

Directional Dependence in Multivariate Distributions

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As Kumar Jogdeo notes in his 1982 article “Concepts of dependence” in the *Encyclopedia of Statistical Sciences*, “Dependence relations between random variables is one of the most studied subjects in probability and statistics. The nature of the dependence can take a variety of forms, and unless some specific assumptions are made about the dependence, no meaningful statistical model can be contemplated.”

One of the simplest ways of measuring dependence is via a measure of association. Scale-invariant or distribution-free measures of association—such as the population versions of Spearman’s rho and Kendall’s tau—are well understood in the bivariate case, but much less well known in the multivariate case. In both the bivariate and multivariate cases, such measures are functions of the *copula* of the random variables, the function that joins or couples a multivariate distribution function to its one-dimensional marginal distribution functions.

Several multivariate extensions of Spearman’s rho have been proposed. But they can fail to detect dependence in some multivariate distributions. In this talk we will propose several “coefficients of dependence” to remedy this situation (in the trivariate case) and study their properties, and also examine similar results for Kendall’s tau and Blomqvist’s beta.