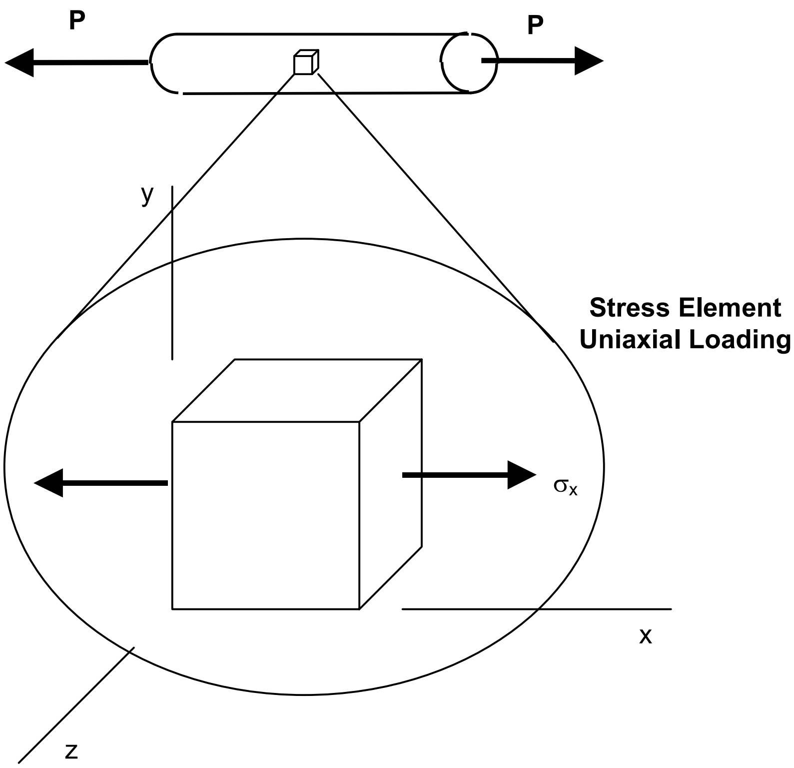
Chapter 1 - Introduction

Normal Stress and Strain

Normal stress and strain are introduced with a uniaxially loaded bar, which is prismatic and has an axially applied load in alignment with the bar centroid, i.e.:



Formulas for Normal Stress and Strain:

σ **= P/A,**

where
σ = normal stress
P = internal axial force
A = cross-sectional area,
units Pa = N/m^2, psi, ksi

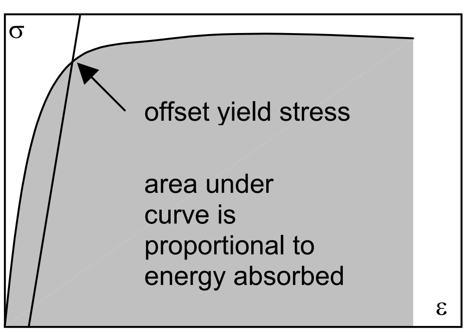
ε **=** δ**/L,**

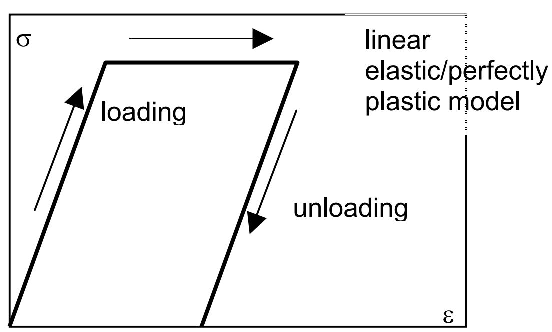
where
ε = normal strain
δ = change in length
L = original length
unitless

Deformation sign convention:
Tension, elongation are positive; compression, shortening are negative.

Stress/Strain Diagrams, Hooke's Law

$\sigma_x = E\varepsilon_x$, Hooke's Law for uniaxial loading and normal stress and strain, from the linear elastic section of the stress/strain curve



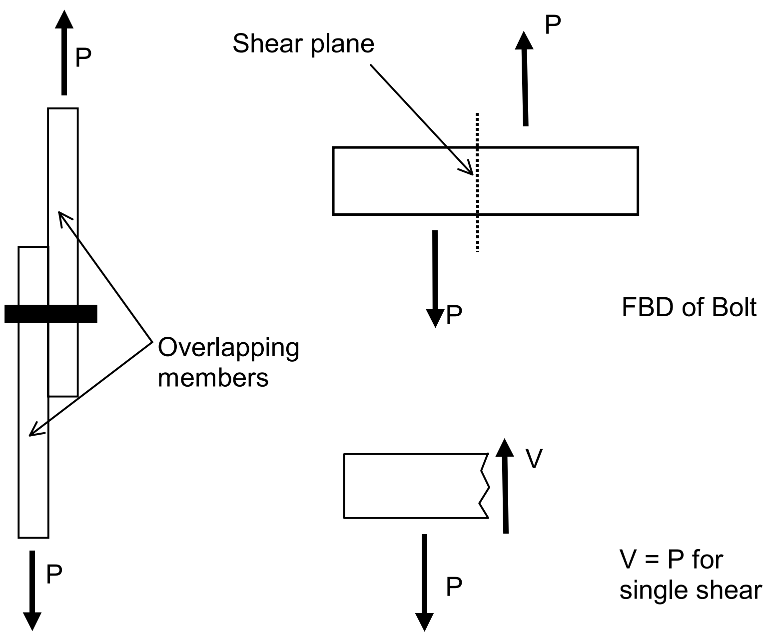


E, the **modulus of elasticity**, is a constant of proportionality between stress and strain. It is also the slope of the stress/strain curve in the linear elastic region. Units are same as stress (GPa, ksi)

Shear Stress and Strain

Direct shear problems are those which have a high internal shear force to bending moment ratio. Shear stress is generally assumed to be uniformly distributed. A classic direct shear problem is that of a pin connecting two plates.

SINGLE SHEAR LAP JOINT



Formulas for Shear Stress and Strain:

τ = V/A,

where
τ = shear stress
V = internal shear force
A = cross-sectional area of the shear plane,
units Pa = N/m^2, psi, ksi

γ,

where
γ = shear deformation angle, radians (unitless)

τ = Gγ, Hooke's Law for shear with the shear modulus G in shear units.

You can also have the load P divided into any number of shear planes, depending on how many pieces are connected.

You can also have to check bearing stress in a pin-type **multiple criteria problem:**

bearing stress $\sigma = P/A_p$, where Ap = projected area of pin on plate, td
t = thickness of plate
d = pin diameter

Factor of Safety n = capacity/demand

Thermal Strain

$$\varepsilon_T = \alpha \cdot \Lambda T$$

Strain/Displacement

$$\delta = \int_0^L \varepsilon(x)dx$$

Chapter 2 – Axially Loaded Members

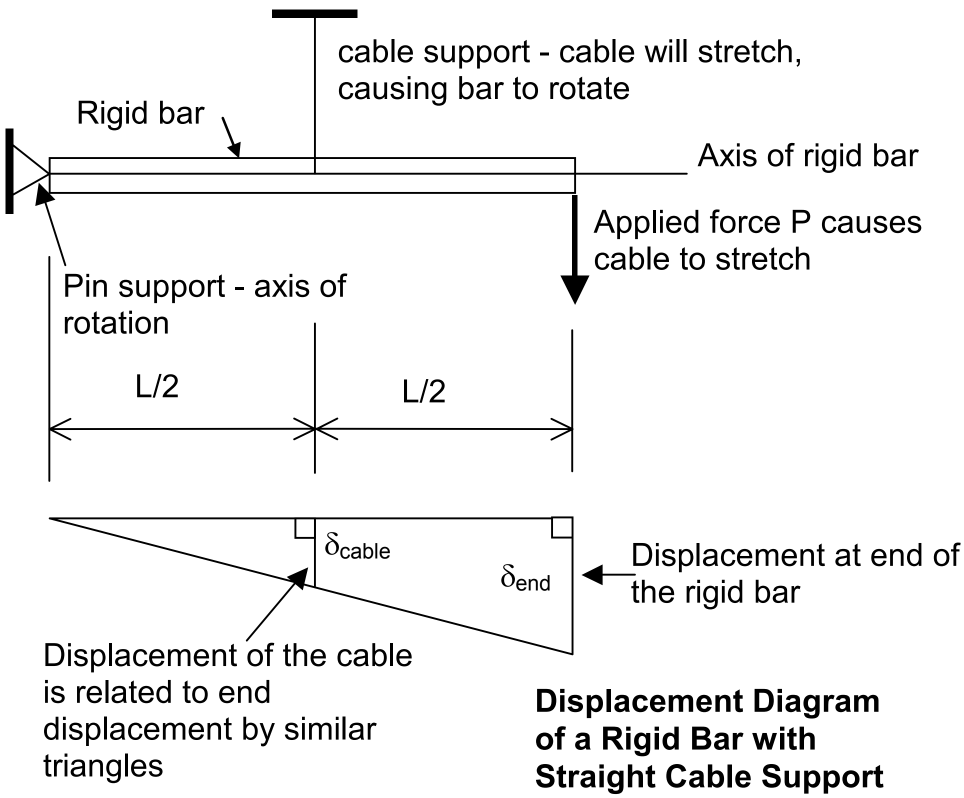
Axial Load/Deformation Equations

Basic L/D Equation (simple bar) $\delta = \dfrac{PL}{EA}$

Discrete, or Stepwise L/D Equation $\delta = \sum_{i=1}^{n} \dfrac{P_i L_i}{E_i A_i}$

Continuous, or Generalized L/D Equation $d\delta = \dfrac{P_x dx}{EA_x}$ and $\delta = \int_0^L \dfrac{P_x dx}{EA_x}$

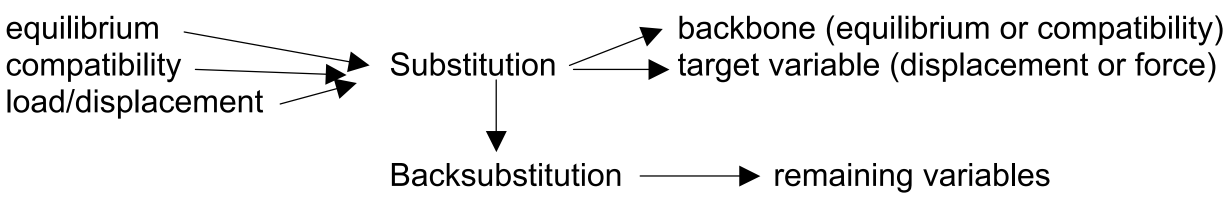
Compatibility/Displacement Diagrams

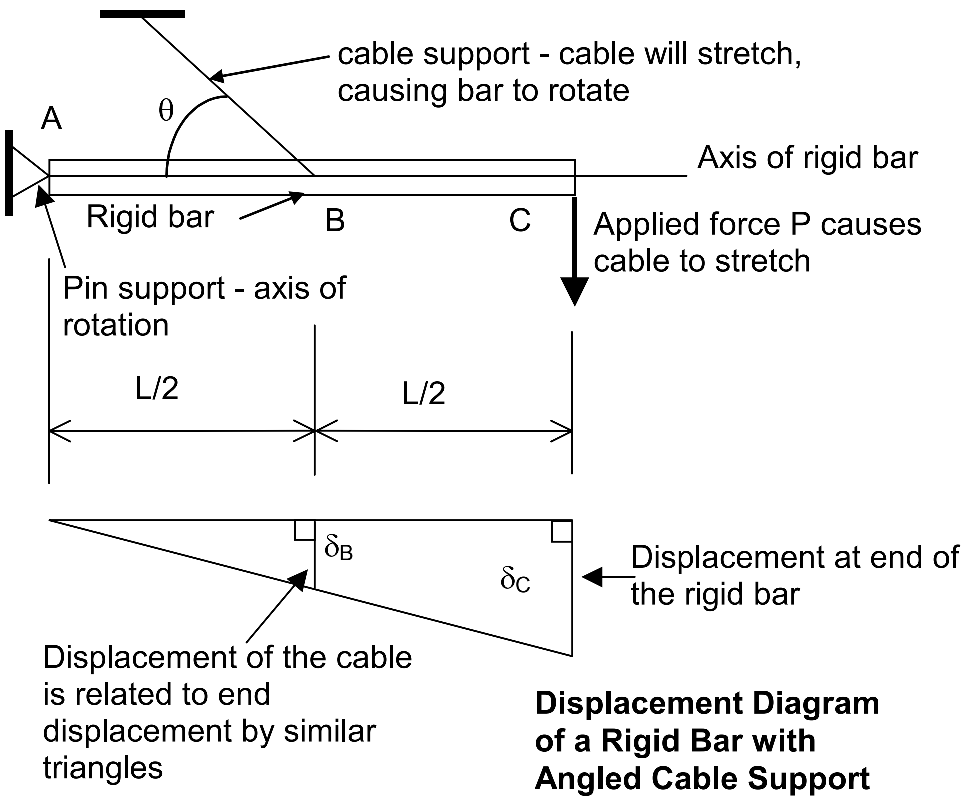


Displacement Diagram of a Rigid Bar with Straight Cable Support

Compatibility equation is $\delta_{cable} = \delta_{end}/2$.

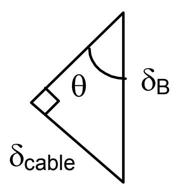
Indeterminate Problems





Displacement Diagram of a Rigid Bar with Angled Cable Support

Relationship between Cable and Bar Displacements



δ_{cable} is computed from L/D equation

Spring Equation

$F = k\Delta$ where
- F = force in spring
- k = spring stiffness (force/length)
- Δ = spring displacement

Geometric properties of circular shapes

Solid circles
$A = \pi r^2$
$J = I_p = \pi/2 * (r^4)$

Hollow circles
$A = \pi(r_o^2 - r_i^2)$
$J = I_p = \pi/2 * (r_o^4 - r_i^4)$

Stresses on inclined sections – Uniaxial loading

$\sigma_\theta = \sigma_x \cos^2\theta = P/A \cos^2\theta$

$\tau_\theta = -\sigma_x \sin\theta \cos\theta = -P/A \sin\theta \cos\theta$

Relationship between E and G: $G = \dfrac{E}{2(1+\upsilon)}$