VECTOR ALGEBRA

Vector Addition, Three Dimensions

$\vec{F} = F_X\hat{i} + F_Y\hat{j} + F_Z\hat{k}$ Cartesian components of a vector **F**,
where $F_X = F\cos(\theta_X), F_Y = F\cos(\theta_Y), F_Z = F\cos(\theta_Z)$.

$\vec{R} = \vec{F} + \vec{S} = (F_X + S_X)\hat{i} + (F_Y + S_Y)\hat{j} + (F_Z + S_Z)\hat{k}$ Vector addition by components.

$R = \sqrt{(R_X)^2 + (R_Y)^2 + (R_Z)^2}, \cos(\theta_X) = \frac{R_X}{R}, \cos(\theta_Y) = \frac{R_Y}{R}, \cos(\theta_Z) = \frac{R_Z}{R}$

Vector Addition, Two Dimensions

$\vec{F} = F_X\hat{i} + F_Y\hat{j}$ Cartesian components of a vector **F**,
where $F_X = F\cos(\theta), F_Y = F\sin(\theta_Y)$, where θ is the angle from the **horizontal**.

$\vec{R} = \vec{F} + \vec{S} = (F_X + S_X)\hat{i} + (F_Y + S_Y)\hat{j}$ Vector addition by components.

$R = \sqrt{(R_X)^2 + (R_Y)^2}, \tan(\theta) = \frac{R_Y}{R_X}$

Unit vector λ

$\lambda = \cos\theta_X\hat{i} + \cos\theta_Y\hat{j} + \cos\theta_Z\hat{k}$ The unit vector is **unit-less**!

Direction vector with start point A and endpoint B.
$\vec{D}_{AB} = \langle B_X - A_X, B_Y - A_Y, B_Z - A_Z \rangle$ Make sure you keep start point and endpoint consistent.

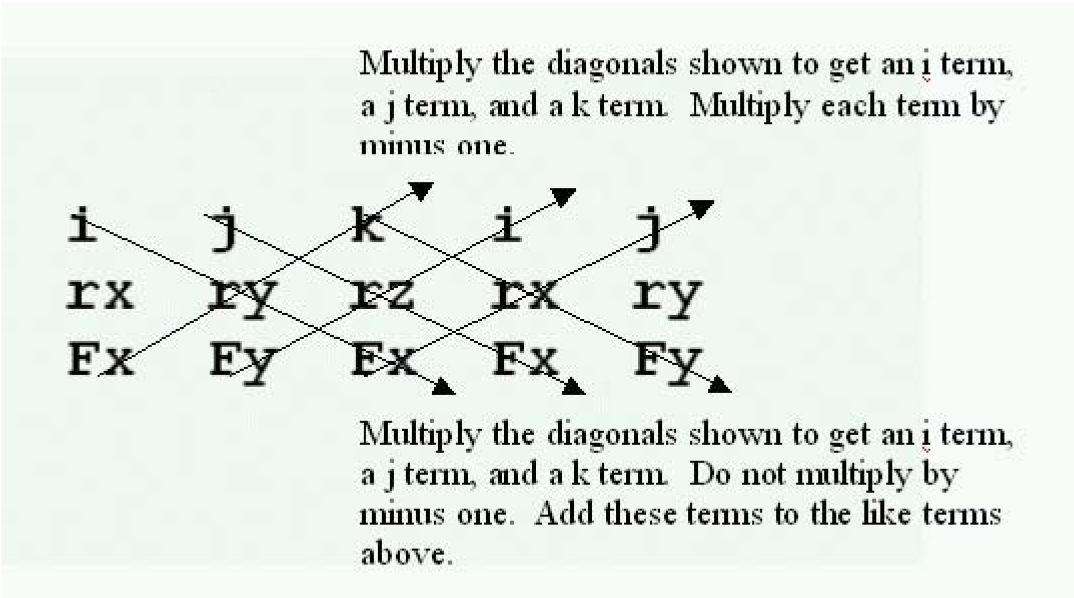
Vector Cross Product (Vector Product)

V = **P** X **Q**, V = PQ sin θ, where θ is the angle between the two vectors **V** and **Q**.

$V_X = P_YQ_Z - P_ZQ_Y$
$V_Y = P_ZQ_X - P_XQ_Z$
$VZ = P_XQ_Y - P_YQ_X$

, or use the determinate expansion method: $V = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ P_X & P_Y & P_Z \\ Q_X & Q_Y & Q_Z \end{vmatrix}$

For the case of finding the cross product for moments:



Shortest distance between a point C and a line AB using the cross product:

$d = \frac{|\vec{AB} \times \vec{AC}|}{|\vec{AB}|}$, where AC is any vector between the point C and the line AB (you could have used BC instead of AC).

Vector Dot Product (Scalar Product)

S = **V * Q** is a scalar whose magnitude is VxQx +VyQy + VzQz.

Also S = VQ cos θ, where θ is the angle between the two vectors **V** and **Q**.

To find the angle between 2 vectors, find cos q = (**V * Q**)/VQ.

Mixed Triple Product

The mixed triple product is a scalar found by doing this to 3 vectors:

M = **S * (P X Q),** which can be done as one step in the determinant expansion of the matrix formed from the 3 vectors:

$$M = \begin{vmatrix} S_X & S_Y & S_Z \\ P_X & P_Y & P_Z \\ Q_X & Q_Y & Q_Z \end{vmatrix}$$

Also, **S * (P X Q) = Q * (S X P) = P * (Q X S)**

MECHANICS EQUATIONS

Newton's Laws

Law One – Equilibrium. Here are the vector equations:

$\sum \vec{F} = 0$ Static rigid body equilibrium, particle equilibrium
$\sum \vec{M} = 0$ Static rigid body equilibrium

These can be broken down into the scalar equations:

$$\sum F_X = 0, \sum F_Y = 0, \sum F_Z = 0$$
$$\sum M_X = 0, \sum M_Y = 0, \sum M_Z = 0$$

Law Two – Acceleration F = m**a.**

The SI mass to weight conversion formula is an example of Newton's second law:

$\vec{W} = m \cdot \vec{g}$. The conversion factor is 9.81 N/kg.

Law Three – Action/Reaction

Action/Reaction pairs of forces at contact points have the following properties:

- Equal in magnitude
- Opposite in direction
- Same line of action

Moments – Definition:

Mo = **r** X **F,** moment of a force about a point. Also Mo = rF sin θ = Fd

where **r** is the distance vector between the axis of rotation o and the line of action of **F**.
F is the force vector. d is the shortest distance between o and the line of action of **F**.

Moment of a couple M = Fd, where d is the shortest distance separating the forces **F** and **–F.**

To find the component of the moment about a particular axis, which is a scalar, use the mixed triple product:

$$M = \begin{vmatrix} \lambda_X & \lambda_Y & \lambda_Z \\ r_X & r_Y & r_Z \\ F_X & F_Y & F_Z \end{vmatrix}$$

, where l is the unit vector for the particular axis, and r is the distance vector between the axis and the line of action of the force (any two points on those lines will work).

Equilibrium Problems

When presented with an equilibrium problem (i.e., one in which some object or system of objects are “at rest” or in constant straight line motion), first determine whether the problem is a **concurrent force system** or not. Concurrent force systems (including three force members) can be solved using **particle equilibrium** methods (i.e., no moment equations).

When solving for the support reactions of a rigid body system in equilibrium, it is often helpful to select a sequence of equations which have the fewest number of unknowns in each equation. The first equation is usually a moment equation, taken about some point where many of the forces in the system meet. For three dimensional systems, the first equation is often taken about some axis through which most of the forces pass or are parallel to.

Supports – 2D

One unknown – Link or Roller
Two unknowns – Pin or Hinge
Three unknowns – Fixed

Supports – 3D

One unknown – 3D Roller, leg on smooth surface
Two unknowns – 2D Roller (non-rotating caster)
Three unknowns – Ball and socket, leg on rough surface
Four unknowns - Universal joint, single hinge without flange, hinge pair without flanges
Five unknowns – Single hinge with flange, hinge pair with one flange
Six unknowns – Fixed support