

At the level of an individual firm it may be reasonable to assume that the supply of inputs is perfectly elastic, and therefore that input prices can be taken as fixed. At the more aggregated industry level, however, input prices are less likely to be exogenous. Since the level of aggregation in this study is that of total United States manufacturing, it may be inappropriate for us to assume that prices are exogenous and that the regressors in (3) are uncorrelated with the disturbances. If our estimation procedure is to provide consistent estimates of the parameters in (3), this possible simultaneity must be taken into account. We have chosen as our estimation procedure the method of instrumental variables: We regress each of the regressors in (3) on a set of variables considered exogenous to the United States manufacturing sector, and employ the fitted values from these "first stage" regressions as instruments in place of the regressors in (3).[4]

Since the disturbance covariance matrix of the four equation model (3) is singular, we could arbitrarily drop the M_K equation and estimate the remaining three equations by three-stage least squares (3SLS). The problem with this procedure is that the 3SLS estimates may not be invariant to the equation deleted. Invariance can be attained, however, by iterating the 3SLS procedure until the estimated coefficients and residual covariance matrix converge. For this reason we employ the iterative three-stage least squares (I3SLS) estimator. The I3SLS es-

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[4] Since linear homogeneity in prices (2) is imposed, we can rewrite the regressors in (3) as logarithms of the price ratios. We form values of the instruments as fitted values from the regressions of ln (P_L/P_K), ln (P_E/P_K), and ln (P_M/P_K) on the exogenous variables.

timator is consistent and asymptotically efficient, but in finite samples it provides coefficient estimates which in general differ numerically from those of the full information maximum likelihood estimator.[5]

III. Data Construction and Sources

The data required for I3SLS estimation of the KLEM translog cost function in United States manufacturing 1947–1971 are the prices and cost shares of the four inputs and values for the exogenous variables used in forming the instruments.

Following procedures outlined by Christensen-Jorgenson (1969), we construct the rental price of capital services P_K from nonresidential structures and producers' durable equipment, taking account of variations in effective tax rates and rates of return, depreciation, and capital gains. We then construct quantity indexes of K by Divisia aggregation of capital services from nonresidential structures and producers' durable equipment. Finally, we compute the value of capital services as the product of the quantity index K and the rental price P_K. A more detailed discussion of procedures used in constructing these capital price and quantity indexes is found in Berndt and Christensen (1973b).

Since data on labor compensation are readily available, we obtain an estimate of P_L by first concentrating on our measure of L. We construct our measure of labor services L as a Divisia index of production ("blue collar") and nonproduction ("white collar") labor manhours, adjusted for quality changes using the educational attainment indexes of Christensen-Jorgenson (1970). Our measure of the value of labor services is total compensation to employees in United States manufacturing, adjusted for the earnings of proprietors. We then compute P_L as adjusted total labor compensation divided by L. A more detailed discussion of methodology and data sources used in the construction of the labor price and quantity indexes is presented in Berndt-Christensen (1974).

We next construct annual price and quantity indexes for energy and other intermediate materials in United States manufacturing 1947–1971. Annual interindustry flow tables measuring flows of goods and services from 25 producing sectors to 10 consuming sectors and five categories of final demand, in both current and constant dollars, are presented in Faucett (1973).[6] Based on these tables, we construct annual quantity indexes of E as Divisia quantity indexes of coal, crude petroleum, refined petroleum products, natural gas, and electricity purchased by establishments in United States manufacturing. We then compute the value of energy purchases as the sum of current dollar purchases of these five energy types. Finally, we form the price index P_E as the value of total energy purchases divided by E.

Annual quantity indexes of M are constructed from the Faucett interindustry flow tables as Divisia quantity indexes of nonenergy intermediate good purchases by establishments in United States manufacturing from agriculture, non-fuel mining, and construction; manufacturing excluding petroleum products; transportation; communications, trade, and services; water and sanitary services; and foreign countries (imports). We then compute the value of total nonenergy intermediate good purchases as the sum of current dollar purchases of all these nonenergy intermediate goods. Finally, we form the price index P_M as the value of total nonenergy intermediate good purchases divided by M.

In tables 1 and 2 we tabulate price, quantity, and cost shares of K, L, E, and M, and total input cost. Looking at the first four columns of table 1, we note that over the 1947–1971 time period the prices of K, E, and M rose less rapidly than the price of L. The slower growth rates of P_E and P_K are due partly to actions of the federal government, particularly the price ceilings on certain energy types and the favorable tax treatment given corporations investing in new plant and equipment (the accelerated

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[5] See Dhrymes (1973).

[6] The interindustry flow tables described in Faucett (1973) are based on data from the annual Bureau of Mines *Minerals Yearbook*, the Census of Minerals Industries (1954, 1958, 1963 and 1967), the Census of Manufacturers (1947, 1954, 1958, 1963 and 1967), the U.S. Department of Commerce Input-Output Tables (1947, 1958, 1963), Annual Surveys of Manufacturers, and a variety of secondary sources. This set of accounts was developed with support from the Ford Foundation Energy Policy Project and the Mathematics and Computation Laboratory, Office of Preparedness, General Services Administration.

Table 1. — Price and Quantity Indexes of Capital, Labor, Energy, and Other Intermediate Inputs — U.S. Manufacturing 1947–1971

Year	P_K	P_L	P_E	P_M	K	L	E	M
1947	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1948	1.00270	1.15457	1.30258	1.05525	1.14103	.97501	.92932	.88570
1949	.74371	1.15584	1.19663	1.06225	1.23938	.92728	1.01990	.94093
1950	.92497	1.23535	1.21442	1.12430	1.28449	.98675	1.08416	1.07629
1951	1.04877	1.33784	1.25179	1.21694	1.32043	1.08125	1.18144	1.13711
1952	.99744	1.37949	1.27919	1.19961	1.40073	1.13403	1.18960	1.17410
1953	1.00653	1.43458	1.27505	1.19044	1.46867	1.20759	1.28618	1.30363
1954	1.08757	1.45362	1.30356	1.20612	1.52688	1.13745	1.29928	1.18144
1955	1.10315	1.51120	1.34277	1.23835	1.58086	1.19963	1.33969	1.32313
1956	.99606	1.58186	1.37154	1.29336	1.62929	1.23703	1.41187	1.35013
1957	1.06321	1.64641	1.38010	1.30703	1.72137	1.23985	1.52474	1.35705
1958	1.15619	1.67389	1.39338	1.32699	1.80623	1.16856	1.44656	1.25396
1959	1.30758	1.73430	1.36756	1.30774	1.82065	1.25130	1.54174	1.41250
1960	1.25413	1.78280	1.38025	1.33946	1.81512	1.26358	1.56828	1.40778
1961	1.26328	1.81977	1.37630	1.34319	1.83730	1.24215	1.59152	1.39735
1962	1.26525	1.88531	1.37689	1.34745	1.84933	1.29944	1.65694	1.48606
1963	1.32294	1.93379	1.34737	1.33143	1.87378	1.32191	1.76280	1.59577
1964	1.32798	2.00998	1.38969	1.35197	1.91216	1.35634	1.76720	1.64985
1965	1.40659	2.05539	1.38635	1.37542	1.98212	1.43460	1.81702	1.79327
1966	1.45100	2.13441	1.40102	1.41878	2.10637	1.53611	1.92525	1.90004
1967	1.38617	2.20616	1.39197	1.42428	2.27814	1.55581	2.03881	1.95160
1968	1.49901	2.33869	1.43388	1.43481	2.41485	1.60330	2.08997	2.08377
1969	1.44957	2.46412	1.46481	1.53356	2.52637	1.64705	2.19889	2.10658
1970	1.32464	2.60532	1.45907	1.54758	2.65571	1.57894	2.39503	2.03230
1971	1.20177	2.76025	1.64689	1.54978	2.74952	1.52852	2.30803	2.18852
Average annual growth rate, 1947–1971	0.8	4.3	2.1	1.8	4.3	1.8	3.5	3.3

depreciation allowances and investment tax credits). As shown in the last four columns of table 1, over the same 1947–1971 time period, the quantities of E and K demanded, grew at a faster rate than the quantities of M and L. In table 2 we present total factor cost and cost shares of the four inputs. The average cost shares of the inputs are approximately 0.0535 for K, 0.2744 for L, 0.0448 for E, and 0.6272 for M.

Our final data construction involves formation of instruments to be used in our I3SLS regressions. We regress the logarithms of the price ratios P_L/P_K, P_E/P_K, and P_M/P_K on ten variables considered exogenous to the United States manufacturing sector, using the fitted values as instruments.[7] The R^2 estimates from these first-stage regressions are 0.8839, 0.7648, and 0.7661, respectively.

IV. Empirical Results

In table 3 we present I3SLS estimates of the KLEM translog cost function for United States manufacturing, 1947–1971, with linear homogeneity in prices imposed. The conventional R^2 figures (computed as one minus the ratio of the residual sum of squares to the total sum of squares in each equation) are 0.4736 for the K equation, 0.8204 (L), 0.6714 (E), and 0.6161 (M). The Durbin-Watson statistics are 1.3087 (K), 2.1516 (L), 1.1907 (E), and 1.8517 (M).[8]

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[7] We assume that the following ten variables are exogenous variables in a more complete but unspecified model of the United States economy: (1) United States population, (2) United States population of working age, (3) effective rate of sales and excise taxation, (4) effective rate of property taxation, (5) government purchases of nondurable goods and services, (6) government purchases of durable goods, (7) government purchases of labor services, (8) real exports of nondurable goods and services, (9) real exports of durable goods, and (10) United States tangible capital stock at the end of the previous year. These exogenous variables have been employed in Berndt (1974) and Berndt-Christensen (1973b, 1974) and are tabled 1929–1968 in (1973b). We are grateful to L. R. Christensen for providing us with revised and updated data through 1971.

[8] The determinant of the estimated Ω is .35268 E-11. The null hypothesis of no autocorrelation against the alternative of a non-zero diagonal autocovariance matrix cannot be re-

TABLE 2. — TOTAL COST AND COST SHARES OF CAPITAL, LABOR, ENERGY, AND OTHER INTERMEDIATE MATERIALS — U.S. MANUFACTURING 1947–1971

Year	Total Input Cost[a]	Cost Shares K	L	E	M
1947	182.373	.05107	.24727	.04253	.65913
1948	183.161	.05817	.27716	.05127	.61340
1949	186.533	.04602	.25911	.05075	.64411
1950	221.710	.04991	.24794	.04606	.65609
1951	255.945	.05039	.25487	.04482	.64992
1952	264.699	.04916	.26655	.04460	.63969
1953	291.160	.04728	.26832	.04369	.64071
1954	274.457	.05635	.27167	.04787	.62411
1955	308.908	.05258	.26465	.04517	.63760
1956	328.286	.04604	.26880	.04576	.63940
1957	338.633	.05033	.27184	.04820	.62962
1958	323.318	.06015	.27283	.04836	.61866
1959	358.435	.06185	.27303	.04563	.61948
1960	366.251	.05788	.27738	.04585	.61889
1961	366.162	.05903	.27839	.04640	.61617
1962	390.668	.05578	.28280	.04530	.61613
1963	412.188	.05601	.27968	.04470	.61962
1964	433.768	.05452	.28343	.04392	.61814
1965	474.969	.05467	.27996	.04114	.62423
1966	521.291	.05460	.28363	.04014	.62163
1967	540.941	.05443	.28646	.04074	.61837
1968	585.447	.05758	.28883	.03971	.61388
1969	630.450	.05410	.29031	.03963	.61597
1970	623.466	.05255	.29755	.04348	.60642
1971	658.235	.04675	.28905	.04479	.61940

[a] Billions of current dollars.

A cost function is well-behaved if it is concave in input prices and if its input demand functions are strictly positive. The translog cost function does not satisfy these restrictions globally. We must check our fitted translog form for positivity and concavity at each observation. Positivity is satisfied if the fitted cost shares are positive. We check the fitted cost shares based on the I3SLS parameter estimates and find that the positivity conditions are satisfied at each annual observation. Concavity of the cost function is satisfied if the Hessian matrix, based on the I3SLS parameter estimates, is negative semidefinite; we find that the concavity condition is also satisfied at each of our annual observations. We conclude that our estimated KLEM translog cost function is well-behaved in the region including the data for United States manufacturing, 1947–1971.

TABLE 3. — I3SLS PARAMETER ESTIMATES OF KLEM TRANSLOG COST FUNCTION — U.S. MANUFACTURING 1947–1971 (asymptotic t-ratios in parentheses)

Parameter	Estimates	Parameter	Estimates
α_K	.0564 (36.571)	γ_{KM}	−.0153 (−1.301)
α_L	.2539 (112.359)	γ_{LL}	.0739 (10.159)
α_E	.0442 (38.078)	γ_{LE}	−.0043 (−1.438)
α_M	.6455 (173.348)	γ_{LM}	−.0697 (−5.942)
γ_{KK}	.0254 (3.455)	γ_{EE}	.0214 (2.393)
γ_{KL}	.0001 (.022)	γ_{EM}	−.0068 (−.527)
γ_{KE}	−.0102 (−2.444)	γ_{MM}	.0918 (3.243)

To measure factor substitution possibilities we compute the estimated Allen partial elasticities of substitution (σ_{ij}) and price elasticities (E_{ij}) as formulated in (4), (5), and (6). Because the estimated σ_{ij} are rather stable over the 1947–1971 time period, we present in table 4 estimated σ_{ij} for only six selected years; E_{ij} appear in table 5.

TABLE 4. — I3SLS ESTIMATED ALLEN PARTIAL ELASTICITIES OF SUBSTITUTION TRANSLOG COST FUNCTION — U.S. MANUFACTURING 1947–1971

AES	1947	1953	1959	1965	1971
σ_{KK}	−8.74	−8.83	−8.75	−8.72	−8.83
σ_{LL}	−1.79	−1.71	−1.66	−1.61	−1.53
σ_{EE}	−10.69	−10.63	−10.70	−10.70	−10.66
σ_{MM}	−.33	−.35	−.36	−.38	−.39
σ_{KL}	1.01	1.01	1.01	1.01	1.01
σ_{KE}	−3.09	−3.25	−3.14	−3.22	−3.53
σ_{KM}	.58	.53	.56	.56	.49
σ_{LE}	.61	.65	.64	.64	.68
σ_{LM}	.57	.59	.59	.60	.61
σ_{EM}	.76	.77	.75	.74	.75

Several important conclusions emerge from tables 4 and 5: (i) energy demand is responsive to a change in its own price — the own price elasticities E_{EE} are about −.47; (ii) energy and labor are slightly substitutable — the estimated σ_{LE} are about 0.65, while the cross-price elasticities E_{LE} and E_{EL} are about 0.03 and 0.18, respectively; (iii) energy and capital display substantial complementarity — the estimated σ_{KE} are about −3.2 while the estimated cross-

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jected; the chi-square statistic is 1.39, while the 0.01 critical value is 6.63. For further details of this test, see Berndt-Savin (1975).

TABLE 5. — I3SLS ESTIMATED PRICE ELASTICITIES OF DEMAND TRANSLOG COST FUNCTION — U.S. MANUFACTURING 1947–1971

Price Elasticity	1947	1953	1959	1965	1971
E_{KK}	−.49	−.46	−.49	−.50	−.44
E_{KL}	.26	.27	.28	.29	.30
E_{KE}	−.14	−.15	−.14	−.14	−.16
E_{KM}	.37	.34	.35	.35	.30
E_{LK}	.06	.05	.06	.06	.05
E_{LL}	−.45	−.46	−.46	−.46	−.45
E_{LE}	.03	.03	.03	.03	.03
E_{LM}	.37	.37	.37	.37	.37
E_{EK}	−.17	−.17	−.18	−.18	−.17
E_{EL}	.16	.17	.18	.18	.20
E_{EE}	−.47	−.49	−.47	−.45	−.49
E_{EM}	.49	.49	.47	.46	.46
E_{MK}	.03	.03	.03	.03	.02
E_{ML}	.15	.16	.16	.17	.18
E_{ME}	.03	.04	.03	.03	.03
E_{MM}	−.21	−.22	−.22	−.23	−.24

price elasticities E_{KE} and E_{EK} are about $-.15$ and $-.18$, respectively; (iv) as found in numerous traditional two-input (capital-labor) studies, capital and labor tend to be quite substitutable — the estimated σ_{KL} are about 1.01, the estimated cross-price elasticities E_{KL} and E_{LK} are about 0.28 and 0.06, and the estimated own price elasticities E_{KK} and E_{LL} are approximately $-.48$ and $-.45$, respectively.

V. Implications of Empirical Results

Virtually all empirical studies of investment demand and capital-labor substitutability have assumed a value added specification of technology. We now consider the validity of such a specification.

The concept of value added has been employed by national income accountants as a device for allocating the origins of national income to the services of the primary inputs capital and labor. Nominal value added is the product $P_V V$,

$$P_V V = P_K K + P_L L,$$

where V is real value added and P_V is the value-added deflator. Given a gross output production function $Y = Y(K, L, E, M)$ or a gross output cost function $G = G(Y, P_K, P_L, P_E, P_M)$, we wish to consider cases in which it is valid to analyze substitution possibilities between K and L while ignoring E and M. Previous researchers have generally made one of the following sufficient assumptions: (a) the quantity ratios E/Y and M/Y always move in fixed proportions (i.e., are perfectly correlated), either because E and M are technologically nonsubstitutable[9] ($\sigma_{EE} = \sigma_{EM} = \sigma_{MM} = \sigma_{KE} = \sigma_{LE} = \sigma_{KM} = \sigma_{LM} = 0$), or because of coincidental shifts in supply and demand. This assumption has traditionally been called the Leontief aggregation condition. In table 6 we present simple correlations between K/Y, L/Y, E/Y, and M/Y.[10] The simple correlation between E/Y and M/Y is 0.3924, which suggests that these ratios have not moved in fixed proportions. We conclude that the Leontief aggregation condition for the value added specification is not satisfied by our United States manufacturing data; (b) that P_E, P_M and P always move in fixed proportions, either because E and M are perfectly substitutable, or because of coincidental shifts of supply and demand.[11] This assumption has traditionally been called the Hicksian aggregation condition. In the second half of table 6 we present simple correlations between the price ratios P_K/P, P_L/P, P_E/P, and P_M/P.[12] If P_E, P_M, and P are perfectly correlated then P_E/P and P_M/P must also be perfectly correlated. The simple correlation between P_E/P and P_M/P is 0.4538, which suggests that for our data P_E, P_M, and P have not moved proportionally. We conclude that the Hicksian aggregation condition for the value added specification is not satisfied by our United States manufacturing data; (c) that the inputs K and L are weakly separable from E, M, i.e., that in the linear homogeneous gross output production function

$$\begin{aligned} Y &= Y_1(K, L, E, M) = Y_2[(K, L), E, M] \\ &= Y_3(V, E, M), \end{aligned}$$

or for its dual unit cost function.

$$\begin{aligned} G/Y &= G_1\,(P_K, P_L, P_E, P_M) \\ &= G_2\,[(P_K, P_L)P_E, P_M] \\ &= G_3\,(P_V, P_E, P_M).^{13} \end{aligned}$$

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[9] For further discussion, see Parks (1971).

[10] The output data are from Faucett (1973).

[11] For a more detailed discussion, see Diewert (1973b).

[12] The output deflator is from Faucett (1973).

[13] This justification for the existence of a value added specification has been discussed by Arrow (1972) in the context of a three input production function — capital, labor, and all intermediate goods. For a further discussion of weak separability, see Berndt-Christensen (1973a) and H. A. J. Green (1964).

TABLE 6. — CORRELATION MATRIX OF INPUT-OUTPUT QUANTITY AND PRICE RATIOS — U.S. MANUFACTURING 1947–1971

	K/Y	L/Y	E/Y	M/Y		P_K/P	P_L/P	P_E/P	P_M/P
K/Y	1.0000	.3933	.8136	.3520	P_K/P	1.0000	.4208	—.1689	.3842
L/Y		1.0000	.3719	.8174	P_L/P		1.0000	—.3029	.3989
E/Y			1.0000	.3924	P_E/P			1.0000	.4538
M/Y				1.0000	P_M/P				1.0000

This condition for the validity of the value added specification can be checked statistically by testing whether the parameter restrictions implied by $[(K, L), E, M]$ weak separability on the translog cost function are satisfied with our KLEM data. For our translog cost function, inputs K and L will be weakly separable from E, M if and only if

$$M_K \gamma_{LE} - M_L \gamma_{KE} = 0$$

and

$$M_K \gamma_{LM} - M_L \gamma_{KM} = 0.$$

Since the M_i are positive, these conditions can be satisfied in two distinct ways:

$$\gamma_{KE} = \gamma_{LE} = \gamma_{KM} = \gamma_{LM} = 0 \tag{7}$$

$$\alpha_K/\alpha_L = \gamma_{KK}/\gamma_{KL} = \gamma_{KL}/\gamma_{LL} = \gamma_{KE}/\gamma_{LE} = \gamma_{KM}/\gamma_{LM}. \tag{8}$$

We refer to (7) as the linear separability restrictions and (8) as the nonlinear separability restrictions.[14]

The assumption of weak separability places restrictions on the AES. For our translog cost function, the linear value added separability restrictions imply that $\sigma_{KE} = \sigma_{LE} = \sigma_{KM} = \sigma_{LM} = 1.0$. Thus, linear separability imposes a partial Cobb-Douglas structure on the translog cost function. The nonlinear value added separability restrictions require that $\sigma_{KE} = \sigma_{LE} \neq 1.0$, $\sigma_{KM} = \sigma_{LM} \neq 1.0$. The two ways in which value added separability can occur for our translog form have quite different implications for the behavior of the cost shares. The linear restrictions imply that the sum of M_K and M_L is a constant, while the nonlinear restrictions imply that the ratio of M_K and M_L is a constant.

We have estimated the KLEM translog cost function input demand equations (3) with first the linear (7) and then the nonlinear (8) restrictions imposed. We obtain a chi-square statistic of 24.88 with the four linear restrictions and 24.21 with the three nonlinear restrictions; the 0.01 critical values are 13.28 and 11.34, respectively. We conclude that the separability conditions for the value added specification are not satisfied by the data for United States manufacturing, 1947–1971.

Virtually all empirical studies of investment demand and capital-labor substitutability in United States manufacturing have assumed a priori that the value added specification is valid. Since the Leontief, Hicksian, and separability conditions for the value added specification do not appear to be satisfied by the data, we must call into question the reliability of investment and factor demand studies for United States manufacturing based on this value added specification.

There is of course no reason to restrict our analysis to the value added specification. Other types of input aggregation may be consistent with our data. From table 6 it appears that no two input/output quantity or price ratios seem to be nearly perfectly correlated. Hence the conditions for Leontief or Hicksian aggregation of any two inputs do not appear to be satisfied. To check whether any of the separability conditions are satisfied, we must perform a number of tests. We have tested for the validity of parametric restrictions implied by all possible types of separability in our KLEM model; the parametric restrictions corresponding with the various types of separability are presented in Berndt-Christensen (1973c, table A1). At the 0.01 level of significance, the only wrong separability condition which our data cannot reject is the $[(K, E), (L, M)]$ linear separability restriction $(\gamma_{KL} = \gamma_{LE} = \gamma_{KM} = \gamma_{EM} = 0)$.[15] In table 7 we present mean I3SLS estimated AES with these separability restrictions imposed.[16]

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[14] For a derivation and further discussion of these separability restrictions, see Berndt-Christensen (1973a,b,c, 1974), and Blackorby, Primont, and Russell (1975).

[15] The chi-square statistic is 9.04 while the 0.01 critical value is 13.28.

[16] The positivity and concavity conditions with these separability restrictions imposed are satisfied at each annual observation.

TABLE 7. — MEAN ESTIMATED AES WITH SEPARABILITY RESTRICTIONS IMPOSED KLEM TRANSLOG COST FUNCTION FOR U.S. MANUFACTURING 1947–1971

	σ_{KK}	σ_{KL}	σ_{KE}	σ_{KM}	σ_{LL}	σ_{LE}	σ_{LM}	σ_{EE}	σ_{EM}	σ_{MM}
Linear $[(K,E)\ (L,M)]$	−11.25	1.00	−6.61	1.00	−1.6	1.00	.55	−12.2	1.00	−.40

Since we are unable to reject the conditions for $[(K,E),\ (L,M)]$ linear separability, we cannot reject the conditions for the consistent aggregation of K and E (heuristically, an index of "utilized capital") and of L and M, i.e.,

$$V = V(K, L, E, M) = V_1\ (K^*, L^*),$$

where K^* is an index of K and E and L^* is an index of L and M; alternatively, the null hypothesis $\sigma_{KL} = \sigma_{KM} = \sigma_{LE} = \sigma_{EM} = \sigma_{K^*L^*} = 1.0$ cannot be rejected.

Our separability results have implications for methods of reliably projecting future energy demand. A large number of energy demand projections have been based on the ratio of energy (in BTU's) to real value added. This projection method would be appropriate only if the value added specification were valid and if all own and cross-price elasticities were zero.[17] Since the conditions for the value added specification are not satisfied and since input demand functions are price responsive, we must conclude that reliable energy demand projections cannot be made on the basis of a value added specification, but must take into account projected output and projected prices of K, L, E, and M.

We conclude by briefly noting some policy implications of our empirical results. Since σ_{EE} and σ_{KE} are negative and σ_{LE} is positive, the lifting of price ceilings on energy types would tend to reduce the energy and capital intensiveness of producing a given level of output and increase the labor intensiveness. Moreover, since investment tax credits and accelerated depreciation allowances reduce the price of capital services, $\sigma_{KE} < 0$ implies that these investment incentives generate an increased demand for capital and for energy. To the extent that energy conservation becomes a conscious policy goal, general investment incentives may become less attractive as fiscal stimulants.

REFERENCES

Allen, R. G. D., *Mathematical Analysis for Economists* (London: Macmillan, 1938), 503–509.

Anderson, K. P., "Toward Econometric Estimation of Industrial Energy Demand: An Experimental Application to the Primary Metals Industry" (Santa Monica: Rand Corporation Report R-719-NSF, Dec. 1971).

Arrow, K. J., "The Measurement of Real Value Added" (Stanford: University Institute for Mathematical Studies in the Social Sciences, Technical Report number 60, June 1972).

Baxter, R. E., and R. Rees, "Analysis of Industrial Demand for Electricity," *Economic Journal* (1968), 277–298.

Berndt, E. R., "Reconciling Alternative Estimates of the Elasticity of Substitution," this REVIEW (forthcoming).

———, and L. R. Christensen, "The Internal Structure of Functional Relationships: Separability, Substitution, and Aggregation," *Review of Economic Studies* (July 1973a), 403–410.

———, "The Translog Function and the Substitution of Equipment, Structures, and Labor in U.S. Manufacturing, 1929–1968," *Journal of Econometrics* (Mar. 1973b), 81–114.

———, "The Specification of Technology in U.S. Manufacturing," Discussion Paper 73-17, University of British Columbia (Nov. 1973c).

———, "Testing for the Existence of a Consistent Aggregate Index of Labor Inputs," *American Economic Review* (June 1974), 391–404.

Berndt, E. R., and N. E. Savin, "Estimation and Hypothesis Testing in Singular Equation Systems with Autoregressive Disturbances," *Econometrica* (forthcoming).

Berndt, E. R., and D. O. Wood, "An Economic Interpretation of the Energy-GNP Ratio," in *Energy: Demand, Conservation, and Institutional Problems,* M. Macrakis (ed.) (Cambridge: MIT Press), chapter 3 (1974).

Blackorby, C., D. Primont, and R. R. Russell, "On the Flexibility of Flexible Functional Forms," University of British Columbia Discussion Paper 75-02 (Jan. 1975).

Christensen, L. R., and D. W. Jorgenson, "The Measurement of U.S. Real Capital Input, 1929–1967," *Review of Income and Wealth* (Dec. 1969), 293–320.

———, "U.S. Real Product and Real Factor Input, 1929–1967," *Review of Income and Wealth* (Mar. 1970), 19–50.

Christensen, L. R., D. W. Jorgenson, and L. J. Lau, "Conjugate Duality and the Transcendental Logarithmic Production Function," *Econometrica* (July 1971), 255–256.

———, "Transcendental Logarithmic Production Frontiers," this REVIEW (Feb. 1973), 28–45.

Darmstadter, J., et al. *Energy in the World Economy,*

---

[17] For a further discussion, see Berndt-Wood (1974).

Resources for the Future, Inc. (Baltimore: Johns Hopkins Press, 1971).

Dhrymes P. J., "Small Sample Asymptotic Relations Between Maximum Likelihood and 3SLS Estimators," *Econometrica* (Mar. 1973), 357–364.

Diewert, W. E., "An Application of the Shephard Duality Theorem: A Generalized Leontief Production Function," *Journal of Political Economy* (May/June 1971), 481–507.

———, "Separability and the Generalized Cobb-Douglas Utility Function" (Ottawa: Department of Manpower and Immigration, Jan. 1973a), mimeo.

———, "Hicks' Aggregation Theorem and the Existence of a Real Value Added Function" (Ottawa: Department of Manpower and Immigration, Jan. 1973b), mimeo.

———, "Applications of Duality Theory" in *Frontiers of Quantitative Economics,* M. Intrilligator and David Kendrick (eds.) (Amsterdam: North-Holland, 1974), vol. 2.

Dupree, W. and J. West, "United States Energy Through the Year 2000," U.S. Department of the Interior (Washington: U.S. Government Printing Office, Dec. 1972).

Jack Faucett Associates, "Data Development for the I-O Energy Model: Final Report" (Chevy Chase, Maryland: Jack Faucett Associates, Inc., May 1973).

Green, H. A. J., *Aggregation in Economic Analysis: An Introductory Survey* (Princeton: Princeton University Press, 1974).

Morrison, W. E., and C. L. Readling, "An Energy Model for the United States," Bureau of Mines, U.S. Department of the Interior, Information Circular 8384 (Washington: U.S. Government Printing Office, 1968).

Mount, T. D., L. D. Chapman, and T. J. Tyrrell, "Electricity Demand in the United States: An Econometric Analysis," ORNL-NSF-EP-49 (Oak Ridge, Tennessee: Oak Ridge National Laboratory (June 1973).

National Energy Board, *Energy Supply and Demand in Canada and Export Demand for Canadian Energy, 1966 to 1990* (Ottawa: Information Canada, 1971).

National Petroleum Council, *U.S. Energy Outlook: An Initial Appraisal, 1971–1985* (Washington: National Petroleum Council, 1–3, 1971).

Parks, R. W., "Price Responsiveness of Factor Utilization in Swedish Manufacturing, 1870–1950," this REVIEW (May 1971), 129–139.

Schurr, S. H., et al. *Energy in the American Economy, 1850–1975,* Resources for the Future, Inc. (Baltimore: Johns Hopkins Press, 1960).

Uzawa, H., "Production Functions with Constant Elasticities of Substitution," *Review of Economic Studies* (Oct. 1962), 291–299.