

Chapter 4 BEAM STATICS

Relationships between Load, Shear, Bending Moment

Slope Formulas, with loads (q) positive downward:

$$\frac{dV}{dx} = -q \quad \frac{dM}{dx} = V$$

Area Formulas

$$V_B - V_A = \int_A^B -q dx \quad M_B - M_A = \int_A^B V dx$$

Deformation Sign Convention for Internal Shear and Moment:

- Shear force: positive shear deforms the element down to the right
- Bending moment: positive bending moment deforms the element in a smile

Method A	Method B	Method C	Method D
Cut Section Method	Method of Integration	Direct Method	Method of Superposition
uses regular method of generating equations of equilibrium from a free body diagram	mathematic way of generating equations from the relationships of q, V, M	uses q, V, M relationships to generate graphs directly	combines graphs of V, M for individual load cases into the total V, M diagrams
"foolproof"	useful for analyzing beams with higher order or unusual load equations	good for multi-segmented beams with point and uniform loads	useful for easily generating V, M values or diagrams on the computer
a good way to check answers for the other methods	ties into method for finding beam deflections	needs experience to be used effectively	eliminates the need for complicated if statements in program

Cut Section Method

Can be used two ways:

- Find values for V, M by making the cut at a particular point
- Find equations for V_x , M_x , by making the cut at x in a continuous beam segment. Consider x to be the distance from the left end of the beam and stay consistent.

Chapter 3 TORSIONAL LOADING

Geometric properties of a hollow circle

$$A = \pi(r_o^2 - r_i^2) \qquad J = I_p = \pi/2(r_o^4 - r_i^4)$$

Torsion Load/Deformation Equations

$$\text{Basic L/D Equation (circular bar)} \quad \phi = \frac{TL}{GJ}, \text{ or } \phi = \frac{TL}{GI_p}$$

$$\text{Discrete, or Stepwise L/D Equation} \quad \phi = \sum_{i=1}^n \frac{T_i L_i}{G_i J_i}$$

$$\text{Continuous, or Generalized L/D Equation} \quad d\phi = \frac{T_x dx}{GJ_x} \text{ and } \phi = \int_0^L \frac{T_x dx}{GJ_x}$$

Shear Stress

$$\text{Maximum shear stress equation: } \tau = \frac{Tr}{J} \text{ or } \tau = \frac{Tr}{I_p}, \text{ where}$$

τ = shear stress

T = internal torsion

r = radius of the bar

J = I_p = cross-sectional polar moment of inertia

stress units Pa = N/m², psi, ksi

Power Transmission

Basic equation for constant torque, angular velocity $P = T\omega$, where:

$$P = \text{power delivered or consumed, in horsepower, } \frac{ft-lb}{s}, \text{ or watt } W = \frac{N-m}{s}$$

$$\text{conversions from horsepower H: } 1H = 550 \frac{ft-lb}{s} \approx 746W$$

ω = angular velocity in rad/s (when x-ing out units, radians are unitless (except to other radians))

T = torque delivered or consumed by the motor or machine

Stresses on inclined sections - Torsion

$$\sigma_\theta = \tau_{xy} \sin 2\theta = (Tr/J) \sin 2\theta \qquad \tau_\theta = \tau_{xy} \cos 2\theta = (Tr/J) \cos 2\theta$$