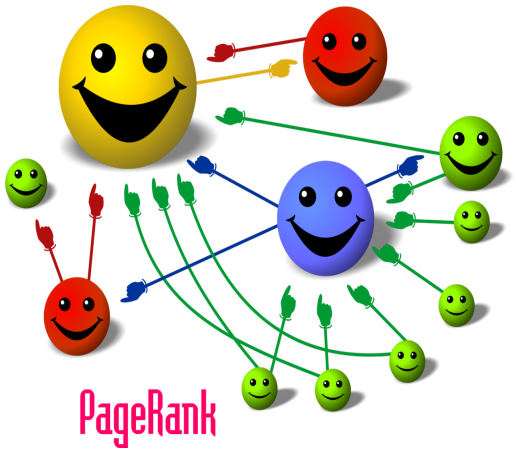
MTH 610: DIRECTED NETWORKS

Instructor: J. J. P. Veerman

In this course we familiarize the student with the principal notions of the algebraic theory of directed networks together with some applications (such as diffusion, consensus, flocking, network flows, chemical reaction networks, social networks, and so on). The course deals with theoretical aspects of directed networks and subsequently delves into some of the common applications of this theory. Here are the topics.
1. (20%) Elementary notions of (directed) graph theory. Connectedness notions, structure of weakly connected digraphs. Definitions and properties of different adjacency and Laplacian matrices, and in particular their spectrum.
2. (40%) We cover the most important basic theorems concerning the algebraic theory of directed graphs. These are: Perron-Frobenius, Jordan Normal Form, Cauchy-Binet, Jacobi's Formula, and the Matrix Tree theorem. All include proofs, except the Jordan normal form.
3. (40%) Using this theory, we look at important examples of dynamical processes on directed networks such as diffusion, consensus, flocking, Ford-Fulkerson, pagerank, contagion, chemical reaction networks, and Markov chains.



Eventually, this course will appear in book form. But for now, all material has been recorded in the form of freely available lecture notes and published papers. Here are the references:
* `http://web.pdx.edu/~veerman/2019-Digraphs1.pdf`
* `http://web.pdx.edu/~veerman/2019-Digraphs2.pdf`
* `http://web.pdx.edu/~veerman/2019-Digraphs-3.pdf`
* `http://web.pdx.edu/~veerman/2019-Digraphs-4.pdf`.
* J. J. P. Veerman, E. Kummel, *Diffusion and Consensus on Weakly Connected Directed Graphs*, **Linear Algebra and Its Applications** 578, 184–206, 2019.
* J. J. P. Veerman, R. Lyons, *A Primer on Laplacian Dynamics in Directed Graphs*, **Nonlinear Phenomena in Complex Systems** No. 2, Vol. 23, 2020. (Proceedings of the 6th Ph.D. School/Conference on Mathematical Modeling of Complex Systems, Pescara (IT), 2019.)
* J. J. P. Veerman, T. Whalen-Wagner, E. Kummel, *Chemical Reaction Networks*, **Submitted**.
* J. J. P. Veerman, *Social Balance and the Bernoulli Equation*, **The American Mathematical Monthly**, Vol 125, Issue 8, 724-732, 2018.

Expected Preparation: MTH 412/512 or MTH 462/562 or MTH 422/522 or consent of the instructor.