G 326 term project ideas

1. decay chains: coupled ODEs

   In the lab we set up a very simple decay series of a parent and one daughter. A term project could build on that start to simulate a longer decay series. Three examples:

   - Industrial refining of uranium ore produced a radioactive waste product U234. You could develop a model to simulate the decay fo the waste and the energy it releases. Youll need to identify a decay series (there is more than one option) and appropriate constants, define how much U234 is stored in a typical waste drum, and the energy released at each decay step. Think about how to handle isotopes with very different half lives.
   - U and Th decay series contain isotopes with distinct geochemical properties. When those properties result in the separation of parent and daughter isotopes (processes collectively called fractionation), the resulting disequilibrium starts a chronometer that be used to study the physical processes involved in the fractionation. Isotopes in the U and Th series have a large range of decay constants, allowing processes that transpire over a wide range of time scales to be studied. U-series disequilibria are used to study such processes as magma differentiation, groundwater flow, and ocean processes. A good place to start in thinking about a project on this topic is the website of the Isotopes and the Environment group at the University of Oxford:
     http://www.earth.ox.ac.uk/ gideonh/research/research.htm

2. linear reservoirs: coupled ODEs

   One of our homework problems considered contaminant content and flow through a single, well-mixed reservoir. That framework can be extended to consider a series of reservoirs. Such linear reservoirs are used to build simple models that often yield significant insights into physical systems. Some possible linear reservoir problems:

   - In alpine settings, subglacial meltwater drains usually drains via linked cavities or conduits. Water moves through these two types of drainage systems differently, with different consequences for glacier flow. One way to study the nature of the subglacial rainage system is to pour dye into a surface conduit that is believed to be connected to the subglacial drainage system and monitor the timing and concentration of its arrival in streams that emerge at the glacier snout. A series of linked linear reservoirs can be used to model a linked cavity system. A possible project would be to build such a model with several linked cavities and compare its predictions with observations from a glacier dye test. You will need to come up with an estimate for the subglacial water flow rate. I can provide plots of dye discharge from actual experiments.

- A linear reservoir framework could also be built to simulate contaminant delivery to the Willamette River via the city sewer system (reservoirs with water flowing through them) due to a storm event (which can be seen as the equivalent to the dye injection in the above description). If you choose this project you will need to come up with estimates for surface runoff volume and flow rate into the sewer system fr the storm event, sewer system volume and flow rate (is one reservoir adequate or does the system work as a series?), and the concentration of surface contaminants that enter the sewer system during a storm.
- Federal and state governments set limits for acceptable contaminant levels in various reservoirs (the atmosphere, lakes, streams, soils, and so on). Polluters (industry, municipalities, and so on) are then mandated by law to ensure that their emissions dont result in contaminant levels that exceed the specified levels. Suppose you are the environmental engineer at the pulp mill we considered in a homework problem earlier this term. Design a model that you could use to tell your boss what concentration of a contaminant in the mill effluent would be acceptable for a range of assumptions about the timing and magnitude of total water flow into and out of the lake. You can use a pretty simple linear reservoir model to study the effect of different seasonal river flow cycles on contaminant concentration and design an emission strategy that would minimize impact on the lake and its residents.

3. predator-prey relationships: coupled ODEs

Relationships among populations was another application of coupled ODEs presented in Chapter 3 of your lab manual. In that chapter, the a logistic equation for a population c was discussed, as were the **Lotka-Volterra** equations for a simple predator-prey relationship:

$$\frac{\mathrm{d}c_1}{\mathrm{d}t} = c_1\,(\alpha - \beta c_2) \tag{1}$$

$$\frac{\mathrm{d}c_2}{\mathrm{d}t} = -c_2\,(\gamma - \delta c_1) \tag{2}$$

in which the negative tells you that c_2 is the predator and the parameters α, β, γ, and δ describe interactions between the two organisms. There is no carrying capacity in these equations but one could be added.

A project could be formulated to investigate the stability of models with more than two species and to build a predator-prey model that includes carrying capacity.

4. oscillators

   Cycles emerge in natural systems for many reasons. In some cases, for example seasonal temperature variations, the cycle is imposed by an external forcing (in this case, the inclination of Earth's spin axis together with Earth's orbit around the Sun). In other cases, for example predator-prey population dynamics, cycles arise internally, due to non-linearities in the relationships among components of a system. A number of interesting oscillator problems have been developed to explain cycles in geologic systems, for example

   - Fitzhugh-Nagumo equations have been used to investigate spiral troughs on Marian ice caps. (Pelletier, *Geology*, 2004)
   - Spring and block arrangements are used to investigate stick-slip behavior along faults. (Carlson and Langer, *Physical Review A*, 1989)
   - Glacial to interglacial cycles have been simulated using fedbacks among albedo, precipitation, and ice sheet size in reduced order models. (Ashkenazy and Tziperman, *QSR*, 2004).

5. diffusion equations: second-order PDEs (or ODEs for steady-state)

   Diffusion is a fundamental concept for many problems in Earth science. Mathematically, it is the time-change in some conserved quantity (for example, temperature or salinity), driven by a spatial gradient in the magnitude of that quantity.

   In one dimension of a cartesian coordinate system, a diffusion equation is written:

$$\frac{\partial \phi}{\partial t} = -\frac{\partial}{\partial x}\left(-K\frac{\partial \phi}{\partial x}\right) \tag{3}$$

   where ϕ represents the conserved quantity, t represents time, x represents the one dimension, and K is a material property called the diffusion coefficient, or diffusivity. If the diffusivity is isotropic:

$$\frac{\partial \phi}{\partial t} = K\frac{\partial^2 \phi}{\partial x^2} \tag{4}$$

   The partial derivative indicates that the dependent variable varies with more than one independent variable. If the system of interest has attained steady state, the time derivative is zero and the equation becomes a second order ordinary differential equation.

- Diffusion equations are found in geochemistry, hydrology, hydrogeology, glaciology, electrostatics and magnetostatics and many other subjects within earth science. A term project could involve finding a diffusion equation that applies to a topic of interest to you (magma cooling, thermal cycles, chemical diffusion in a crystal, asthenosphere response to crustal loading etc.) and solving it for a specified set of boundary conditions.

- Many sediment diagenesis problems involve diffusion. A relatively simple example involving sea floor sediments considers the relationship between methane and sulfate content of un-bioturbated sediments, investigated by Martens and Berner, 1977 (Interstitial Water Chemistry of Anoxic Long Island Sound Sediments. 1. Dissolved Gases, *Limnology and Oceanography*, 22 (1), 10-25). Those authors were interested in bacterial activity in anoxic marine sediments and observed that in core samples where methane content was high, sulfate content was low. They developed some simple mathematical models to test their ideas regarding microbial activity. You could reproduce their analysis of bacterial consumption of methane using the steady state diagenetic equaiton:

$$D\frac{\partial^2 C}{\partial z^2} - \omega\frac{\partial C}{\partial z} - kC = 0 \tag{5}$$

  in which D represents the methane diffusion coefficient, C represents the methane concentration at any depth z, ω represents the sediment burial rate, and k represents a consumption rate. I have a copy of the Martnes and Berner paper if you are interested. This is a general class of problem, you could find other examples as well.

- The dominant heat sources within Earth's crust are diffusion from the mantle to the surface and the decay of radiogenic isotopes concentrated within the crust. Measurements of heat flux through the surface of continental crust and of the heat produced by radiogenic decay in rock samples from near the top of the crust indicate that radiogenic isotopes must be concentrated near the surface (this is due to redistribution by partial melting)

  In one (vertical) dimension, the depth-variation in radiogenic heat production can be expressed:

$$\frac{dH}{dz} = -\frac{H(z)}{h_r} \tag{6}$$

  where H represents the heat production per unity mass at any depth z and h_r is an empirically derived scaling constant. If the crustal temperature is at steady state, which would be true for old continental crust, the heat produced within the crust (according to equation (6)) and the heat diffused through the crust must balance:

$$k\frac{d^2T}{dz^2} + \rho H = 0 \tag{7}$$

in which T represents the temperature of the crust k is a thermal diffusivity constant, and H varies with z. dT/dz is the geothermal gradient and kdT/dz is the heat flux due to diffusion. The heat flux q due to radiogenic heating is computed:

$$q = H\rho h \tag{8}$$

where ρ is the density of the crust and h is its thickness. Heat flux into the base of the crust from the mantle must also be defined and used as a boundary condition.

Using this general framework and some measured variables, a model can be built to compute the continental geotherm. Start by deriving an analytical solution equation (6) that can be used in equation (7) if $H = H_s$ is measured at the surface $z = 0$. Integration with respect to z of the resulting equation, using the boundary condition that $q = q_m$ as z goes to infinity, results in an ODE that can be solved numerically with measured ρH_s , an estimated q_m, and an empirically-derived h_r. When you have derived the governing equation, let me know and Ill find some values for those quantities.