Chapter 3

An ice sheet model

3.1 introduction

Ice sheets are key components of Earth's climate system. They contain nearly all of the planet's fresh water, changes in their volume have an immediate effect on sea level, changes in their area and surface characteristics affect global albedo, and they play a role in the circulation of both the atmosphere and the ocean. Through the latter half of the Quaternary, ice sheets have modulated the planetary response to orbitally-driven insolation cycles. Looking forward, the Greenland and Antarctic ice sheets have the potential to play important roles in climate change. In this lab we will use a numerical ice sheet model to examine ...

A numerical model is an equation, or system of equations, written in such a way that it can be solved approximately, using appropriate initial and boundary conditions. Such models may be very simple (for example, a model to simulate permafrost temperature given an annual surface temperature cycle) or very complicated (as in the general circulation model EdGCM). The model we will use in the lab, called GLIMMER, is a set of programs and climate "drivers" built over many years by a research group at the University of Bristol, led by Tony Payne. Recently, a collaborative project has created a user interface that makes GLIMMER accessible to non-specialists.

3.2 conservation equations

The mathematical descriptions of physical systems we encounter in Earth science problems have at their hearts statements requiring the conservation of energy, momentum, or mass. We see this fundamental principle written in many different forms for many different quantities but strictly, they all derive from one place.

Conservation involves the change in a quantity ϕ (with dimension of *something*/L^3) in an arbitrary inertial volume V enclosed by a surface S. A unit vector $\hat{n}$ normal to S, defined to be positive outward, is used to identify the regions of space within and outside of S.

The value of ϕ within V may change over time t if there is a flux through S or creation of ϕ within V. The flux may have two parts, one due to diffusion and another due to advection. The change in ϕ within V is written:

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \phi \mathrm{d}V \;=\; -\int_S \mathbf{F}\cdot\hat{n}\ \mathrm{d}S \;-\; \int_S \phi\mathbf{V}\cdot\hat{n}\ \mathrm{d}S \;+\; \int_V H\mathrm{d}V \tag{3.1}$$

where $\mathbf{F}$ represents the flux due to diffusion, $\phi\mathbf{V}$ represents advection of ϕ in a fluid flow with velocity $\mathbf{V}$, H represents a source (or sink) of ϕ within V and boldface indicates a vector-valued quantity. The negative signs in front of the first two terms on the right-hand side of (3.1) indicate that an outward flux results in a decrease of ϕ in the volume enclosed by S.

Equation (3.1) is a statement of conservation of ϕ for the unit volume V. This statement is always true, independent of the size of V and even if the fields enclosed by S are not continuous. This is the case because we integrate over the whole volume. It is important to note, however, that the integration also means that information on spatial scales smaller than V is not available to us.

One way to visualize the importance of the integration in equation (3.1) is to consider rice being poured from a bag. The rice is a discontinuous mixture of grains and air. If we focus in on a small volume of the flowing rice, in any instant of time we might see rice, we might see air, or we might see a mixture of the two. If we zoom out, and look again, we see the flow. If we integrate over that larger volume, the pouring rice looks like a fluid flow but we can't say anything about what is happening to individual grains.

In order to make progress for situations in which ϕ varies in space, we must be able to write equation (3.1) as a partial differential equation. This requires the derivatives of ϕ to exist within V. That is, our field ϕ must be nearly constant within within smaller, subdivided regions of V and vary smoothly from little region to little region. It is important to recognize that there is still some lower limit at which our assumption of a locally continuous field breaks down (recall the example of rice pouring from a bag). That limit varies from system to system and among quantities within a given system. When we discuss the variation of ϕ in space, we are really talking about the average ϕ's in small regions of space. We can make no assertions about anything that happens on a smaller scale and we assume that no smaller scale processes are important to the problem we wish to address. (The assumption about "sub-grid" processes is sometimes a poor one, in which case we might attempt to parameterize them, as we did for radiative transfer in the energy balance model.)

With the assumption of a locally continuous field, we can rewrite equation (3.1) as a (local) partial differential equation. We begin by applying the (very handy) divergence theorem which, for a vector-valued quantity, states:

$$\int_S \mathbf{F}\cdot n\ \mathrm{d}S \;=\; \int_V \nabla\cdot\mathbf{F}\ \mathrm{d}V \tag{3.2}$$

Using index notation, equation (3.2) is written:

$$\int_S \mathbf{F}_j \; n_j \; \mathrm{d}S \;=\; \int_V \frac{\partial \mathbf{F}_j}{\partial x_j} \; \mathrm{d}V \tag{3.3}$$

Using equation (3.2), the surface integrals in equation (3.1) may be replaced:

$$-\int_S \mathbf{F}\cdot \mathrm{d}S \;-\; \int_S \phi \mathbf{V}\cdot \mathrm{d}S \;=\; -\int_V \nabla\cdot(\mathbf{F} \;+\; \phi\;\mathbf{V})\,\mathrm{d}V \tag{3.4}$$

In our Eulerian reference frame it must be the case that

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_V \phi \;\mathrm{d}V \;=\; \int_V \frac{\partial \phi}{\partial t} \;\mathrm{d}V \tag{3.5}$$

Substituting (3.4) and (3.5) into (3.1), we have

$$\int_V \left\{ \frac{\partial \phi}{\partial t} \;+\; \nabla\cdot(\mathbf{F} \;+\; \phi \mathbf{V}) \;-\; H \right\} \mathrm{d}V \;=\; 0 \tag{3.6}$$

Because V is an arbitrary volume, equation (3.6) can only be true if the term in brackets is zero for our little subdivided volumes (note that this "*zero everywhere*" does not necessarily apply at scales smaller than the limiting resolution discussed earlier). At scales we care about, we can write:

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} \;+\; \nabla\cdot(\mathbf{F} \;+\; \phi \mathbf{V}) \;-\; H \;=\; 0 \tag{3.7}$$

This is the general form for all conservation laws in continuum mechanics.

3.3 mathematical model for the flow of glacier ice

3.3.1 conservation of momentum

Starting from Newton's second law of motion, conservation of momentum is

$$\frac{d}{dt}\int_V \rho u_i \; dV \;=\; \int_V \frac{\mathrm{d}\sigma_{ij}}{\mathrm{d}x_j} \; dV + \int_V \rho g_i \; dV \tag{3.8}$$

where t represents time, ρ represents density, u represents velocity, σ_{ij} represents the stress tensor, g represents the acceleration due to gravity, V represents the volume of an arbitrary fluid element, and $(i, j) = \{x, y, z\}$ in a cartesian coordinate system. Equation 3.8 tells us that a fluid element of arbitrary size experiences a "body force" $\rho g_i \delta V$ due to gravity and a force $\frac{\mathrm{d}\sigma_{ij}}{\mathrm{d}x_j}\delta V$ due to the surrounding fluid.

Making the assumptions that we have continuous fields and that ice is incompressible (that is, its density ρ does not change under conditions of interest to us), we can write

$$\rho \frac{Du_i}{Dt} \;=\; \frac{\mathrm{d}\sigma_{ij}}{\mathrm{d}x_j} + \rho g_i \tag{3.9}$$

in which D is a material derivative. The statement of conservation may be further simplified by recognizing that the ice deformation is a viscous flow at low Reynolds number (no vorticity)

$$\frac{\mathrm{d}\sigma_{ij}}{\mathrm{d}x_j} + \rho g_i \;=\; 0 \tag{3.10}$$

We are left with the very simple statement that the gravitational driving force is balanced by forces resulting from the stresses σ_{ij}.

The stress tensor σ_{ij} has nine components in our three dimensional cartesian coordinate system

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix}$$

The components along the diagonal are called normal stresses and the off-diagonal components are called shear stresses. Deformation results not from the full stress but from the deviatoric stress

$$\tau_{ij} \;=\; \sigma_{ij} \;-\; \frac{1}{3}\sigma_{kk}\delta_{ij} \tag{3.11}$$

in which δ_{ij} is the Kroneker delta.

3.3.2 conservation of mass

Making the assumption that there are neither sources nor sinks for mass within our glacier or ice sheet, conservation of mass in some arbitrary region dV is stated

$$\int_V \frac{d\rho}{dt}\, dV \;+\; \int_V \nabla \cdot \rho u_i dV \;=\; 0 \tag{3.12}$$

If we again make the assumption of incompressibility, the equation for local mass continuity is

$$\nabla \cdot u_i \;=\; 0 \tag{3.13}$$

Writing out all of the terms in our cartesian coordinate system

$$\frac{\mathrm{d}u_x}{\mathrm{d}x} \;+\; \frac{\mathrm{d}u_y}{\mathrm{d}y} \;+\; \frac{\mathrm{d}u_z}{\mathrm{d}z} \;=\; 0 \tag{3.14}$$

To make use of this statement, we need to integrate

$$\int_b^s \left(\frac{\mathrm{d}u_x}{\mathrm{d}x} + \frac{\mathrm{d}u_y}{\mathrm{d}y} + \frac{\mathrm{d}u_z}{\mathrm{d}z} \right) dz = 0 \tag{3.15}$$

from the base b to the upper surface s of the ice mass. The integral of $\frac{\mathrm{d}w}{\mathrm{d}z}$ is simply the difference between the vertical component of the velocity at the upper and lower surfaces, so

$$u_z(s) - u_z(b) = -\int_b^s \frac{\mathrm{d}u_x}{\mathrm{d}x} dz - \int_b^s \frac{\mathrm{d}u_y}{\mathrm{d}y} dz \tag{3.16}$$

Changing the order of integration using Leibnitz rule

$$\begin{aligned} u_z(s) - u_z(b) = & - \frac{\mathrm{d}}{\mathrm{d}x} \int_b^s u_x dz + u_x(s) \frac{\mathrm{d}s}{\mathrm{d}x} - u_x(b) \frac{\mathrm{d}b}{\mathrm{d}x} \\ & - \frac{\mathrm{d}}{\mathrm{d}y} \int_b^s u_y dz + u_y(s) \frac{\mathrm{d}s}{\mathrm{d}y} - u_y(b) \frac{\mathrm{d}b}{\mathrm{d}x} \end{aligned} \tag{3.17}$$

The vertical velocity at the upper surface is the result of motion down the surface slope, the rate of new accumulation $\dot{a}$ and any time-change in surface height

$$u_z(s) = \frac{\mathrm{d}s}{\mathrm{d}t} + u_x(s) \frac{\mathrm{d}s}{\mathrm{d}x} + u_y(s) \frac{\mathrm{d}s}{\mathrm{d}y} - \dot{a} \tag{3.18}$$

recognizing that a negative accumulation rate indicates ablation. Similarly, the vertical velocity at the lower surface is

$$u_z(b) = \frac{\mathrm{d}b}{\mathrm{d}t} + u_x(b) \frac{\mathrm{d}b}{\mathrm{d}x} + u_y(b) \frac{\mathrm{d}b}{\mathrm{d}y} - \dot{b} \tag{3.19}$$

in which $\dot{b}$ represents the basal accumulation rate.

Substituting equations 3.18 and 3.19 into equation 3.17, we find that many terms cancel

$$\frac{\mathrm{d}s}{\mathrm{d}t} - \dot{a} - \frac{\mathrm{d}b}{\mathrm{d}t} + \dot{b} = - \frac{\mathrm{d}}{\mathrm{d}x} \int_b^s u_x dz - \frac{\mathrm{d}}{\mathrm{d}y} \int_b^s u_y dz \tag{3.20}$$

Finally, making the simplification $H = s - b$ we have

$$\frac{\mathrm{d}H}{\mathrm{d}t} = - \frac{\mathrm{d}}{\mathrm{d}x} \int_b^s u_x dz - \frac{\mathrm{d}}{\mathrm{d}y} \int_b^s u_y dz + \dot{a} - \dot{b} \tag{3.21}$$

The vertically-integrated form

$$\frac{\mathrm{d}H}{\mathrm{d}t} = - \nabla \cdot (U_i H) + \dot{a} - \dot{b} \tag{3.22}$$

in which U_i represents the integral from b to s is often useful. This equation is prognostic. We use the current geometry of the ice to compute a future time-change in that geometry.

3.3.3 conservation of energy

The first law of thermodynamics is used to make a basic statement of conservation of energy in a volume of ice V within a surface S is

$$\frac{d}{dt}\int_V E\ dV \;=\; -\int_S F_i\cdot\hat{n}\ dS \;-\; \int_S Eu_i\cdot\hat{n}\ dS \;+\; \int_V W dV \tag{3.23}$$

in which E represents the energy of the volume, F_i is a flux due to diffusion, and W represents any sources or sinks of energy within the volume. The term Eu_i is a flux through S due to advection. Following the steps laid out earlier, we use the divergence theorem and the assumptions of continuous fields and incompressibility, such that

$$\frac{dE}{dt} \;+\; \nabla\cdot(F_i + Eu_i) \;-\; W \;=\; 0 \tag{3.24}$$

Our goal is to use the first law of thermodynamics in order to compute the temperature of the ice and any change it may undergo over time.

The energy E is the product of density and the specific internal energy of the ice e, which is itself the product of the specific heat capacity c_p and temperature T because there is no transfer between internal energy and pressure for an incompressible fluid. Thus,

$$\begin{aligned}\frac{dE}{dt} &= \frac{d\,(\rho e)}{dt}\\ &= \rho\frac{de}{dt} \;+\; e\frac{d\rho}{dt}\\ &= \rho c_p\frac{dT}{dt}\end{aligned}$$

The flux due to diffusion follows Fourier's "law" for heat conduction so

$$\begin{aligned}\nabla\cdot F_i &= \nabla\cdot(-k\ \nabla T)\\ &= -k\ \nabla^2 T\end{aligned}$$

in which k represents the thermal diffusivity of ice and we assume gradients in its magnitude to be negligible.

Using progress made above and assuming that $\nabla\cdot u_i$ is small with respect to other terms, we can write the advection term

$$\nabla\cdot(Eu_i) \;=\; \rho c_p\ u_i\cdot\nabla T$$

Two quantities must be considered as energy sources, the work done on the system by internal deformation and the latent heat associated with phase changes. The former is the product of strain rate and the deviatoric stress $\dot{\epsilon}_{ij}\tau_{ij}$. The latter is the product of the latent heat of fusion and the amount of material subject to melting (freezing) per unit volume per unit time, $L_f M_f$.

At last, we are able to write equation 3.24 in terms of temperature

$$\frac{\mathrm{d}T}{\mathrm{d}t} = \frac{k}{\rho c_p}\nabla^2 T - u_i \cdot \nabla T + \frac{1}{\rho c_p}\dot{\epsilon}_{ij}\tau_{ij} + \frac{1}{\rho c_p} L_f M_f \tag{3.25}$$

It is often the case that horizontal terms $\frac{\mathrm{d}^2T}{\mathrm{d}x^2}$ and $\frac{\mathrm{d}^2T}{\mathrm{d}y^2}$ are small enough to be ignored.

3.3.4 constitutive relationship

Strain rates $\dot{\epsilon}_{ij}$ are related to the stress tensor τ_{ij} by the generalized Glen flow law

$$\dot{\epsilon}_{ij} = A(T^*)\tau_e^{n-1}\tau_{ij} \tag{3.26}$$

in which T^* is the absolute temperature corrected for the pressure dependence of the melt temperature, τ_e is the second invariant of the stress tensor and the exponent n is 3. The rate factor A follows the Arrhenius relationship

$$A\left(T^*\right) = EA_o e^{-Q/RT^*} \tag{3.27}$$

in which A_o is a constant, Q represents the activation energy for crystal creep, R is the gas constant, and E is a tuning parameter used to account for the effects of impurities and anisotropic ice fabrics. The homologous temperature is

$$T^* = T + \rho g H \Phi \tag{3.28}$$

in which Φ is 9.8 $\times 10^{-8}$ K Pa^{-1}, about 8.7 $\times 10^{-4}$ K m^{-1}. The pressure-dependent melt temperature is simply the triple point temperature less the product $\rho g H \Phi$.

3.4 numerical model

3.4.1 GLIMMER

The ice sheet model GLIMMER uses a finite difference method to solve the governing thermodynamic equations for ice using the *shallow ice approximation.* This is the approach generally adopted for modeling large ice masses. The assumption is made that slopes at the upper and lower surfaces are sufficiently small that normal stress components can be neglected. This leads to a *local* balance between the gravitational driving stress and the basal shear stress and expressions for the shear stresses

$$\begin{aligned}
\tau_{xz}(z) &= -\rho g\,(s-z)\,\frac{\mathrm{d}s}{\mathrm{d}x} \\
\tau_{yz}(z) &= -\rho g\,(s-z)\,\frac{\mathrm{d}s}{\mathrm{d}y}
\end{aligned} \tag{3.29}$$

Evolution of the ice thickness uses equation 3.22 and the temperature solver uses a version of equation 3.25 simplified to neglect horizontal diffusion (a typical simplification).

The model equations are solved on a regular grid using the Glen flow law (equation 3.26) and appropriate boundary conditions for the upper and lower surfaces. These include the surface ice accumulation rate and temperature and a geothermal gradient (applied at the base of a bedrock layer with specified thermal properties). Basal traction may also be specified, in the situation where ice is at the melt temperature at the base. Isostatic adjustment of the land surface beneath the ice sheet, not discussed here, is also included.

3.4.2 numerical scheme

The continuous functions represented by the model governing equations cannot be solved exactly. Instead, they are discretized so that finite approximations of their solutions may be made. There are a variety of numerical techniques available for this purpose, GLIMMER makes use of a finite difference method.

In brief, the model domain (a region of Earth's surface, for example, Greenland) is subdivided into a regularly-spaced horizontal grid and derivatives are approximated along the grid directions. The grid is fixed in space over the course of the model run. Model variables such as ice thickness are updated at each time step according to the numerical approximations of the governing equations. The vertical dimension is treated using a non-dimensional "stretch" coordinate so that an evolving ice thickness may be accommodated. The scaling is:

$$\zeta = \frac{s - z}{H} \tag{3.30}$$

so that $\zeta = 1$ at the surface s and $\zeta = 0$ at the base. The governing equations must be re-written in the new, (x, y, ζ) coordinate system.

If you would like to read more about the inner workings of GLIMMER, its documentation is available at the class website. This is not necessary for the present lab exercise.

3.4.3 ISIS

The Interactive System for Ice Sheet Modeling (ISIS) is a user interface to GLIMMER. It is still in development, as part of an *International Polar Year* collaboration among groups at the University of Montana, Portland State, UC Santa Cruz, Auburn, and the University of Texas at El Paso. We will use ISIS as a means to set up and run experiments involving the Greenland Ice Sheet. Most of our analysis of model output will be done using MATLAB scripts.

ISIS installs with a number of pre-defined model domains and experiments. We will use a few of the Greenland setups. The Greenland initialization, used to generate a "modern" steady state, from which experiments may be started, requires about two hours of run time. To save time, the results of that simulation, stored in netCDF-format files titled gland−ClimateEvo.2ka.nc, gland−ClimateEvo.hot2.nc, and gland−ClimateEvo.1ka.nc, have been prepared for you and will be distributed in class. You should store them in the ISIS UserOutput folder.

The basic procedure for initiating a pre-defined model run are outlined here. You are encouraged to dig more deeply into the ISIS interface.

1. start ISIS: you will see a File pull-down menu, a Help pull-down menu and four tabbed pages. The tabbed pages are Configuration, Excecution, Visualization, and Analysis. If you click on the Configuration tab you will see a nested list of model parameters that may be set by the user.

2. Choose **Select Scenario** in the **File** pull-down menu. This opens a new interface window with a list of pre-defined scenarios. Open the Greenland scenario list. The first option, Greenland Climate Evolution, has already been run for you. Choose the third setup in the list, **Greenland 500 year climate warming.**

3. Choose **save as** in the **File** pull-down menu and save the configuration file in the **UserConfig** directory. *This step is important. If you don't do this, you will overwrite the ISIS configuration file for this simulation when you run the model.*

4. Click the **Configuration** tab again. Open **CF output** from the list of options and save each of the three output files for this model run to the **UserOutput** directory. You will need to do this each time you set up a new model experiment.

5. Choose the **Excecution** tab and push the run button. You will see text from the runtime log file appear in the window at the right.

6. **Visualization** provides some simple tools for inspecting the model output.

3.4.4 Matlab scripts for data visualization: steps for you to follow

A group of Matlab scripts are available at the class website. Using these tools, with a few simple modifications, will allow you to answer all of the questions associated with this lab. You are welcome to modify and expand the scripts.

ISIS stores data at selected time steps as the model runs. With the default settings, ISIS produces output files with two temporal resolutions, 20 years and 100 years. The output file names indicate the temporal resolution, so that gland−500−1.100.nc is a 500 year long run with saved data at every 100th year.

3.4.4.1 reading the output data

ISIS generates netCDF output data files. NetCDF (Network Common Data Form) is a machine-independent, self-describing, binary data format that is a widely-used standard for exchanging scientific data. You will read these files into the Matlab workspace using two scripts, netcdf.m and prepare_output.m, provided at the class website. prepare_output.m uses the first to read a netCDF file and pull out variables to be used in the later analysis. The values are stored in a data structure, a compact but somewhat obscure framework. You can read more about data structures in the MATLAB help. You will need to make sure that two data file names are correct in prepare_output.m: the model output file, with a .nc extension, and a MATLAB native format file, with a .mat extension. The latter file will contain the set of variables to be used by the next script. The lines you will need to modify are

```
test=netcdf('gland-500-1.100.nc');
outname='gland-500-1-100.mat';
```

Save and run the script. This is the list of variables stored in the MATLAB file generated for gland−500−1.100.nc:

name	dimension	description	units
N	1x1	saved time steps	
T	6x11x141x83	ice temperature	°C
b	6x141x83	bed elevation	m
bdot	6x141x83	basal melt rate	m a^{-1}
h	6x141x83	ice thickness	m
ux	6x11x141x83	velocity, x-component	m a^{-1}
uy	6x11x141x83	velocity, y-component	m a^{-1}
uz	6x11x141x83	velocity, z-component	m a^{-1}
x1	83x1	x grid node locations	m
y1	141x1	y grid node locations	m
zeta	11x1	vertical coordinate	

As you can see, the output variables are multi-dimensional, including both space and time. Some variables, such as the thickness h have three dimensions: time and two horizontal dimensions while others, such as the temperature T have four: time, vertical coordinate, and two horizontal coordinates. This 500 year run with data saved every 100 years yields 6 saved times, the initial time and 5 steps during the model run. The horizontal coordinates correspond to the horizontal grid that defines the model domain. The vertical coordinate is in the ζ system, which itself is described in the variable zeta.

3.4.4.2 a simple map

Next, make a simple color map of ice thickness at the start of the model run by typing the following commands at the prompt in the MATLAB command window. You may notice that the array h requires some special handling before we can render the thickness field. The MATLAB function squeeze is used to convert the an array with three dimensions, one a sigleton, into an array with only two dimensions that can be handled by the **pcolor** routine. The MATLAB function double is used to convert the data type from single to double, also a requirement for rendering the color map. The data type single requires less storage space than does the type double. Here are the lines you need to type:

```
>> clear
>> load gland-500-1-100.mat
>> figure(1)
>> pcolor(double(squeeze(h(1,:,:)))), axis equal
>> xlabel('column number')
>> ylabel('row number')
>> colorbar
```

The black lines in the figure you just created show you the resolution of the numerical model. To eliminate them, type:

```
>> shading flat
```

Here's another set of commands to plot the surface temperature at the start of a model run:

```
>> clear
>> load gland-500-1-100.mat
>> figure(2)
>> pcolor(double(squeeze(T(1,1,:,:)))), axis equal
>> xlabel('column number')
>> ylabel('row number')
>> colorbar
```

The plotting scripts provided for you at the class website select a particular latitude along which to plot several sections through the ice sheet. You may wish to investigate other sections. If so, the map in the figure you just created can be used to find new row and column numbers for a different section.

3.5 Questions

Select a Greenland future warming scenario in ISIS and run the ice sheet model. When the model run is complete, extract output data from the resulting netCDF file using prepare_output.m, as described in section 3.4.4.

1. Run the scripts plot_slices.m and plot_vertprof.m with the output from the model initialization. To do this you will need to set that data file gland−ClimateEvo−2ka.mat as the file to load in line 6 of the script. Change the plotting time step number n to 126 at line 26 of plot_slices.m.
   (a) Describe and explain the differences in the horizontal speed profiles at sites near the center of the ice sheet and near the ice sheet margin.
   (b) Describe and explain the differences in the temperature profiles at sites near the center of the ice sheet and near the ice sheet margin.
   (c) What's up with the temperature of the ice near the base of the ice sheet ice at the margin site?

2. Run the complete suite of plotting routines with the output from your climate warming model. You should run the script plot_slices .m first, followed by the scripts plot_vertprof .m and plot_change.m. Be sure to set the file name for the file you wish to load and the time step number to plot in the script plot_slices .m. Please indicate at the start of your answers which ISIS scenario you chose.
   (a) How does ice speed change over the course of the simulation along the two sections across the ice sheet? What changes in the ice are responsible for the change you observe?
   (b) What physical processes are responsible for the thick core of cold ice in the interior of the ice sheet at the end of the 500 year warming run?
   (c) Suppose you continued running forward in time from here with no further change in the surface temperature. Describe what would happen to the temperature profiles at the center and margin sites.
3. Why is the ice divide not in the center of the ice sheet?
4. **graduate students** Keeping the warming scenario the same, how do the temperature and velocity fields change when the basal traction is reduced (in essence, the basal traction parameter in the model scales the effectiveness of basal water at lubricating the bed and facilitating fast flow)? Explain the physical processes responsible for the changes you document.