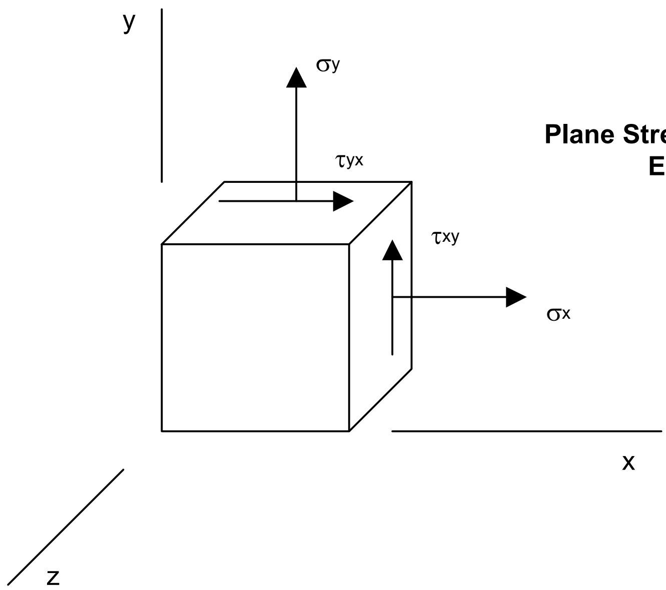
Chapter 7 - ANALYSIS OF STRESS AND STRAIN

Stress and Strain Tensors - Plane Stress and Plane Strain

For the remainder of the chapter, we will be concentrating on **plane stress** conditions. For plane stress, the stress and strain tensors have the following patterns:

$$\begin{bmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_y & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad \begin{bmatrix} \varepsilon_x & \frac{\gamma_{xy}}{2} & 0 \\ \frac{\gamma_{yx}}{2} & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix}$$



Plane Stress on a Stress Element

Notice that although the stresses are confined to the x and y faces of the element (equal and opposite stresses act on the negative x and y faces), there are normal strains which exist on the z face.

Transformation Equations for Plane Stress

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x_1 y_1} = -\frac{(\sigma_x - \sigma_y)}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$$\sigma_{y_1} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

Average, Principal, and Maximum Shear Stresses

$$\sigma_{x_1} + \sigma_{y_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x + \sigma_y}{2} = \sigma_x + \sigma_y$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

$$\sigma_1 = \sigma_{avg} + R$$

$$\sigma_2 = \sigma_{avg} - R$$

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}{}^2} = R$$

Hooke's Law for Plane Stress

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) \qquad \varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) \qquad \varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\sigma_x = \frac{E}{1-\nu^2}(\varepsilon_x + \nu\varepsilon_y) \qquad \sigma_y = \frac{E}{1-\nu^2}(\varepsilon_y + \nu\varepsilon_x)$$

Pressure Vessels

Spherical Stress:

any direction $\sigma = \frac{pr}{2t}$

Cylindrical Stress:

hoop $\sigma = \frac{pr}{t}$ axial $\sigma = \frac{pr}{2t}$

Combined Stresses

beam-columns $\sigma = \frac{P}{A} \pm \frac{My}{I}$. M = Pe for eccentrically loaded columns

Beam Deflections

Integration Method EIv” = -M

Chapter 5 BEAM STRESSES

Relationships between Moment and Curvature and Normal (Bending) Stress and Moment

Based on this statics sign convention,

$$K = \frac{1}{\rho} = -\frac{M}{EI}$$

$$\varepsilon_x = -Ky$$

$$\sigma_x = -EKy = \frac{My}{I} = \frac{M}{S}$$

Moment of Inertia, Section Modulus Formulas

$I = \int y^2 dA \qquad S = \dfrac{I}{y_{extreme\ fiber}}$ composite section $I = \Sigma I_{ci} + \Sigma A(y_i - \bar{y})^2$

Solve for S after computing I for composite sections.

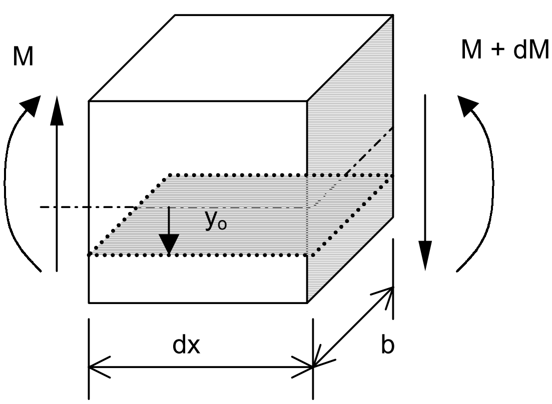
Solid rectangle formulas:

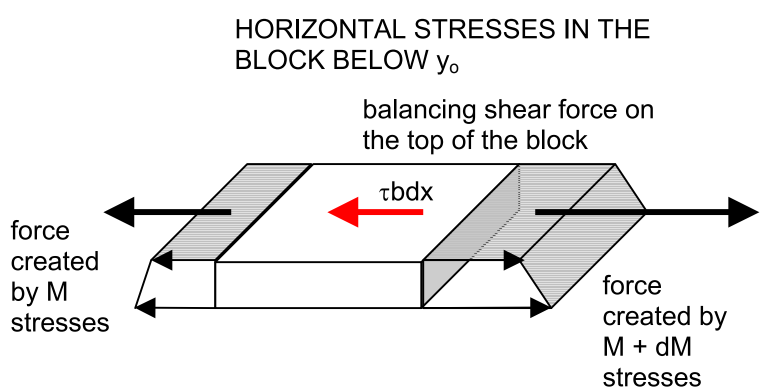
$$I_c = \frac{bh^3}{12} \qquad S = \frac{bh^2}{6}$$

Solid circle formulas:

$$I_c = \frac{\pi r^4}{4} = \frac{\pi d^4}{64} \qquad S = \frac{\pi r^3}{4} = \frac{\pi d^3}{32}$$

Beam Shear Stresses (Horizontal Shear Stress)





Generic beam shear stress formula: $\tau = \dfrac{VQ}{Ib}$, where V is the design shear force, Q is the first moment of the area, taken either above or below the location of shear stress, I is the moment of inertia for the entire cross section of the beam, and b is the beam width at the location of shear stress.

First moment of the area Q (various formulas), $Q = \int y dA = \Sigma(y_{ci} - \bar{y})\Delta A = yA$

Derived Shear Stress Formulas

Solid Simple Rectangle, $\tau = \frac{V}{2I}\left[\frac{h^2}{4} - y^2\right]$ and $\tau_{max} = \frac{3V}{2A}$

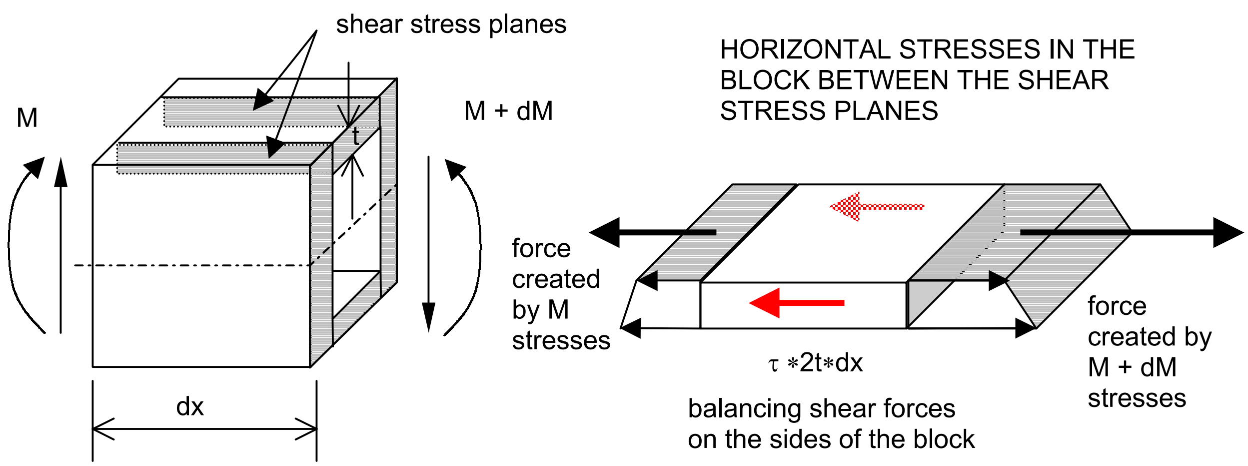
Solid Simple Circle, $\tau_{max} = \frac{4V}{3A}$, and Hollow Simple Circle $\tau_{max} \cong \frac{2V}{A}$

Shear Flow formula, $f = \frac{VQ}{I}$, in units of force/length.

Thin-Walled Sections

$$\tau = \frac{VQ}{It}$$

Built-up Beams



$$\textit{Connection Capacity} \geq \textit{Demand}$$

Built-up beam Table

Fastener Type (example)	**Units**	**Demand**	**Capacity**
area (glue)	force/area (stress)	$\tau = \frac{VQ}{It}$	allowable shear stress
line (weld)	force/length (shear flow)	$f = \frac{VQ}{I}$	allowable shear stress x thickness of fastening line (x2 for double-lines as per plate girder flange welds)
discrete (nails, bolts, screws, rivets, dowels)	force (shear flow x spacing)	$fs = \frac{VQ}{I}s$	allowable shear force on connector (x2 for a box beam or similar connection)